INTEGRATED PRODUCTION AND DISTRIBUTION PLANNING IN MULTI-SITE MANUFACTURING SCENARIO

Ph.D. Thesis

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April 2019

Respectfully dedicated to

"My Family"

&

"My Teachers"

DECLARATION

I, **Gaurav Kumar Badhotiya**, declare that this thesis entitled "**INTEGRATED PRODUCTION AND DISTRIBUTION PLANNING IN MULTI-SITE MANUFACTURING SCENARIO**" and the work presented in it are my own.

I confirm that:

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CERTIFICATE

This is to certify that the thesis entitled "**INTEGRATED PRODUCTION AND DISTRIBUTION PLANNING IN MULTI-SITE MANUFACTURING SCENARIO**" being submitted by **Gaurav Kumar Badhotiya (2014RME9523)** is a bonafide research work carried out under my supervision and guidance in fulfillment of the requirement for the award of the degree of **Doctor of Philosophy** in the Department of Mechanical Engineering, Malaviya National Institute of Technology Jaipur, India. The matter embodied in this thesis is original and has not been submitted to any other University or Institute for the award of any other degree.

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ABSTRACT

In today's competitive globalised market, it is imperative for enterprises to improve their efficiency and provide better service to customers. To do so, organisations are opting for producing at multiple manufacturing sites situated at geographically distinct locations. The generation of effective production and distribution planning is necessary for success of this multi-site network. Production planning is concerned with quantity of production of different products and aims to find a cost effective plan, while distribution planning is related with computing transportation quantity so as to fulfil the demand of customers. Most of the literature focuses on solving these two planning problems independently called as two phase approach, which leads to suboptimal solutions. The purpose of this research work is to deal with the complex scenario and aspects of integrated production and distribution planning in two echelon supply chain network in which demand of multiple selling locations is served by multiple manufacturers. The study covers integrated production and distribution planning problem considering single as well as multiple objectives, optimize in deterministic as well as under uncertain environment. The problems are formulated using mathematical programming, especially mixed-integer linear programming (MILP) technique, and to solve these mathematical models, several solution approaches are implemented.

Initially, a mixed integer linear programming (MILP) model is formulated for an integrated production and distribution planning problem in a multi-product, multi-period and multi-site manufacturing environment. Three important aspects of production and distribution planning; set up cost/time for different products at manufacturing sites, capacities of the heterogeneous transport vehicles and backordering for unfulfilled demand are considered in an integrated manner to represent the real life scenario. An illustrative example inspired from an Indian automobile manufacturing company is taken and analytical results are presented to indicate the performance of the proposed model. The problem is solved by branch and cut approach using CPLEX solver of IBM CPLEX 12.7 software. The

outcome of the proposed mathematical model determines optimal quantities of production, inventory and trasnportation.

Real life problems of multi-site integrated production and distribution planning is having large number of variables and this complexity makes it difficult to be solved by computational solver in reasonable time due to limitation of memory. Consequently, Lagrangian relaxation based heuristics approaches are implemented, which are able to generate feasible solutions in less computational time. The hard constraints are relaxed and added into objective function, resulting in an easy to solve subproblem. This relaxation makes the original problem infeasible. To maintain the feasibility; lagrangian heuristics algorithms are proposed. The performance of lagrangian relaxation approach is demonstrated using various random data based problem instances and compared solution of them with commercial solver results.

For a successful production plan, there exists more than one criterion. The proposed mathematical model is extended to include multiple criterions or objectives. Three conflicting objectives that need to be minimized are total cost, delivery time, and backorder level. A variant of Goal programming method, known as Preemptive goal programming is used to solve the proposed multi-objective mathematical model. Input parameter data of automobile manufacturing company is considered and further analysis is also conducted to visualize effect of changing priority level on objective functions and deviation variables.

The practical integrated production and distribution planning problem in supply chain has multiple conflicting objectives which are often fuzzy or uncertain due to unavailability or improper information. Therefore, it is necessary to consider this variability while modeling production and distribution plans. A fuzzy multi-objective mixed integer linear programming model is formulated for a multi-product, multi-period and multi-site manufacturing environment. The fuzzy objectives are represented by piecewise linear membership function. Sensitivity analysis on objective function values is conducted and important findings are drawn from analytical result of the study.

The fuzzy approach only considers truth side of a problem but do not consider false side and aspect of indeterminacy, which occurs due to unexpected parameters hidden in some propositions. To consider all the three aspects, concept of Neutrosophy is implemented. Neutrosophic sets have been introduced as a generalization of fuzzy and intuitionistic fuzzy sets to represent imprecise, vague and incomplete information about a problem. Mathematical model for multi-objective multi-site integrated production and distribution planning problem is formulated in neutrosophic environment. Each objective function of the proposed mathematical model is represented by membership functions of neutrosophic environment i.e. truth, indeterminacy and falsity. A neutrosophic model is constructed using these membership functions to find the best compromised solution. The outcome of the neutrosophic model is compared with Intuitionistic fuzzy programming approach which shows superiority of the results obtained through the neutrosophic approach.

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LIST OF ABBREVIATIONS

SCM	Supply Chain Management
PD	Production and Distribution
HPP	Hierarchical production planning
IPDP	Integrated production and distribution planning
MSPDP	Multi-site production and distribution planning
MSIPDP	Multi-site integrated production and distribution planning
MO-MSIPDP	Multi-objective multi-site integrated production and distribution planning
FMO-MSIPDP	Fuzzzy multi-objective multi-site integrated production and distribution planning
DM	decision maker
CLSP	Capacitated lot sizing problem
LP	Linear programming
MIP	Mixed integer programming
MILP	Mixed integer linear programming
LR	Lagrangian Relaxation
PGP	Preemptive goal programming
FGP	Fuzzy goal programming
IFGP	Intuitionistic fuzzy goal programming
IFO	Intuitionistic fuzzy optimization
NS	Neutrosophic set

Chapter 1

Introduction

1.1 Problem contextualization

Organizations nowadays are configured into network of manufacturing and distribution facilities called as supply chain. Supply chain can be defined as group of entities that interact with each other to procure and transform raw material into finished goods and distribute them to the end customer (Chopra and Meindl, 2004). There are three levels of activities in supply chain depending upon the time horizon and decisions; strategic, tactical and operational (Ballou, 1992). Strategic level decisions are related with facility location design, supply chain network design, production technologies and capacities of plants and made over a time horizon of more than one year. Once the location of facilities is determined, the focus shifts to tactical level decisions, which made over a span of few weeks to a year and are related with assembly policy, lot sizing, production, inventory and distribution; ensuring effective utilization of resources. At the operational level, decision related with operations sequence and schedules are made over a short time horizon of a day to a week.

Successful supply chain requires effective coordination among various members and functions of the supply chain. Performance of the supply chain can be improved through effective integration of production and distribution (PD) functions (Vercellis, 1999). Traditionally, these planning decisions were made sequentially, leading to an infeasible or sub-optimal plan in terms of inventory and capacities (Torabi and Hassini, 2009). To overcome this drawback, organizations are working towards coordinating the PD planning decisions in an integrated framework.

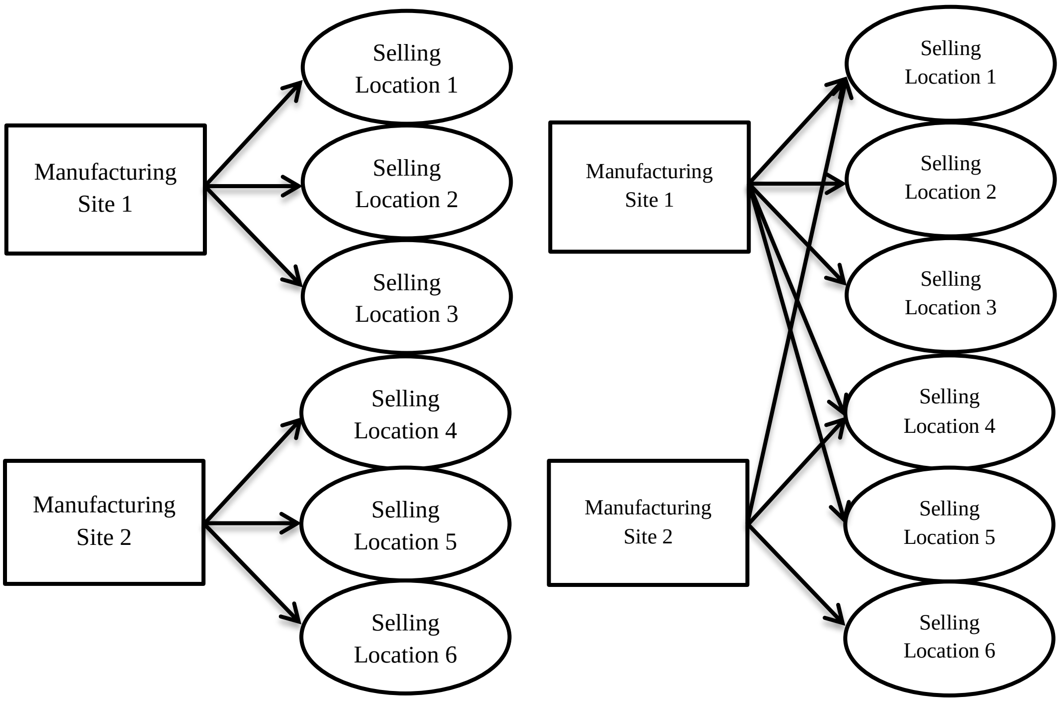
In this research, the interaction between production and distribution decisions of two echelons of supply chain; manufacturer and selling location are analysed. Integration of these two functions in multi-site manufacturing scenario is demonstrated through mathematical modelling and illustrated through real life industrial example problem. The production and distribution plans are generated for single as well as multi-criteria deterministic and uncertain scenarios.

This chapter presents overview of the thesis. First section discusses about background of the research work and two echelon supply chain scenario. After that, the motivation for conducting this research work is presented in terms of research gaps analysed from the critical examination of literature. To fulfil these research gaps, research objectives are formulated. Next section demonstarte about the research methodology employed in this thesis. The chapter ends with explaination of structure of this thesis.

1.1.1 Multi-site manufacturing

In this current trend of globalization, there is a competitive environment for organizations to capture the market by quickly responding to customer needs. This globalisation of market has motivated organizations to move close to the customer by opting for producing at multiple manufacturing sites situated at geographically distinct locations to save cost/time of distribution, focus on few product categories, achieve better quality and to provide better service to customer (Chan et al., 2005). The goal is to deliver the right product to the right place at the right time.

In multi-site manufacturing environment, two scenarios can be possible i.e. series and parallel (Kanyalkar and Adil, 2005). A serial setup produces intermediate products which are then assembled into final products by other manufacturing sites. The manufacturing sites in this setup are non-identical and assembling sites are located close to customer (Guinet, 2001). In parallel setup, each manufacturing site is producing final product and distributing them to customer, as shown in Figure 1.1. There are two ways to deal with the parallel setup, one in which each manufacturing site is allocated with fixed number of selling locations or customers called as fixed channel matrix (Figure 1.1 a). Another way is dynamic allocation of selling locations to manufacturing sites (Figure 1.1 b).



(a) Fixed channel matrix layout (b) Dynamic allocation layout

Figure 1.1: Layouts of Manufacturing site and selling location

Production planning is easy in the fixed channel, but isolating manufacturing sites from each other is not helpful when customer demand reaches beyond each plants fixed production capacity, consequently there is loss of customer due to backordering and lost sales. In dynamic allocation, shifting of excess requirement to other manufacturing site is possible, which leads to effective utilization of production capacity of all sites. In this study, parallel manufacturing sites with dynamic allocation scenario is considered.

1.1.2 Hierarchal and Integrated planning

Hierarchical production planning uses separate mathematical models for every level. It decomposes a complex problem into smaller and easier subproblems and solves them independently. The solution obtained from one level is imposed to next level in decision hierarchy. According to Bitran and Tirupati (1993) "hierarchical planning represents a philosophy to address complex problems, rather than a specific solution technique."

There are some advantages of hierarchical planning. It takes less time to collect data and getting the solution for even a large size industrial problem. Beside these advantages, there are some major disadvantages. The procedure decomposes the original problem into subproblems. Results obtained from the first subproblem are used to optimise second subproblem, which leads to sub-optimal solution. In addition to that, sometimes the solutions obtained are infeasible.

Integration and coordination of decisions related with production, inventory and distribution, yield economic as well as competitive advantages (Darvish and Coelho, 2016). Over the past decades, much research both in industry and academia have focused on increasing flexibility, reducing lead time and minimising total system costs simultaneously through integration of different functions in supply chain. Integrated production and distribution planning (IPDP) is one among them and is the key to success of any industry. The decision variables of different functions are dealt simultaneously in the integrated approach. Traditionally, manufacturing organisations manage production and distribution functions sequentially and independently leading to sub-optimal planning (Zegordi and Nia, 2009). However, in today's competitive global market the interaction between these two functions is a crucial step towards systematic and synchronous production and distribution planning. Several articles in the literature have shown the merits and demerits of integrated approach over hierarchal approach (Park, 2005; Torabi and Hassini, 2009; Darvish and Coelho, 2016, Darvish et al, 2016). A real world example of this integration is IBM, who has developed an integrated plan of PD functions and could gain 2–4% increase in resource utilisation, 15% increase in on-time delivery of products, and 25–30% decrease in inventory (Degbotse et al. 2013).

1.1.3 Multi-site integrated production and distribution planning

To achieve the advantages of multi-site manufacturing i.e. maximise service to the customer and minimise total system cost, a coordination of production and distribution planning is important. Multi-site integrated production and distribution planning (MSIPDP) involves decisions related to determining production and inventory level at different facilities and quantity to be transported between facilities to fulfil the demand of customer. The problem can be viewed as an optimization model that integrates production, inventory and distribution decisions. It is difficult to solve an IPDP problem to optimality due to its combinatorial nature (Lei et al. 2006).

In a multi-site manufacturing scenario, proper allocation of demand from various customers to the manufacturing sites is important. In this thesis, MSIPDP problem is considered incorporating production cost, inventory holding cost at manufacturing site as well as selling location, backorder cost, setup cost and transportation cost, so as to minimize total system cost for an automotive manufacturing industry in India.

1.2 Motivation for research

The key factor for success of a manufacturing orgainization to be competitive in present globalized market is to fulfill customer demand on time with desired product quality. This is a challenging mission because of short product life cycle, high variety of products, short customer lead time and uncertain customer demand. Gaining competitive advantage requires smooth communication and effective coordination between various departments to ensure effective utilization of resources. IPDP functions helps in achieving this challenging mission. To achieve this target, it is required that the PD decisions should be made simultaneously to balance production, setup, inventory carrying and transportation costs in supply chain. Most of the models presented in the literature have treated subsystems of PD network separately, or attempted to coordinate only parts of the whole network; however, their integration can have a significant impact on the overall system.

The integration of production and distribution planning though has advantages over hierarchal planning; it makes the problem computationally complex. Several studies in the literature have dealt with integrated problem and methods used so far have found good quality solutions by oversimplifying or considering too many assumptions. Most of the studies in the literature have investigated simplified problem such as consideration of single item, single period, without setup and capacities, homogeneous transportation etc. in their mathematical model formulation, in order to solve them for large size problem instances. These studies though helpful in understanding the structure and complexity of problem; they present limited real life applications. Implementation of heuristics and metaheuristics approaches is helpful in dealing with large size industrial problems.

The practical IPDP problem in supply chain often has trade-off among multiple conflicting objectives which need to be simultaneously optimized by the decision maker (DM). Apart from the performance measure of production and distribution planning based on financial aspects such as profit, cost, etc., other measures such as customer service level and responsiveness are also critical and should be considered.

The literature on multi-site production and distribution planning (MSPDP) is oblivious to uncertainty. Most of the models are assuming that planning system operates in a deterministic manner and the parameters are exactly as predicted in forecast. In the present environment where organizations are trying to implement the concept of zero inventory and focusing on removing the backordering situation, there is a need to work on optimization models under uncertain environment. There are two ways to handle unceratainty; through probabilistic programming and possibilistic programming. A good amount of research in the literature is dealing with probabilistic or stochastic programming approach to handle uncertainty. Another approach of handling uncertainty which is possibilistic programming is still remains an area to be worked upon.

1.3 Research Objectives

This research attempts to give new insights into the existing literature by presenting novel mathematical programming models and solution approaches regarding production planning, distribution planning and integration of the two in multi-site manufacturing scenario. The main objective of this thesis is to formulate and solve IPDP mathematical model for a two echelon supply chain in deterministic as well as in uncertain environment. Initially, a mixed integer linear programming model is formulated for MSIPDP problem and solved using CPLEX 12.7 solver which works based on branch and cut algorithm. Three important aspects of production and distribution planning: setup cost and time, backordering and capacity of heterogeneous transportation vehicles are incorporated in the formulation of mathematical model.

In order to be able to solve large size problem instances, a Lagrangian relaxation based heuristics approach is implemented. The approach provides lower bound on the optimal solution since the objective is of minimization. After every iteration, a new lower bound value is obtained based on the most recent upper bound value. Two different formulations are made which are relaxing hard constraints and adding them into objective function, thus decomposing the original problem into subproblems. To maintain the feasibility of the solution, two heuristics algorithms are implemented.

For a successful production plan, there exists more than one criterion. Performance measure of production and distribution planning based on three important criteria i.e. cost, delivery time and backorder level are considered for formulation of multi-objective mathematical model and a variant of goal programming method, known as preemptive goal programming is used to handle the proposed mathematical model.

Another important contribution of this study is incorporation of uncertainty. In real life situation, the parameters of production and distribution are often fuzzy or uncertain due to several factors such as variation in human performance, changing environmental conditions, and unavailability or improper information (Liang, 2007; Azadegan et al., 2011). Therefore, it is necessary to consider this variability while modeling production and distribution plans. Mathematical model with multiple objectives are formulated and solved using possibilistic programming approaches (Fuzzy, Intuitionistic fuzzy and Neutrosophic) to deal with impreciseness.

The main objectives of this study that fill the gaps in existing research are given as follows:

1. Mathematical model formulation for multi-site integrated production and distribution planning (MSIPDP) problem and implementation of an effective solution approach to handle large size practical problems.

2. Multi-objective mathematical model formulation and solution for MSIPDP problem.

3. Analysing the multi-objective MSIPDP problem under uncertainty.

The major contribution of this research is an in-depth analysis of integration of production and distribution planning problem in multi-site manufacturing scenario. The findings of this research will be helpful for researchers and practitioners to formulate a real life problem into mathematical model, solve the problem using exact as well as heuristic approach and in deterministic as well as uncertain environment. The research methodology proposed in this study is discussed in the next Section.

1.4 Research Methodology

The research methods employed in this thesis are: literature review, mathematical modeling and optimization in deterministic as well as uncertain environment.

Literature review

The literature review (Chapter 2) provides overview and fundamentals of production and distribution planning in multi-site manufacturing scenario and discuss issues regarding mathematical modeling and solution approaches. On the basis of literature review, major research gaps in this area are found out. Critical analysis of literature reveals that existing work in this area does not take into account important production and distribution aspects simultaneously.

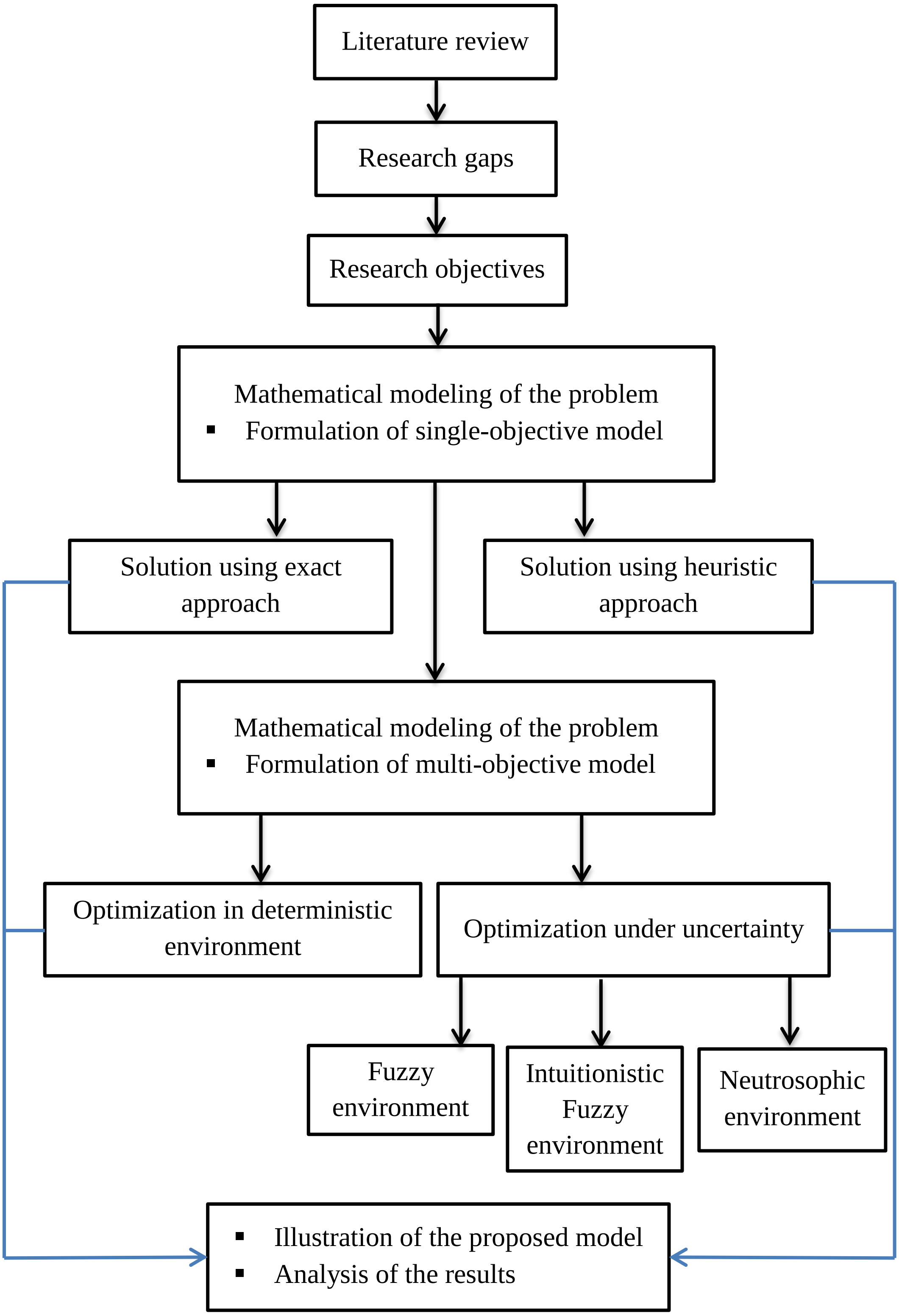


Figure 1.2: Research methodology adopted in current research

Mathematical modeling in deterministic environment

The research work formulates single as well as multi-objective mathematical models for MSIPDP problem under deterministic environment. The proposed mathematical models are described and illustrated using real life problem of an Indian automotive industry. Appropriate solution approaches are identified and implemented to get desired results.

Mathematical modeling under uncertainty - Possibilistic programming

The proposed multi-objective mathematical model is optimized under uncertain environment. Possibilistic programming approaches are employed to address improper, imprecise and indeterminate information in parameters, constraints and objective functions of the proposed mathematical model. The crisp formulations of mathematical models are solved using CPLEX solver provided by IBM ILOG CPLEX.

1.5 Organisation of the thesis

The thesis is structured into eight chapters as described below:

Chapter 1- Introduction: This chapter provides general introduction about multi-site manufacturing and integration of production and distribution planning. The motivation for conducting this research and obectives of research are discussed. A step by step research methodology is also described in the chapter which will be able to achieve the objectives.

Chapter 2- Literature Review: Chapter 2 discusses about earlier research work done in the area of MSPDP. Based on the structured literature review, few research gaps are suggested.

Chapter 3- Mathematical modeling - solution using exact approach: The IPDP problem of multi-product, multi-period, multi-site manufacturing with condiseration of setup, heterogeneous transportation and backordering is addressed in chapter 3. A MILP model is formulated and discussed with illustration of real life industrial problem.

Chapter 4- Mathematical modeling – solution using heuristic approach: Chapter 4 deals with the complexity of the proposed MILP model. To handle the complexity of the problem, Lagrangian relaxation (LR) based heuristics approach is implemented. Two heuristics are proposed for handling complexity of production and transportation part of the problem and solutions are compared with exact optimization results of MILP solver.

Chapter 5- Mathematical modeling – multi-objective formulation: Chapter 5 extends the problem to multi-objective formulation. The objectives are to minimize total system cost with minimum backorder level and distribution time of products. The proposed mathematical model is solved using preemptive goal programming method.

Chapter 6- Multi-objective optimization in Fuzzy environment: In Chapter 6, multi-objective optimization of MSIPDP problem under uncertainty is addressed. Fuzzy programming approach is implemented to handle the ambiguity of parameters and impreciseness of objective functions.

Chapter 7- Multi-objective optimization in Neutrosophic environment: Chapter 7 also deals with uncertainty in multi-objective formulation, but in Neutrosophic environment. To analyze the performance of the neutrosophic approach, results are compared with intuitionistic fuzzy programming approach.

Chapter 8- Conclusion: Finally, Chapter 8 is devoted to conclude the thesis and provide future research directions in this area.

Chapter 2

Literature Review

2.1 Introduction

The purpose of this chapter is to provide a state-of-the-art review on production and distribution planning in multi-site manufacturing environment and to establish link between background and this research work. The contributions of this chapter are, to review the mathematical models proposed in the literature, classify the literature on the basis of problem formulation approach, aspects of the mathematical model, solution approaches and real-life applications, and to provide important findings and research gaps. The systematic literature review methodology of Badhotiya et al. (2016), Soni and Kodali (2011) and Fahimnia et al. (2013) has been adopted. Initially, PD planning problem in multi-site manufacturing environment is investigated to find research gaps. A separate review on MSPDP problem under uncertainty is conducted after following observations from first review process. The next section provides analysis of the collected literature, in which methodology for conducting review is discussed along with literature classification and focused literature review on uncertainty. Section 2.3 provides research gaps identified through critical examination of literature and Section 2.4 presents chapter summary.

2.2 Analysis of Literature

Integrated decision making is one of the most important aspects of supply chain management. Over the past years, there have been an increasing number of articles published on the IPDP problem. Earlier works in this area attempts to deal with this problem in single site manufacturing environment. However, in this current global competitive environment, it is imperative for organizations to shift from single site to multi-site manufacturing. Therefore, over the last two decades, many researchers have worked upon Multi-site integrated production and distribution planning (MSIPDP) problem.

Few researchers in the past have conducted review on PD planning problems and mathematical models, summary of which is given in Table 2.1. Bhatnagar et al. (1993) addressed the issue of coordination in organizations. Two levels of problem are defined, coordination between functions, called as General Coordination problem and coordination within the same function at different echelons in an organization, called as Multi-Plant Coordination problem. Vidal and Goetschalckx (1997) reviewed strategic PD planning models. The articles were classified based on optimisation models, modelling issues and case studies and applications. The focus of the study was on mixed integer programming models for global logistics system. The articles were analysed for lack of features and on the basis of that, opportunities for future research were identified. Sarmiento and Nagi (1999) reviewed articles on integrated PD system with the focus on analysis of transportation system. Strategic and tactical planning level was also a perspective for review. The articles were classified based on fixed and infinite time horizon of production-inventory-distribution models.

Table 2.1 Summary of literature review articles on production and distribution planning

Articles	**Article Scope**	**#**	**Time range**	**Content description**
Gelders and Wassenhove (1981)	Production planning	UN	UN	Focus on resource utilization, capacity allocation, lot sizing and scheduling
Bhatnagar et al. (1993)	Production planning	UN	UN	Articles classified on the basis of level of coordination as general and multi-plant
Vidal and Goetschalckx (1997)	Production-distribution planning	UN	UN	Articles were classified into four groups: Existing reviews, optimization models, additional issues for modelling, and case studies and applications.
Sarmiento and Nagi (1999)	Integrated Production-distribution system	UN	UN	Classified on the basis of type of decision to be taken in the model

Articles	Article Scope	#	Time range	Content description
Erengüç et al. (1999)	Integrated Production/ distribution planning	UN	UN	Articles were analysed according to the supply chain network and nature of relationship between each stage
Schmidt and Wilhelm (2000)	Strategic, tactical and operational level logistics	UN	UN	Discussed strategic, tactical and operational level modelling issues
Bilgen and Ozkarahan (2004)	Strategic, tactical and operational levels of SCM	UN	UN	literature survey on production and distribution models of SCM
Fahimnia et al. (2008)	Integrated Production-distribution planning	UN	UN	Emphasis on optimization and simulation studies. Described characteristics of the models.
Mula et al. (2010)	supply chain production and transportation planning	44	1989-2009	Articles were classified based on supply chain structure, decision level, modelling approach, purpose, shared information, limitations, novelty and application
Fahimnia et al. (2013)	Integrated Production-distribution planning	UN	1988-2009	Articles were classified based on degree of complexity and solution approaches applied.
Díaz-Madroñero et al. (2015)	Integrated Production and transport routing planning	22	1994-2013	Articles were classified based on modelling aspects and solution approaches.

Note: UN – Unknown

An invited review on integrated PD planning in supply chain was provided by Erengüç et al. (1999). After defining the supply chain, the articles provide PD planning function at each stage of the supply chain. Based on review, some future research directions were also provided. Multi-national logistics network is taken into consideration by Schmidt and Wilhelm (2000). Literature on Strategic, tactical and operational level decisions and their modelling issues was reviewed in the paper. In 2004, Bilgen and Ozkarahan reviewed supply chain management PD planning literature at strategic, tactical and operational level. The literature was classified on the basis of solution methodology applied. Fahimnia et al. (2008) reviewed PD models of supply chain with focus on optimisation and simulation studies. Issue of demand uncertainty in production-distribution models was also analysed. Mula et al. (2010) presented an invited review of 44 reference articles on supply chain production and transportation planning. The emphasis was given on mathematical programming models and articles were classified based on supply chain structure, decision level, modelling approach, purpose, shared information, limitations, novelty and application.

Complexity and solution based classification of integrated PD planning models was provided by Fahimnia et al. (2013). The complexity was considered in terms of product, plant, warehouse, transport path and time period. Literature was classified in four categories of solution based classification: mathematical techniques, heuristics techniques, simulation modelling and genetic algorithms. Based on the classification of literature research gaps were identified and future research trends were suggested. Recently, in 2015, Díaz-Madroñero et al. reviewed tactical optimisation models for integrated PD planning decisions. Mathematical programming models on production transportation planning presented in the literature are discussed on emphasis on production, inventory and routing aspects.

Earlier review articles on PD planning were focused on complete supply chain irrespective of number of manufacturers. As the market is spreading and become competitive, manufacturing organisations are shifting from single to multiple manufacturing sites. The first article on MSPDP was published in 1997 and since then, there is continuous growth in the number of articles on this area. This chapter provides review of articles in the field of MSPDP and classifies them on the basis of modelling approach, aspects of the model, solution approach and practical application.

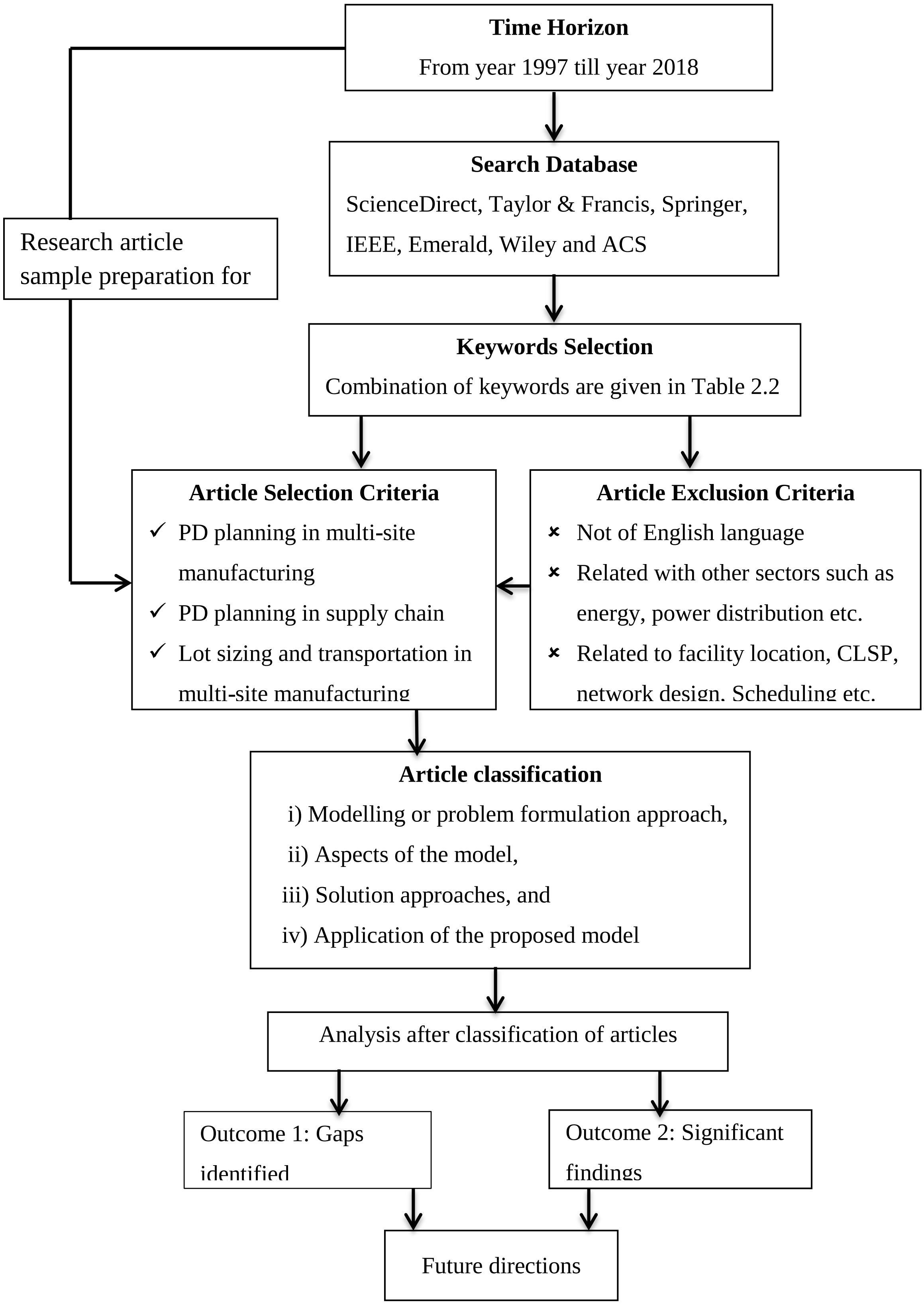


Figure 2.1: Literature Review methodology adopted in current study

2.2.1 Review Methodology

This Section highlights some important research work done in this field. The review methodology is based on collection, exclusion, classification and critical analysis of literature to draw out meaningful research gaps. Articles in the literature are organised and classified to draw out meaningful interpretations. Figure 2.1 presents review methodology followed in this research. Initially, articles were collected from major scientific publishing databases and applying exclusion criterias yielded desired relevant pool of research articles. These articles were then classified into categories and critically analysed to draw out meaningful findings and research gaps. The complete procedure of this review methodology in detail is explained in the subsequent Sections.

2.2.1.1 Reference collection

The articles published in the field of MSPDP are collected from major scientific publishing databases including Elsevier, Taylor & Francis, Springer, ACS (American Chemical Society), Wiley and Inderscience. The combinations of search keywords used for extracting the articles are given in Table 2.2. References and search results used in earlier review articles were also followed for complete collection of relevant articles. Only those articles written in English language were selected for further classification and analysis. Initially, 221 articles were collected by following the above mentioned search scheme. From this collection, relevant review articles were scrutinised and irrelevant review articles and articles related with scheduling and sequencing, network design, closed loop supply chain, facility location and lot sizing problem were excluded. After the scrutiny and exclusion process, a total of 65 research articles remained for further classification and review. The distribution of collected articles published in journals is enlisted in Table 2.2.

Table 2.2: Keywords used for article collection

S. No.	Constructs	Keywords
1	Manufacturing	Multi-site; Multi-plant; Supply chain
2	Planning	Planning; Production; Distribution; Transportation
3	Integration	Integration; Coordination; Collaboration
4	Solution	Optimization; modeling; Uncertainty; multi-objective

It can be seen from Table 2.3 that there exists a large numer of articles availale in this area published in reputed international journals. According to the collected literature, the first article on MSPDP was published in 1997 by McDonald and Karimi which shows that MSPDP is a promising research area.

Table 2.3: Journal wise distribution of articles

S.No.	Journal	Publisher	Frequency
1	International Journal of Production Research	Taylor & Francis	13
2	International Journal of Production Economics	Elsevier	5
3	Computers & Industrial Engineering	Elsevier	5
4	Computers & Operations Research	Elsevier	4
5	Computers and Chemical Engineering	Elsevier	4
6	IIE Transactions	Taylor & Francis	4
7	European Journal of Operational Research	Elsevier	4
8	Industrial & Engineering Chemistry Research	ACS	4
9	Omega	Elsevier	2
10	Information Sciences	Elsevier	2
11	Expert Systems with Applications	Elsevier	2
12	International Journal of Management Science and Engineering Management	Taylor & Francis	2
13	OR Spectrum	Springer	2

2.2.1.2 Literature classification

The classification scheme of this study is adopted from Mula et al. (2010). The literature is classified under four schemes i) Modelling or problem formulation approach, ii) Aspects of the model, iii) Solution approaches, and iv) application of the proposed model. The detailed discussion of each classification scheme is given in subsequent sections.

Problem formulation approach

This classification tells about the echelon or supply chain entities considered, type of mathematical representation or modelling approach, objective of the mathematical model and the time formulation of the proposed model. Table 2.4 classify the literature based on problem formulation approach.

Supply chain consists of five entities viz. supplier, manufacturer, distributor, retailor and customer. Considering complete supply chain and focusing on all the aspects will make the resulting model very complex and difficult to solve. Therefore, a combination of echelons is taken into consideration by several researchers. For example, Sabri and Beamon (2000), Jang et al. (2002) and Chen and Chang (2006) have considered four echelons; supplier, manufacturer, distributor and customer while article by Miller and Matta (2003) focused on two geographically distant plants only. The majority of articles in the literature have considered two echelons of supply chain to reduce complexity of the problem.

A mathematical programming model can be formulated as single objective or multi-objective. In the collected literature, 22 articles are formulated based on multiple objective functions and 43 articles are on single objective. In the single objective, 32 articles were formulated with total cost minimisation as objective function and remaining 11 articles taken profit maximisation as objective function which shows that cost minimisation is the most preferred objective function.

Table 2.4: Literature classification by problem formulation

Author	Supply chain echelons considered	Modelling approach	Objective function			Time representation
			MTC	MTP	MO	
McDonald and Karimi (1997)	Manufacturer-Customer	MILP		✓		Continuous
Dogan and Goetschalckx (1999)	Supplier-Manufacturer-Distributor	MILP	✓			Discrete
Vercellis (1999)	Manufacturer-Distributer	MILP	✓			Discrete
Sabri and Beamon (2000)	Supplier-Manufacturer-Distributor- Customer	MOMILP			✓	Discrete
Timpe and Kallrath (2000)	Manufacturer-Customer	MILP		✓		Discrete
Dhaenens-Flipo (2000)	Manufacturer-Customer	ILP	✓			Discrete
Dhaenens-Flipo and Finke (2001)	Manufacturer-Customer	ILP	✓			Discrete
Sakawa et al. (2001)	Manufacturer-Customer	MILP	✓			Discrete
Jayaram and Pirkul (2001)	Supplier-Manufacturer-Customer	MILP	✓			Single period
Jang et al. (2002)	Supplier-Manufacturer-Distributor- Customer	LP	✓			Discrete
Kallrath (2002)	Manufacturer-Customer	MILP		✓		Discrete
Miller and Matta (2003)	Manufacturer	MILP	✓			Discrete
Chen et al. (2003)	Manufacturer-Distributer-retailor	MOMINLP			✓	Discrete
Jackson and Grossmann (2003)	Manufacturer-Customer	NLP		✓		Discrete
Ryu et al. (2004)	Manufacturer-Distributer	LP	✓			Single period
Jolayemi and Olorunniwo (2004)	Manufacturer-Distributer	MILP		✓		Discrete

Author	Supply chain echelons considered	Modelling approach	Objective function			Time representation
			MTC	MTP	MO	
Chen and Lee (2004)	Manufacturer-Distributer-retailor	MOMINLP			✓	Discrete
Chan et al. (2005)	Manufacturer-Customer	LP	✓			Single period
Kanyalkar and Adil (2005)	Manufacturer-Customer	MILP	✓			Discrete
Park (2005)	Manufacturer-Retailor	MILP		✓		Discrete
Gen and Syarif (2005)	Manufacturer-Customer	LP	✓			Discrete
Chen and Chang (2006)	Supplier-Manufacturer-Distributor- Customer	LP	✓			Discrete
Ek¸sio˘glu et al. (2006)	Manufacturer-Retailor	MILP	✓			Discrete
Lei et al. (2006)	Manufacturer-Distributer	MILP	✓			Discrete
Yılmaz and Çatay (2006)	Supplier-Manufacturer-Distributor	MILP	✓			Discrete
Ek¸sio˘glu et al. (2007)	Manufacturer-Retailor	MILP	✓			Discrete
Roghanian et al. (2007)	Manufacturer-Distributer	MOLP			✓	Single period
Kanyalkar and Adil (2007)	Supplier-Manufacturer-Customer	MOMILP			✓	Discrete
Aliev et al. (2007)	Manufacturer-Distributer- Customer	LP		✓		Discrete
Liang (2007)	Manufacturer-Distributer	MOLP			✓	Single period
Liang (2008a)	Manufacturer-Distributer	MOLP			✓	Discrete
Liang (2008b)	Manufacturer-Distributer	MOLP			✓	Single period
Selim et al. (2008)	Manufacturer-Distributer-retailor- Customer	MOLP			✓	Discrete

Author	Supply chain echelons considered	Modelling approach	Objective function			Time representation
			MTC	MTP	MO	
Liang and Cheng (2009)	Manufacturer-Distributer	MOLP			✓	Discrete
Torabi and Hassini (2009)	Supplier-Manufacturer-Distributor	MOMILP	✓			Discrete
Verderame and Floudas (2009)	Manufacturer-Distributer	MILP			✓	Discrete
Alemany et al. (2010)	Supplier-Manufacturer-Distributor	MILP	✓			Discrete
Bilgen (2010)	Manufacturer-Distributer	MILP		✓		Discrete
Bilgen and Günther (2010)	Manufacturer-Distributer	MILP	✓			Discrete
Kanyalkar and Adil (2010)	Supplier-Manufacturer-Customer	MILP	✓			Discrete
Safaei et al. (2010)	Manufacturer-Distributer-retailor	MILP	✓			Discrete
Mula et al. (2010)	Manufacturer-Customer	MILP	✓			Discrete
You et al. (2010)	Manufacturer-Customer	MILP	✓			Discrete
Calvete et al. (2011)	Manufacturer-Retailor	MILP				Single period
Jolai et al. (2011)	Manufacturer-Distributer-retailor-Customer	MOLP	✓			Discrete
Kanyalkar and Adil (2011)	Supplier-Manufacturer-Customer	MILP			✓	Discrete
Terrazas-Moreno et al. (2011)	Manufacturer-Customer	MILP		✓		Discrete
Lidestam and Rönnqvist (2011)	Supplier-Manufacturer-Distributor	MILP		✓		Discrete
Liang (2011)	Manufacturer-Distributer	MOLP			✓	Discrete
MirzapourAl-e-hashem et al. (2011)	Supplier-Manufacturer-Customer	MOMINLP			✓	Discrete

Author	Supply chain echelons considered	Modelling approach	Objective function			Time representation
			MTC	MTP	MO	
Pathak and Sarkar (2012)	Supplier-Manufacturer-Retailor- Customer	MOLP			✓	Discrete
Amorim et al. (2012)	Manufacturer-Distributer	MOMILP			✓	Discrete
Fahimnia et al. (2012)	Manufacturer-Customer	MINLP	✓			Discrete
Jung and Jeong (2012)	Manufacturer-Distributer- Customer	LP		✓		Discrete
Yuan et al. (2012)	Manufacturer-Customer	MILP	✓			Discrete
Torabi and Moghaddam (2012)	Manufacturer-Distributer	MOLP			✓	Discrete
Melo and Wolsey (2012)	Manufacturer-Customer	MILP	✓			Discrete
Nasiri et al. (2014)	Supplier-Manufacturer-Distributor	MINLP	✓			Continuous
Camacho-Vallejo et al. (2015)	Manufacturer-Distributer-retailor	LP	✓			Single period
Gholamian et al. (2015)	Supplier-Manufacturer-Customer	MOMINLP			✓	Discrete
Gholamian et al. (2016)	Supplier-Manufacturer-Customer	MOMINLP			✓	Discrete
Khalili-Damghani and Tajik-Khaveh (2015)	Supplier-Manufacturer-Distributor	MOMILP			✓	Discrete
Darvish et al. (2016)	Manufacturer-Customer	ILP	✓			Discrete
Entezaminia et al. (2016)	Supplier-Manufacturer-Customer	MOMILP			✓	Discrete
Rafiei et al. (2018)	Supplier-Manufacturer-Distributor-Customer	MOMINLP			✓	Discrete

Time is an important factor in the modelling of PD planning problems. There are few articles in the literature who have formulated the mathematical model in single time period (Jayaram and Pirkul, 2001; Ryu et al., 2004; Chan et al., 2005; Roghanian et al., 2007; Calvete et al., 2011; Camacho-Vallejo et al., 2015). Articles considered multiple time periods can be classified on the basis of discrete and continuous time representation. Discrete time representation was the most preferred one while formulating the mathematical model. Mathematical model based on continuous time was formulated by McDonald and Karimi (1997) and Nasiri et al. (2014). The major difference in discrete and continuous time representation is the occurrence of an event. In the discrete time representation an event can happen on the boundary of the time period while in continuous time representation an event can occur anytime in a period. A detailed description on discrete and continuous time models can be referred form Floudas and Lin (2004).

Aspects of the problem

This classification scheme tells about dimensions or aspects considered in the literature while formulating a problem in mathematical terms. There exists three important aspects of PD planning; production, inventory and transportation. In production aspect, dimension of the problem is investigated which is defined by number of products, consideration of setup cost/time and capacity of production. Inventory aspect deals with available storage capacity and inventory policies. Transportation aspects deals with fleet of vehicle, transport modes, capacity of transport mode, number of trips, time windows, number of routing etc. Apart from this, other aspect can be consideration of backordering. When the unfulfilled demand or shortage is added in the next period demand then it is called backorder.

Table 2.5 classify the literature according to the above described problem aspects. A majority of the articles considered multiple products in their mathematical formulation. Very few articles have considered single product (Chan et al. 2005; Lei et al. 2006; Ekşioğlu et al. 2006; Yılmaz and Catay 2006; Ekşioğlu et al. 2007; Alemany et al. 2010; Camacho-Vallejo et al. 2015; Darvish et al. 2016; Rafiei et al. 2018).

Table 2.5: Literature classification by problem aspects

Author	Product		Setup			Production	Storage	Transportation	Backorder
	Single	Multiple	Setup cost	Setup time	Sequence dependent	Capacity	Capacity	limit	
McDonald and Karimi (1997)		✓				✓			✓
Dogan and Goetschalckx (1999)		✓				✓			
Vercellis (1999)		✓				✓			✓
Sabri and Beamon (2000)		✓	✓	✓		✓			✓
Timpe and Kallrath (2000)		✓				✓		✓	
Dhaenens-Flipo (2000)		✓			✓	✓			
Dhaenens-Flipo and Finke (2001)		✓			✓	✓		✓	
Sakawa et al. (2001)		✓				✓	✓		
Jayaram and Pirkul (2001)		✓				✓			
Jang et al. (2002)		✓				✓			
Kallrath (2002)		✓				✓		✓	
Miller and Matta (2003)		✓	✓						
Chen et al. (2003)		✓					✓	✓	✓
Jackson and Grossmann (2003)		✓				✓			
Ryu et al. (2004)		✓				✓		✓	
Jolayemi and Olorunniwo (2004)		✓		✓		✓		✓	

Author	Product		Setup			Production Capacity	Storage Capacity	Transportation limit	Backorder
	Single	Multiple	Setup cost	Setup time	Sequence dependent				
Chen and Lee (2004)		✓					✓	✓	✓
Chan et al. (2005)	✓								
Kanyalkar and Adil (2005)		✓				✓		✓	
Park (2005)		✓	✓	✓		✓	✓	✓	
Gen and Syarif (2005)		✓							
Chen and Chang (2006)		✓					✓	✓	
Ek¸sio˘glu et al. (2006)	✓		✓						
Lei et al. (2006)	✓					✓	✓	✓	
Yılmaz and C¸ atay (2006)	✓					✓	✓		
Ek¸sio˘glu et al. (2007)	✓		✓			✓			
Roghanian et al. (2007)		✓				✓			
Kanyalkar and Adil (2007)		✓				✓		✓	
Aliev et al. (2007)		✓							
Liang (2007)	✓					✓	✓	✓	
Liang (2008a)		✓				✓	✓	✓	✓
Liang (2008b)	✓					✓	✓	✓	
Selim et al. (2008)		✓				✓	✓	✓	

Author	Product		Setup			Production Capacity	Storage Capacity	Transportation limit	Backorder
	Single	Multiple	Setup cost	Setup time	Sequence dependent				
Liang and Cheng (2009)		✓				✓	✓	✓	
Torabi and Hassini (2009)		✓				✓		✓	
Verderame and Floudas (2009)		✓	✓	✓		✓		✓	
Alemany et al. (2010)	✓								
Bilgen (2010)		✓	✓	✓		✓		✓	✓
Bilgen and Günther (2010)		✓	✓	✓		✓	✓		
Kanyalkar and Adil (2010)		✓		✓			✓	✓	
Safaei et al. (2010)		✓				✓		✓	
Mula et al. (2010)		✓	✓	✓		✓	✓	✓	
You et al. (2010)		✓				✓			
Calvete et al. (2011)									
Jolai et al. (2011)	✓					✓	✓		
Kanyalkar and Adil (2011)		✓					✓	✓	✓
Terrazas-Moreno et al. (2011)		✓			✓	✓		✓	
Lidestam and Rönnqvist (2011)		✓	✓			✓	✓	✓	
Liang (2011)		✓				✓		✓	✓
MirzapourAl-e-hashem et al. (2011)		✓				✓		✓	✓

Author	**Product**		**Setup**			**Production Capacity**	**Storage Capacity**	**Transportation limit**	**Backorder**
	Single	**Multiple**	**Setup cost**	**Setup time**	**Sequence dependent**				
Pathak and Sarkar (2012)		✓						✓	✓
Amorim et al. (2012)		✓	✓	✓	✓	✓			
Fahimnia et al. (2012)		✓				✓	✓	✓	✓
Jung and Jeong (2012)		✓	✓			✓	✓	✓	
Yuan et al. (2012)		✓				✓		✓	
Torabi and Moghaddam (2012)		✓						✓	✓
Melo and Wolsey (2012)		✓	✓	✓		✓	✓	✓	
Nasiri et al. (2014)		✓				✓		✓	
Camacho-Vallejo et al. (2015)	✓					✓			
Gholamian et al. (2015)		✓				✓		✓	
Gholamian et al. (2016)		✓				✓		✓	
Khalili-Damghani and Tajik-Khaveh (2015)		✓				✓		✓	✓
Darvish et al. (2016)	✓		✓			✓		✓	
Entezaminia et al. (2016)		✓				✓	✓	✓	✓
Rafiei et al. (2018)	✓					✓	✓		

When multiple products or services are provided using common resource, there is need for changeover and setup activities (Allahverdi and Soroush, 2008). Consideration of setup or change over in multi-product manufacturing adds to complexity of the problem. There exist four elements of setup; setup cost, setup time, sequence dependent setup, and setup carryover. Some of the articles have considered the cost and time of setup in their model e.g. Guinet (2001), Sambasivan and Schmidt (2002), Gnoni et al. (2003), Verderame and Floudas (2009), Bilgen (2010), Safaei et al. (2010), Melo and Wolsey (2012), Darvish et al. (2016). Guinet (2001) implemented a primal-dual approach to solve multi-site production planning problem. The fixed setup cost and time was considered which depends on products and sites. Sambasivan and Schmidt (2002) employed heuristic procedure to solve capacitated lot sizing problem considering setup time to determine production capacity consumption. Jolayemi and Olorunniwo (2004) and Kanyalkar and Adil (2010) were considered setup time without considering setup cost. Gnoni et al. (2003) dealt with lot sizing and scheduling problem considering setup cost and time with a case of automotive industry. In the model proposed by Park (2005), a fixed setup cost was considered on lot for lot basis, unrelated with quantity produced. Melo and Wolsey (2012) considered a two echelon supply chain of production sites and clients and setup cost and time was considered. A lot sizing and distribution problem with setup time was considered by Darvish et al. (2016). Sequence dependent setup was considered by Dhaenens-Flipo (2000), Dhaenens-Flipo and Finke (2001) and Terrazas-Moreno et al. (2011). There was no article found considering setup carryover. Article by Amorim et al. (2012) have considered both sequence dependent and sequence independent setup time and cost. Sequence dependent setup for changeover form a block to other block and sequence independent setup for changeover of a product on a production line.

The capacity constraints in the mathematical model are related with production, inventory and distribution activities. These constraints are limited production resources like production capacity of production line or total available time in a period, limited storage space of inventory and constraint on either capacity of vehicle used for transportation or maximum transport limit between source to destination. All of the above explained capacity constraints were taken in to formulation by Park (2005), Lei et al. (2006), Liang (2007), Liang (2008a), Liang (2008b), Selim et al. (2008), Liang and Cheng (2009), Mula et al. (2010), Lidestam and Rönnqvist (2011), Fahimnia et al. (2012), Jung and Jeong (2012), Melo and Wolsey (2012) and Entezaminia et al. (2016).

In inventory system, when there is a stock out situation, the unfulfilled demand is considered as backorder. It may be added to the next period demand or considered as lost. Backorder can increase the distribution cost on expedited shipping, increased production cost due to overtime and the customer service level. Backordering is considered by few articles in the literature such as Vercellis (1999), Alemany et al. (2010), Torabi and Moghaddam (2012), Khalili-Damghani and Tajik-Khaveh (2015) and Entezaminia et al. (2016). Vercellis C (1999) proposed multi-site production planning problem in which unfulfilled demand or backlogged amount was considered as lost sales. Alemany et al. (2010) calculated cost and quantity of backorder without coinciding with the demand for a given time period. Khalili-Damghani and Tajik-Khaveh (2015) and Torabi and Moghaddam (2012) considered backlog cost and backlogging quantity with limits on backorder quantities in their production and logistics planning model.

Few articles in the literature have incorporated different issues in their mathematical models such as delivery time window, heterogeneous transportation, and limited transportation capacity. Multi-plant capacitated lot sizing problem with distribution is presented by Darvish et al. (2016). Motivated by a real case of a furniture company, the concept of delivery time window was proposed and solved using branch and bound algorithm. Park (2005) and Steinrücke, (2015) considered fixed as well as variable transportation cost with limited vehicle capacity Lei et al. (2006) considered heterogeneous transportation fleet where each manufacturing site has its own number of vehicles and each vehicle has different maximum loading capacity. Zegordi and Nia, (2009) assumed the sharing of vehicle by group of suppliers to reduce the transportation cost. H'Mida and Lopez, (2013) have considered two different type of vehicles; tractors and trailers, each have its maximum transportation capacity. Different transportation modes and shipment lead times was considered by Entezaminia et al. (2016). Multi-plant capacitated lot sizing problem with distribution is presented by Consideration of heterogeneous transportation with limited number of vehicle having a fixed loading capacity is presented by Feng et al. (2017).

Solution approach

In this scheme, solution techniques used to handle MSPDP models are classified based on single objective and multi objective model solution approaches. Table 2.6 summarize the solution approaches used in the literature to handle single objective and multi-objective mathematical programming models.

Table 2.6: Approaches used to solve MSPDP models

Solution approaches	**Authors**
Single objective model solution approaches	
Mathematical programming based approaches	McDonald and Karimi (1997), Timpe and Kallrath (2000), Dhaenens-Flipo and Finke (2001), Kallrath (2002), Jolayemi and Olorunniwo (2004), Ryu et al. (2004), Kanyalkar and Adil (2005), Verderame and Floudas (2009), Alemany et al. (2010), Bilgen and Günther (2010), Kanyalkar and Adil (2010), Kanyalkar and Adil (2011), Yuan et al. (2012), Darvish et al. (2016), Miller and Matta (2003)
Heuristics techniques	Dogan and Goetschalckx (1999), Vercellis (1999), Dhaenens-Flipo (2000), Jayaram and Pirkul (2001), Jackson and Grossmann (2003), Park (2005), Ek¸sio˘glu et al. (2006), Lei et al. (2006), Yılmaz and C¸ atay (2006), Ek¸sio˘glu et al. (2007), You et al. (2010), Terrazas-Moreno et al. (2011), Lidestam and Rönnqvist (2011), Nasiri et al. (2014), Camacho-Vallejo et al. (2015)

Solution approaches	Authors
Metaheuristics approach	Calvete et al. (2011), Fahimnia et al. (2012)
Fuzzy optimisation approach	Sakawa et al. (2001), Chen and Chang (2006), Aliev et al. (2007), Bilgen (2010), Jung and Jeong (2012)
Hybrid approaches	Jang et al. (2002), Chan et al. (2005), Gen and Syarif (2005), Safaei et al. (2010), Melo and Wolsey (2012)
Multi-objective model solution approaches	
Goal programming	Kanyalkar and Adil (2007), Khalili-Damghani and Tajik-Khaveh (2015)
LP-metrics method	MirzapourAl-e-hashem et al. (2011), Entezaminia et al. (2016)
Weighted programming	Amorim et al. (2012)
Epsilon constraint method	Sabri and Beamon (2000)
Elastic constraint method	Rafiei et al. (2018)
Fuzzy decision making	Chen et al. (2003), Chen and Lee (2004), Roghanian et al. (2007), Liang (2007), Liang (2008a), Liang (2008b), Selim et al. (2008), Liang and Cheng (2009), Torabi and Hassini (2009), Mula et al. (2010), Liang (2011), Pathak and Sarkar (2011), Jolai et al. (2011), Torabi and Moghaddam (2012), Gholamian et al. (2015), Gholamian et al. (2016)

Most of the approaches for solving the PD planning problem are sequential in nature commonly based on decomposition (Dhaenens-Flipo, 2000; Terrazas-Moreno et al., 2011; Steinrücke, 2015), two phase approach (Lei et al., 2006) and bi-level approach (Camacho-Vallejo et al., 2015). These sequential approaches break the problem into smaller and easier ones. These approaches though are capable of solving the large size problems; they may not produce a global optimal solution (Zegordi and Nia, 2009). Solving the problem in an integrated manner, however, has found to be useful in reducing inventory holding, shortage and transportation cost along with improvement in customer service level (Park, 2005).

The MSIPDP problem in the literature have dealt with solution approaches like mathematical programming based exact optimisation, Linear programming based heuristic approaches, Lagrangian relaxation heuristics, Lagrangian decomposition, two phase approach and metaheuristics approaches like genetic algorithm and hybrid genetic algorithm as shown in Table 2.5. The exact optimisation approaches were solved using optimisation modelling software's such as CPLEX, AIIMS, GAMS, XPRESS-MP, AMPL, LINGO etc. When size of the problem is small, optimal solution can be obtained in less computation time but as the size of the problem gets large, sub-optimal solutions obtained at longer computation times, which generates the need to develop heuristics and metaheuristics approaches which are able to obtain feasible solutions in comparatively less computational time.

Single objective model solution approaches

Single objective model solution approaches implemented in the literature are mathematical programming based approaches, heuristic approaches, metaheuristic approaches, Fuzzy optimisation approaches and Hybrid approaches.

Mathematical programming approaches in the collected literature are branch and bound, parametric optimisation and exact optimisation approaches which can be solved to optimality using any mathematical programming software. There are few articles in the literature that have used mathematical programming based exact optimization approaches. Miller and Matta (2003) implemented branch and bound technique for solution of production and distribution scheduling model. Ryu et al. (2004) formulated a bi-level optimisation model under uncertainty and solved using parametric optimization technique. Kanyalkar and Adil (2011) proposed a robust optimisation model for multi-site procurement-production-distribution planning problem and solved using GLPK (Gnu Linear Programming Kit) software.

Mathematical programming techniques are applicable to solve small size problem to optimality but as the problem size increases so does the complexity of the problem and it takes longer computational time and computer memory to solve the problem to optimality. To overcome this issue, heuristics techniques are used to rapidly find a feasible or suboptimal solution. Heuristics techniques implemented in the literature includes decomposition techniques such as Benders decomposition (Dogan and Goetschalckx, 1999), Lagrangean decomposition (Jackson and Grossmann, 2003; Ek¸sio˘glu et al., 2007; Terrazas-Moreno et al., 2011; Lidestam and Rönnqvist, 2011; You et al., 2010), Spatial decomposition (Dhaenens-Flipo, 2000), LP based heuristics (Vercellis, 1999; Yılmaz and C¸atay, 2006; Camacho-Vallejo et al., 2015), Lagrangian relaxation (Jayaram and Pirkul, 2001; Nasiri et al., 2014), Two phase heuristics (Park, 2005; Lei et al., 2006), and primal dual heuristics (Ek¸sio˘glu et al., 2006).

Metaheuristics approaches are used to overcome the local search phenomenon of heuristics methods. Two articles in the literature have applied metaheuristics approaches. Few more articles have applied metaheuristics with combination of other approaches; those articles are placed in hybrid approaches. Calvete et al. (2011) formulated a bilevel programming model for hierarchical PD planning problem and solved using ant colony optimisation approach. Fahimnia et al. (2012) applied genetic algorithm optimisation approach for solution of PD planning problem of a two level supply chain network.

Apart from the single heuristics and metaheuristics approach, few article have presented hybrid solution approaches such as Chan et al. (2005) proposed a linear programing model for multi-factory supply chain production-distribution planning problem solved using hybrid analytical hierarchy process - genetic algorithm (AHP-GA) approach. For solution of multi-time period production/distribution planning problem, Gen and Syarif (2005) proposed a hybrid spanning tree-based genetic algorithm with the fuzzy logic controller approach. Safaei et al. (2009) proposed a MILP model that takes an integrated view of MSPDP. A hybrid mathematical simulation approach was implemented and computational results indicate the benefits of integrated approach over two phase sequential approach. Melo and Wolsey (2012) proposed a MIP formulation for a two level production-distribution problem solved using hybrid heuristics based on relaxation induced neighborhood search and local branching.

The parameters and objective function of a mathematical model are subjected to uncertainty in real life. To handle this impreciseness, fuzzy optimisation approach is used. Few articles in the literature have implemented fuzzy optimisation approach in single objective mathematical model such as Chen and Chang, (2006); Bilgen, (2010); Jung and Jeong, (2012).

Multi-objective model solution approaches

Production and distribution planning problems in literature are represented by mathematical models based on the concepts of operations research. The objective is to determine best production and distribution plan at minimum total cost or maximum total profit or other objective functions. Most of the models in the literature have considered single criterion for production and distribution planning, such as cost. The practical production and distribution planning decisions consists of more than one conflicting objectives which needs to be simultaneously optimised by decision maker. Multi-objective model solution approaches implemented in the literature are Goal programming, LP-metrics method, Weighted programming, Epsilon constraint method, Elastic constraint method and Fuzzy decision making.

In the past decade, a large number of studies have been done considering multiple objectives in PD planning problems as shown in Table 2.7. Chen et al (2003) proposed a multi-objective optimization of PD planning of a supply chain considering profit, customer service level and safe inventory level as multiple objectives. A multi-objective model for multi-site aggregate production planning is proposed by Leung et al. (2003). Profit maximisation and workforce level are the multiple objectives simultaneously optimised using goal programming method. Kanyalkar and Adil (2007) formulated mixed integer goal programming model considering three objective functions; minimisation of inventory storage volume, forward coverage policy and total cost. Amorim et al. (2012) considered the case of shelf life of perishable goods and formulated a MOMILP model. A decoupled and integrated PD planning model was proposed and solved by weighted programming method.

Table 2.7: Literature on deterministic multi-objective production-distribution planning models

Author (year)	**Objective functions**	**Solution approach**
Chen et al (2003)	Maximising profit, customer service level and safe inventory level	Two phase approach
Leung et al. (2003)	Maximising profit, minimising cost of hiring and laying off and utilization of import quota.	Goal programming
Kanyalkar and Adil (2007)	Minimising inventory storage volume, forward coverage policy and total cost	Goal programming
Amorim et al. (2012)	Minimising cost, maximising remaining shelf life	Weighted linear programming
Liu and Papageorgiou, (2013)	Minimising cost, flow time and lost sales	ε-constraint and lexicographic minimax
Khalili-Damghani and Tajik-Khaveh (2015)	Minimising logistics cost and maximising service level	Weighted goal programming
Ayadi et al. (2017)	Minimization of total cost, maximization of product quality and customer service level	ε-constraint method and AHP
Entezaminia et al. (2016)	Minimization of total cost, maximise environmental criteria.	LP-metrics method
Rafiei et al. (2018)	Minimising of total cost and maximising customer service level	Elastic constraint method

Liu and Papageorgiou, (2013) proposed a multi-objective MILP model considering cost, responsiveness and service level to address PD planning problem of a supply chain. To solve the proposed multi-objective model, ε-constraint and lexicographic minimax methods were employed. Khalili-Damghani and Tajik-Khaveh (2015) proposed a multi-objective MIP model for logistic planning and design problem for a three-echelon supply chain. Logistics cost and service level were the two objective functions used to formulate mathematical model. To solve the proposed multi-objective mathematical model, a weighted goal programming method is applied. The applicability of the proposed model was illustrated by taking illustrative example of a dairy industry supply chain.

Ayadi et al. (2017) dealt with multi-site supply chain planning problem and formulated a multi-objective mathematical model considering minimization of total cost, maximization of product quality and customer service level. These conflicting objectives generate a set of Pareto-optimal solution obtained by solving the model using ε-constraint method. To select the best Pareto-optimal solution, analytical hierarchy process method was applied. Entezaminia et al. (2016) considered environmental aspects while formulating MOMILP model and solved using LP-metrics method. Rafiei et al. (2018) investigated an IPDP problem within a four echelon supply chain considering competitiveness of market. Minimising total cost and maximising customer service level are the two objective functions used to formulate two multi-objective MILP model for competitive and non-competitive market situation. Elastic constraint method is used to solve the proposed multi-objective models.

In the practical PD planning problems in a supply chain, because of the conflicting nature of the multiple objectives and the vagueness in parameters, conventional mathematical programming methods are not suitable for obtaining an effective solution. To overcome this problem, possibilistic programming approaches are used.

Practical application

The imitation of real life application in the mathematical model support and validate the study. This classification describes practical application and illustrative examples proposed in the literature to validate the mathematical model.

Among the 65 research articles in the literature, 33 articles have taken random instances or illustrative examples to analyse the performance of their mathematical models. Rest of the literature reported various discrete and continuous manufacturing industries such as in health care (Pirkul and Jayaram, 1996), construction (Sakawa et al., 2001), automotive manufacturing (Gnoni et al., 2003; Torabi and Hassini, 2009; Fahimnia et al., 2012), ceramic tile industry (Alemany et al., 2010), consumer goods manufacturing (Kanyalkar and Adil, 2010; Kanyalkar and Adil, 2011), furniture manufacturing (Darvish et al., 2016), can production (Dhaenens-Flipo, 2000; Dhaenens-Flipo and Finke, 2001), general appliance company (Aliev et al., 2007), precision machinery and transmission components producer (Liang, 2008a; Liang and Cheng, 2009; Liang, 2011), electronics company (Jung and Jeong, 2012), surface and materials science Company (Torabi and Moghaddam, 2012), wood and paper industry (MirzapourAl-e-hashem et al., 2011; Lidestam and Rönnqvist, 2011; Gholamian et al., 2015, Gholamian et al., 2016) and other process industry (Vercellis, 1999) like chemical (Timpe and Kallrath, 2000; Kallrath, 2000; Lei et al., 2006; Mula et al., 2010), soft drink manufacturer (Liang, 2007; Liang, 2008b; Bilgen and Günther, 2010), dairy industry (Khalili-Damghani and Tajik-Khaveh, 2015).

2.2.1.3 MSIPDP problems under Uncertainty

The practical IPDP problem in supply chain has multiple conflicting objectives which are often uncertain due to several factors such as variation in human performance, changing environmental conditions, and unavailability or improper information (Liang 2007). To incorporate and handle the impreciseness, possibilistic programing approaches are implemented in this study.

Fuzzy programming

This Section examines earlier contributions in handling MSIPDP problem based on fuzzy programming approach. The Literature review is divided into two sub-sections addressing production and distribution planning problems formulated as single objective and multi-objective mathematical model and solved using fuzzy optimisation approach.

The benefit of applying fuzzy set theory is that it allows imprecise aspiration of the DM to be quantified (Hannan 1981). The concept of fuzzy set theory of Zadeh (1965) was first implemented by Zimmermann (1976) for solving a linear programming model having fuzzy objectives as well as fuzzy constraints based on the fuzzy decision-making concept of Bellman and Zadeh (1970). Later Zimmermann (1978) implemented the fuzzy linear programming approach to transform fuzzy multi-objective model into single objective using fuzzy min operator. Many researchers and practitioners followed Zimmermann's fuzzy linear optimization methods for solving IPDP problems in fuzzy environments. Utilization of piecewise linear and continuous functions to represent imprecise aspiration levels in FGP problem was proposed by Hannan (1981). Few researchers have followed Zimmermann's fuzzy linear optimisation methods and Hannan's piecewise linear membership function for solving IPDP problems in fuzzy environments.

Fuzzy optimisation and decision making approach was implemented in MSIPDP literature for both single as well as multiple objective function formulations. In single objective function, Sakawa et al. (2001) formulated a MIP model for production and transportation planning considering real problem of housing material manufacturer. Impreciseness in objective function and parameters of demands and production capacities are represented by membership function of Bellman and Zadeh (1970) and aggregation operator of Zimmermann (1978). Chen and Chang (2006) formulated a mathematical model for supply chain PD planning with fuzzy total cost minimization objective function. To convert the fuzzy model into crisp one, α cut and extension principle of Zadeh (1999) was applied. Bilgen (2008) addressed blending and maritime transport planning problem considering uncertainty in objective functions, shipment and customer demand. Bilgen (2010) addressed fuzzy IPDP problem in a supply chain system of a consumer goods company. Vagueness in objective function and capacity constraints are represented by membership function of Zimmermann (1976). To convert the fuzzy model into crisp model, three different aggregation operators were applied. Jung and Jeong (2012) addressed supply chain planning problem under demand uncertainty in case of a Korean electronics company. Sharahi et al. (2018) dealt with location-allocation and production-distribution problem of a three echelon supply chain. Uncertainty in supply, process and demand were modeled using type-II fuzzy sets.

The usefulness of fuzzy decision making approach also extends to multi-objective models where impreciseness comes in parameters, constraints and in multiple conflicting goals. A two phase fuzzy decision making approach for multi-objective multi-site supply chain optimisation problem is applied by Chen et al. (2003). A multi-objective mixed integer non-linear programming model was formulated and vagueness in profit maximization objective function is addressed. Chen and Lee (2004) formulated a multi-objective mixed integer non-linear programming model for three level supply chain considering uncertainty in demand and prices. Roghanian et al. (2007) formulated a fuzzy programming technique to convert bi-level probabilistic multi-objective programming model into crisp model for enterprise wide supply chain planning. Liang (2007) considered production/transportation planning problem solved by FGP approach to deal with fuzzy multiple goals. Liang (2008a) and Liang (2008b) developed FMOLP model to solve IPDP problem with fuzzy objectives, represented by piecewise linear membership function. Peidro et al. (2009) proposed a fuzzy mathematical model for supply chain planning considering uncertainties in supply, demand and process. Torabi and Hassini (2009) proposed an interactive fuzzy goal programming (IFGP) approach for multi-site procurement, production and distribution planning problem. Liang (2011) handled production and distribution planning decision by using fuzzy linear programming approach based on possibility theory. Sahebjamnia et al. (2016) applied fuzzy programming concepts on capacitated lot sizing problem considering uncertain demand and process parameters. Mohammed and Wang (2017) formulated and solved a fuzzy multi-objective programming model for production distribution plan of a green meat supply chain network. The benefit of applying fuzzy set theory is that it allows imprecise aspiration of the DM to be quantified.

Few articles in the literature have considered environmental issues in the PD planning model solved using fuzzy programming method. Mohammed and Wang (2017) developed a production distribution plan for a green meat supply chain network. Four fuzzy objective functions are addressed simultaneously and solution obtained using LP-metrics, ε-constraint and goal programming method. Mokhtari and Hasani (2017) formulated a multi-objective model for cleaner production-transportation planning and solved using FGP and simulated annealing based heuristics.

Intuitionistic fuzzy programming

Fuzzy sets only consider belongingness of an element of set while intuitionistic fuzzy set considers both belongingness and non-belongingness of sets. Atanassov (1986) proposed the concept of intuitionistic fuzzy sets which is a generalization of the fuzzy sets. Further, Angelov (1997) illustrated the concept of intuitionistic fuzzy optimization (IFO) by solving a simple transportation problem. Jana and Roy (2007) solved a multi-objective transportation model using IFO approach. Chakrabortty et al. (2013) implemented IFO technique for solution of an inventory model with fuzzy cost and demand rate. De and Sana (2014) considered a multi-plant, multi period production inventory model and provided a comparison of solutions of fuzzy and intuitionistic fuzzy optimization approaches. Various multi-objective programming models were also solved using IFO approaches. Bharati et al. (2014) demonstrated application of IFO approach in solution of multi objective linear programming model. Chakrabortty et al. (2015) solved multi-objective intuitionistic fuzzy transportation problem using chance operator. Pareto optimal solution of a multi-objective programming problem using IFGP is obtained by Razmi et al. (2016).

Neutrosophic programming

Uncertainty in MSIPDP problem has been solved in the literature using fuzzy and intuitionistic fuzzy approaches. Fuzzy sets only consider belongingness of an element of set while intuitionistic fuzzy set considers both belongingness and non-belongingness of sets. These approaches consider truth and false sides but do not consider the aspect of indeterminacy, which occurs due to unexpected parameters hidden in some propositions (Abdel-Baset et al, 2016). Neutrosophic set (NS) is generalization of fuzzy and intuitionistic fuzzy set based on the philosophy of Neutrosophy (Smarandache, 1999). There are three components of neutrosophic sets: truth-membership degree, indeterminacy-membership degree, and falsity-membership degree, which are able to represent indeterminate or inconsistent information.

The neutrosophic programming approach was used to solve several mathematical programming models such as linear programming, mixed integer programming, and multi-objective models. Abdel-Baset et al. (2016) introduced the concept of neutrosophic optimization for solving a goal programming problem having multiple conflicting objectives. Mohamed et al. (2017) introduced neutrosophic integer programming problem, where parameters of integer program was represented by triangular numbers and solved using neutrosophic programming approach. Abdel-Basset et al. (2018) solved a linear programming problem using neutrosophic approach. The parameters of proposed mathematical model were represented by trapezoidal numbers.

The concept of NS is also applied in various fields such as transportation modelling, production planning, multi-objective optimization etc. Thamaraiselvi and Santhi (2016) solved a real-life transportation problem in neutrosophic environment using single valued trapezoidal neutrosophic number. Rizk-Allah et al (2018) found a best compromised solution of a multi-objective transportation model in neutrosophic environment and found better results comparing the solutions with fuzzy optimization solutions.

2.3 Research Gaps

The current review helps in identifying potential areas for future research, which received less attention from the researchers in past and need further exploration by engineering management scholars. The key research areas identified for future research based on the research gaps discussed above are summarised below:

- In the entire literature, articles have considered production, inventory and distribution aspects in several combinations but there is no article available considering all these aspects in single mathematical model and in an integrated manner. Formulating a mathematical model considering all the aspects such as setup time/cost, production capacity, storage capacity, transportation capacity, different modes or types of vehicles, backordering etc. would reflect closeness to the real life scenario.
- In the distribution phase, majority of recent articles have considered homogeneous transportation with unlimited number of vehicle availability or one trip transportation and without any loading capacity constraint which simplifies the solution but are generally not feasible in real world distribution networks.
- Incorporating all the production and distribution aspects will result in a complex problem. It is evident from literature that mathematical models were solved either for small size problems or considering too many assumptions to simplify their mathematical models. There is need to work on an effective algorithm which can handle large size practical problems.
- It is evident from the literature that instead of working on real life imitation in mathematical models, researchers focused more on developing or implementing efficient solution approaches. There is a need to work on real life problems or problems which illustrate real life scenarios.
- A large number of articles in the literature have formulated their mathematical modes using variants of linear programming i.e. LP, MIP, MILP. There is a need to work on non-linear programming approaches as well.

- Only a few articles in the literature have formulated mathematical model based on continuous time representation. Continuous time formulation represents closeness to real life situation because parameter change can occur at any time.
- Apart from the performance measures based on financial aspects e.g. cost, profit; other measures such as customer service level, responsiveness, are also critical but have received very less attention.
- The literature on MSPDP is oblivious to uncertainty. Most of the models are assuming that planning system operates as deterministic and the parameters are exactly as predicted in forecast. In the present dynamic environment, there is a need to work upon optimization models under uncertain environment.

2.4 Chapter Summary

The significance of production and distribution planning in multi-site manufacturing environment has been recognised by researchers in last two decades. In this chapter, the existing knowledge based on production and distribution planning in the multi-site manufacturing is consolidated and classified based on different schemes. It becomes clear that integration of PD planning function is crucial to obtain a cost effective optimal plan. The classification of literature presented in this chapter can help decision makers in choosing suitable modelling and solution approach for integrating their production and distribution functions.

Chapter 3

Multi-Site Production and Distribution Planning: An Integrated Approach

3.1 Introduction

In this chapter, two echleon supply chain scenario along with description and formulation of mathematical model is presented. It was observed form the literature review that the three important aspects of MSIPDP problem: set up cost for different products at manufacturing site, capacity of the heterogeneous transport vehicles and backorder cost for unfulfilled demand, have not been considered in an integrated manner yet. Incorporating these three components represents closeness to the real life situation. This chapter fulfils this gap and presents a mathematical model for MSIPDP scenario of an automobile manufacturing company located in India.

The MSIPDP problem considered in this study is inspired from an automobile manufacturing company located in India. The company has multiple geographically dispersed plants producing multiple products (Two wheeler motorcycles). The demand for various products is originating from geographically scattered district level dealers called as selling locations. Each manufacturing site is producing final products and has a limited storage space from where products are distributed to selling locations. Whenever production starts at site, a fixed setup time and cost is incurred. All the manufacturing sites and selling locations have their own storage space of limited capacity. The transportation between manufacturing sites to selling locations is done by a heterogeneous fleet of vehicles each having a fixed amount of loading capacity. Backordering is considered whenever demand of a selling location is not fulfilled. The problem is formulated as MILP model and the analytical results of the proposed model are discussed.

This chapter is formulated in four sections. Section 3.2 provides description about the problem considered, notations used and assumption taken to formulate the mathematical model. The proposed model is a deterministic multi-product, multi-period, two echelon supply chain structure with consideration of setup, heterogeneous transportation and backordering. Section 3.3 presents the input parameter values and discussion of results obtained after solving the proposed model. The summary of this chapter is provided in Section 3.4.

3.2 Problem description

This section describes the MSIPDP problem considered and formulates the mathematical model. Figure 3.1 shows a typical network of manufacturing sites and selling locations to provide an overview of the problem. Multiple products are produced at multiple manufacturing sites which are then distributed to multiple selling locations. Each manufacturing site has a limited production capacity, a fixed setup time/cost for each production run of a product, and known processing time per unit. After production, the finished products are transported to various selling locations through heterogeneous fleet of vehicles each having a fixed loading capacity. Fixed transportation cost for each pair of manufacturing site and selling location and variable cost for transporting one unit of a product is known. There is a limited storage at each manufacturing site and selling location. When demand of any selling location is not fulfilled, it is considered as backorder and added to the next period demand but up to a predecided fraction of demand. The objective is to plan production and distribution quantities in order to satisfy the demand of products at each selling location at minimum total cost.

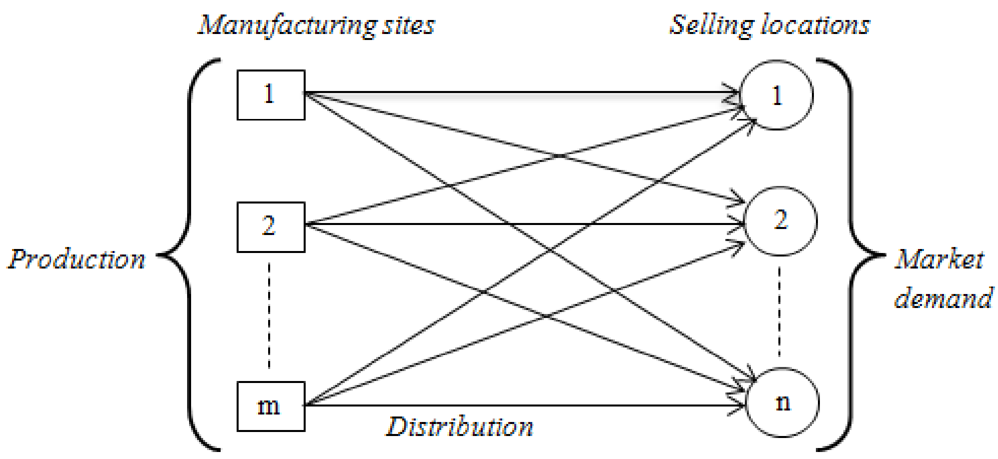


Figure 3.1: Cross supply network of manufacturing sites and selling location

3.2.1 Assumptions

Following assumptions are taken to convert the real life problem in to the mathematical model:

1. The demands of all the products originating from fixed number of selling locations are deterministic and known in advance.
2. The associated capacities of each manufacturing site and selling location are known.
3. There is direct transportation between manufacturing sites and selling locations and transportation lead times are negligible.
4. There are no quantity based discounts.
5. All the products produced and transported are assumed as defect free, hence the products are produced on the basis of exact order quantity.

3.2.2 Deterministic formulation of multi-site integrated production and distribution planning model

3.2.2.1 Nomenclature

Following sets, parameters and variables will be used for formulation of mathematical model.

Sets

I	Set of products (i = 1,……,I)
S	Set of manufacturing sites (s = 1,……,S)
L	Set of selling locations (l = 1,……,L)
G	Set of vehicle types (g = 1,…..,G)
T	Set of time periods (t = 1,……,T)

Parameters:

$D_{i,l,t}$	Demand of product *i* at selling location *l* in period *t*
$St_{i,s}$	Setup time of product *i* at manufacturing site *s*
$Pt_{i,s}$	Processing time of product *i* at manufacturing site *s*
$Pc_{i,s}$	Production cost per unit of product *i* in manufacturing site *s*
$Sc_{i,s}$	Setup cost of product *i* at manufacturing site *s*
$Hs_{i,s}$	Inventory carrying cost of product *i* at manufacturing site *s* per time period
$Hm_{i,l}$	Inventory carrying cost of product *i* at selling location *l* per time period
$Vt_{i,g,s,l}$	Variable transportation cost of vehicle type *g* for transporting product *i* from manufacturing site *s* to selling location *l*
Ft_g	Fixed transportation cost per vehicle of type *g*
$Bc_{i,l}$	Backordering cost of product *i* at selling location *l*
$Pcp_{s,t}$	Production capacity of manufacturing site *s* in period *t*
Ss_s	Maximum storage capacity of manufacturing site *s*
Sm_l	Maximum storage capacity of selling location *l*
Tc_g	Maximum capacity of transport vehicle of type *g*
θ_i	Fraction of demand of product *i* that is allowed to be backordered

M	A sufficiently large number

Decision variables:

$x_{i,s,t}$	Quantity of product *i* produced at manufacturing site *s* during period *t*
$Qt_{i,g,s,l,t}$	Quantity of product *i* transported by vehicle type g from manufacturing site *s* to selling location *l* in time period *t*
$Qb_{i,l,t}$	Backordered quantity of product *i* at selling location *l* at the end of the period *t*
$Nt_{g,s,l,t}$	Number of vehicles of type g used to transport between manufacturing site *s* to selling location *l* in period *t*
$Qs_{i,s,t}$	Quantity of product *i* carrying at manufacturing site *s* at the end of time period *t*
$Qm_{i,l,t}$	Quantity of product *i* carrying at selling location *l* at the end of time period *t*

$$y_{i,s,t} = \begin{cases} 1 & \text{if manufacturing site s is setup for producing product i in period t} \\ 0 & \text{otherwise} \end{cases}$$

3.2.2.2 The objective function

The objective function of the proposed mixed integer linear programming (MILP) model formulation of MSIPDP problem is consisting of various cost elements as follows.

Total production cost = (Production cost per unit of product) * (quantity of products produced)

i.e. $\sum_i \sum_s \sum_t (Pc_{i,s} x_{i,s,t})$

Setup cost = (Setup cost of product at manufacturing site)*(binary variable of setup)

i.e. $\sum_i \sum_s \sum_t (Sc_{i,s} Y_{i,s,t})$

Total Inventory carrying cost at site = (Inventory carrying cost per unit of product at manufacturing site) * (quantity of products carrying at manufacturing site at the end of period)

i.e. $\sum_i \sum_s \sum_t (Hs_{i,s} Qs_{i,s,t})$

Total Inventory carrying cost at selling location = (Inventory carrying cost per unit of product at selling location) * (quantity of products carrying at selling location at the end of period)

i.e. $\sum_i \sum_s \sum_t (Hm_{i,l} Qm_{i,l,t})$

Total fixed transportation cost = (fixed cost of vehicles) * (number of vehicle used for transportation)

i.e. $\sum_g \sum_s \sum_l \sum_t (Ft_g \, Nt_{g,s,l,t})$

Total variable transportation cost = (variable cost of transportation) * (quantity transported)

i.e. $\sum_i \sum_g \sum_s \sum_l \sum_t (Vt_{i,g,s,l} \, Qt_{i,g,s,l,t})$

Total backorder cost = (Backordering cost of product) * (quantity backordered)

i.e. $\sum_i \sum_l \sum_t (Bc_{i,l} Qb_{i,l,t})$

The complete objective function can be written as follows:

$$\textit{Minimum total cost} = \sum_i \sum_s \sum_t Pc_{i,s} x_{i,s,t} + \sum_i \sum_s \sum_t Sc_{i,s} y_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_g \, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l} \, Qt_{i,g,s,l,t} + \sum_i \sum_l \sum_t Bc_{i,l} Qb_{i,l,t} \tag{3.1}$$

The objective function shown in equation (3.1) seeks to minimise the total cost comprising production cost, inventory carrying cost at different production site and selling locations, fixed and variable transportation cost between plants and selling locations, setup cost at manufacturing sites and backorder cost.

3.2.2.3 Constraints

The proposed MILP model is subjected to following constraints.

Inventory balance constraints: Constraint (3.2) and (3.3) states inventory balance equation at manufacturing site and selling locations respectively. Constraint (3.2) relates the inventory quantity at start and end of period with the production and transported quantity at each manufacturing site. Constraint (3.3) relates inventory and backorder level at the start and end of period to the demand and transported quantity in that period at each selling location. It shows that in a particular period if both inventory and backorder quantity is greater than zero then there is no optimal solution.

$$Qs_{i,s,t} = Qs_{i,s,t-1} + x_{i,s,t} - \sum_g \sum_l Qt_{i,g,s,l,t} \quad \forall\, i,s,t \qquad (3.2)$$

$$Qm_{i,l,t} - Qb_{i,l,t} = Qm_{i,l,t-1} - Qb_{i,l,t-1} + \sum_g \sum_s Qt_{i,g,s,l,t} - D_{i,l,t} \quad \forall\, i,l,t \qquad (3.3)$$

Setup constraint: Constraint (3.4) force the binary setup variable.

$$x_{i,s,t} \le M.y_{i,s,t} \quad \forall\, i,s,t \qquad (3.4)$$

Production capacity constraint: Constraints (3.5) shows that production and setup time at each site should be less than or equal to the available production capacity of that period.

$$\sum_i (x_{i,s,t} Pt_{i,s} + St_{i,s,t}\, y_{i,s,t}) \le Pcp_{s,t} \quad \forall\, s,t \qquad (3.5)$$

Transportation capacity constraint: Constraint (3.6) provides limit on transported quantity, quantity transported from manufacturing site to selling location should be less than or equal to transportation capacity of the vehicle.

$$\sum_i Qt_{i,g,s,l,t} \le Tc_g.Nt_{g,s,l,t} \quad \forall\, g,s,l,t \qquad (3.6)$$

Backorder constraint: Constraint (3.7) limits the backordering quantity in each period at each selling location to be less than some fraction of demand.

$$Qb_{i,l,t} \le \theta_i D_{i,l,t} \quad \forall\, i,l,t \qquad (3.7)$$

Storage space constraint: Constraint (8) and (9) are storage capacity constraints at each manufacturing site and selling location respectively.

$$\sum_i \sum_t Qs_{i,s,t} \leq Ss_s \qquad \forall\, s \qquad (3.8)$$

$$\sum_i \sum_t Qm_{i,l,t} \leq Sm_l \qquad \forall\, l \qquad (3.9)$$

Nature of variables: Constraint (3.10) and (3.11) defines the nature of the variables.

$$y_{i,s,t} \in \{1, 0\} \qquad (3.10)$$

$$x_{i,s,t},\ Qs_{i,s,t},\ Qm_{i,l,t},\ Qt_{i,g,s,l,t},\ Qb_{i,l,t}, Nt_{g,s,l,t} \geq 0 \text{ and integer} \qquad (3.11)$$

Initial inventory and backorder value: Constraint (3.12) defines that initial inventory at manufacturing site and selling location and backorder quantity is assumed as zero.

$$Qs_{i,s,0} = Qm_{i,l,0} = Qb_{i,l,0} = 0 \qquad (3.12)$$

3.3 Illustrative Example

This Section illustrates the problem instances of case company and report analytical results computed by solver. The case company considered in this study is an automobile manufacturing company producing two wheeler motorcycles. The company has three plants geographically located at northern, western and southern parts of India. The company produces fifteen different models of the two wheelers at its manufacturing facilities, but to avoid complexity of the mathematical model only three product categories that are manufactured in all the three plants have been considered. The three products are distributed to four selling locations located at different parts of the country. Each manufacturing site is producing final products and has a limited storage space. The motorcycles are transported from manufacturing sites to selling locations by a heterogeneous fleet of vehicles each having a fixed loading capacity.

3.3.1 Input data values

The problem considered here is demonstrating a real world problem of small problem size and formulated in a stylised manner. There are three manufacturing sites producing three types of products and distributing them to four selling locations. The planning horizon is taken as three months having discrete time periods of one month. Due to confidentiality of exact data and for proper representation, input parameter values are scaled down and considered in a uniformly distributed manner. The value of different input parameters is shown in Table 3.1.

Table 3.1: Input data values of parameters

Parameter		**Value**
	Product 1	U (200-250)
Demand (units)	Product 2	U (220-300)
	Product 3	U (150-200)
Setup time (hour/setup)		U (0.75-1.5)
Processing time (hour)		U (1.9-2.1)
Production cost (Rs.)		U (10000-13000)
Carrying cost at a manufacturing site (Rs.)		U (50-60)
Carrying cost at selling location (Rs.)		U (40-50)
Variable Transportation cost (Rs.)		U (300-400), U(250-350)
Fixed transportation/vehicle cost (Rs.)		U (2000-10000), U(6000-30000)
Setup cost (Rs.)		U (13000-15000)
Backorder cost (Rs.)		U (11000-12000)
Transportation capacity of vehicles (units)		{40, 84}
Max storage at site and selling location (units)		{50, 100}
Fraction of demand backorder		0.5

3.3.2 Computational results and discussion

The proposed mathematical model was written and solved using CPLEX solver provided via IBM ILOG CPLEX 12.7 on a PC Intel Core i5 1.7 GHz and 4GB RAM. The solution of the proposed model identifies periods in which production occurs, optimal production quantity in each manufacturing site, optimal inventory quantity at manufacturing site and selling location, optimal transportation quantity and number of vehicles sufficient to fulfil the demand at minimum total cost.

Table 3.2: Computational complexity and results

Object		**Value**
Objective function (Total Cost)		91620983
Production cost		72025150
Backorder cost		14604991
Fixed transportation cost		2551125
Variable transportation cost		2104148
Setup cost		330420
Carrying cost at manufacturing site		1746
Carrying cost at selling location		3403
Number of variables	Binary	27
	Integer	447
Number of constraints		279
Solution time in CPU seconds		3813.5
Number of nodes		8519277
Number of iterations		39836323

The computational results are obtained to investigate the performance of the proposed mathematical model. Table 3.2 shows the computational complexity in terms of number of variables, constraints, iterations, nodes and solution time in CPU seconds and cost computations. It can be seen from Table 3.2 that after visiting 8519277 nodes, performing 39836323 iterations, and in 3813 CPU seconds; optimal solution is obtained. Total cost of the proposed model is calculated as Rs. 91620983 in which production, transportation and backorder cost are having the major contribution. Because of the high demand of the products it is observed that backorder quantity is more and inventory quantity at both manufacturing sites and selling locations is very less. Table 3.3 to Table 3.7 shows the optimum values of decision variables.

Table 3.3: Optimum value of production quantity at manufacturing site

Manufacturing Site	**M1**			**M2**			**M3**		
Period / Product	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**
P1	307	307	307	310	265	-	308	285	110
P2	304	304	304	310	310	310	313	313	-
P3	315	315	315	304	302	-	198	190	-

Table 3.3 shows the amount of products produced at each manufacturing site in each time period. For example the quantity of product 1, produced in manufacturing site 2 in first time period is 310.

Table 3.4: Optimum value of backorder quantity at selling location

Selling Location	**L1**			**L2**			**L3**			**L4**		
Period / Product	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**
P1	6	-	111	-	-	104	8	-	101	6	-	110
P2	-	8	-	38	61	116	-	-	111	41	-	100
P3	-	-	112	-	-	92	-	-	72	-	-	104

Table 3.4 shows the optimum quantity of backorder level at selling location of each product in the end of each time period. It can be observed from the table that product 2 is having maximum demand among all and that is why maximum

backorder quantity is for product 2. It can be seen that there is some backorder for each product which states that the demand is not fulfilled which results in a low amount of inventory at each selling location.

Table 3.5: Optimum value of quantity transported from manufacturing site to selling location

Vehicle	**Product**	**Manufacturing site**	**Selling Location**											
			L1			**L2**			**L3**			**L4**		
			Period											
			t1	**t2**	**t3**	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**
G1	**P1**	**M1**	-	-	-	-	-	11	68	100	101	-	-	-
		M2	-	-	-	237	211	-	73	54	-	-	-	-
		M3	-	-	-	-	-	-	74	61	-	-	-	26
	P2	**M1**	-	-	-	75	110	59	-	-	-	-	-	21
		M2	-	-	-	65	111	119	161	199	111	-	-	80
		M3	-	-	-	-	-	-	114	59	-	199	254	-
	P3	**M1**	80	160	112	5	10	88	92	140	59	-	-	59
		M2	-	-	-	67	200	-	142	55	-	-	-	-
		M3	-	-	-	-	-	-	12	-	-	-	66	12
G2	**P1**	**M1**	239	207	111	-	-	84	-	-	-	-	-	-
		M2	-	-	-	-	-	-	-	-	-	-	-	-
		M3	-	-	-	-	-	-	-	-	-	234	224	84
	P2	**M1**	229	194	224	-	-	-	-	-	-	-	-	-
		M2	-	-	-	84	-	-	-	-	-	-	-	-
		M3	-	-	-	-	-	-	-	-	-	-	-	-
	P3	**M1**	120	19	1	-	-	-	-	-	-	-	-	-
		M2	-	-	-	-	-	-	-	-	-	-	-	-
		M3	-	-	-	-	-	-	-	-	-	186	112	-

The amount of quantity transported from manufacturing site to selling location in each time period is shown in Table 3.5. It can be observed from the table that the maximum amount of product transported is equal to the maximum transportation quantity limit. For example the quantity of product 1 transported from manufacturing site 1 to selling location 1 in first time period is 100.

Table 3.6: Optimum number of vehicle type used for transportation

Vehicle	**Manufacturing site**	**Selling Location**											
		L1			**L2**			**L3**			**L4**		
		Period											
		t1	**t2**	**t3**	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**	**t1**	**t2**	**t3**
G1	**M1**	2	4	3	2	3	4	4	6	4	-	-	2
	M2	-	-	-	12	14	3	9	8	3	-	-	2
	M3	-	-	-	-	-	-	5	3	-	5	8	-
G2	**M1**	7	5	4	-	-	1	-	-	-	-	-	-
	M2	-	-	-	1	-	-	-	-	-	-	-	-
	M3	-	-	-	-	-	-	-	-	-	5	4	1

Table 3.6 shows the optimal number of both vehicle types and it is observed that vehicle type 1 is used more than the type 2 for minimum total cost.

One important thing in linear programming models of PD planning is the size of the problem. In this study, size of the problem is defined by number of products, number of sites, number of selling locations, number of vehicles and number of time periods. The problem instance solved in this study is demonstrating the real life situation but having a small problem size. To solve larger problems, either a suboptimal solution can be obtained by changing the % gap of CPLEX solver or a heuristic approach can be implemented.

In the proposed mathematical model, there was limit on total transported quantity according to vehicle capacity, but there was no limit on minimum transported quantity which results in high fixed transportation cost due to small amount of loading of vehicles. In practical scenario, increasing the vehicle transportation capacity decreases the requirement of number of vehicles and thus reduces the fixed transportation cost but it would not be consistent because the amount of quantity transported depends upon the product demand, production quantity and availability of storage at selling location.

3.4 Chapter Summary

This chapter considered a MSIPDP problem of a two echelon supply chain, formulated as a MILP model to minimise total cost including production, inventory, setup, backorder and transportation cost. A real life case of an automobile industry is considered and a stylised data set is generated with the purpose of providing details and demonstrates the performance of the model. The model includes most of the characteristics of the automobile industry such as consideration of setup, backorder and heterogeneous fleet of vehicles with distinct capacities. The computational results obtained using CPLEX solver for objective function and decision variables are tabulated. The outcomes of analytical results are discussed to drawn out some important managerial implications.

This chapter tries to fill the gaps by formulating a mathematical model considering major cost and capacity aspects of production as well as distribution processes. The scenario is related with the actual case of an automotive company for analysis of the problem and computational results. The proposed mathematical model and solution reflects the actual scenario up to a sufficient degree of reality and can be helpful to decision makers to effectively plan production and distribution activities in a multi-site manufacturing scenario.

To achieve better solutions in reasonable computational time and to solve large size problems, heuristics and metaheuristics algorithms can be applied. In addition to that, stochastic programming or fuzzy approach can be applied to handle uncertainty in objective function and parameters. The proposed mathematical model can be extended to include multiple conflicting objectives related with customer service level. Some of these issues are addressed in upcoming chapters.

Chapter 4

Lagrangian Relaxation for Multi-Site Production and Distribution Planning

4.1 Introduction

Integrated decision making is one of the most important aspects of supply chain management. Over the past years, there have been an increasing number of articles published on the IPDP problem. Chapter 3 presented the formulation of mathematical model for MSIPDP problem and solution using exact optimization technique with illustrative example. The results indicate that for large size problem, computational solver takes too much time or does not generate an optimal solution. For solving these large size industrial problems, a Lagrangian relaxation based heuristic algorithm is implemented in this study.

There are different approaches employed in the literature to solve the complex problems. There is a tradeoff between solution quality and acceptable computational time. Exact optimization methods such as Branch & Bound can generate optimal solutions for MIP problems, but it takes too much time to solve a complex problem. On the other hand, feasible solutions in less computational time can be obtained by heuristics approaches, which compromise on solution quality. Heuristic approaches are used to investigate two important problem aspects, generation of feasible solution and finding strong lower bounds. The purpose of this study is to formulate and solve large size MSIPDP problem. The heuristic algorithms implemented in this study are based on Lagrangian relaxation (LR) technique that incorporates the hard constraints into the objective function, resulting in an easy to solve subproblem. The feasibility of these subproblems is maintained by using two heuristic algorithms. The computational results are tested using problem instances generated by uniform distribution. The performance of these heuristics is evaluated by comparing with exact optimization results.

The rest of the chapter is structured as follows. Section 4.2 is devoted to describe background and overview of the problem considered. Section 4.3 discuss about the problem considered and formulation of mathematical model. Section 4.4 presents the methodology of implementation of Lagrangian heuristics and algorithms for feasibility establishment. Computational results using exact and heuristics approaches are reported in Section 4.5. Section 4.6 concludes the chapter.

4.2 Background and overview

Due to the complexity of the MSIPDP mathematical formulation, several heuristic algorithms have been implemented in literature to solve large size industrial problems. Kuno & Utsunomiya (2000) solved a production-transportation problem using Lagrangian relaxation based branch and bound algorithm. The computational results indicate that the algorithm can solve large size problems. Wu and Golbasi (2004) proposed a multi-product multi facility supply chain planning problem. A Lagrangian decomposition scheme was implemented where the original model is decomposed into single item subproblems. Effectiveness of shortest path algorithm in comparison to subgradient search algorithm was analysed. Park (2005) dealt with the problem of IPDP in a multi-plant, multi-retailer, two echelon supply chain. The proposed model was solved using two phase heuristics approach having local improvement procedure coupled with load shifting among periods. Advantage of integrated approach over decoupled one was shown in computational results. Lei et al. (2006) proposed a MIP model for single item, multi-period, multi-plant integrated production and distribution routing problem with heterogeneous transportation. The computational results of two phase heuristics approach was compared with CPLEX solver results and it was found that heuristics gives same/better results in less computational time. Ekşioğlu et al. (2007) presented an IPDP problem in a two echelon supply chain formulated as MILP model. The proposed model was solved using Lagrangian decomposition based heuristics. Primal-dual and dynamic programming algorithms were used to solve subproblems and results were compared with optimal solution of CPLEX solvers for small size problems.

Lidestam and Ronnvist (2011) applied a lagrangean decomposition heuristic for IPDP problem of a pulp company supply chain. The problem was formulated as MILP including tactical decision related with transportation of raw material, production and distribution of products. Kanyalkar and Adil (2011) dealt with multi-site integrated procurement, production and distribution planning problem and formulated a mixed integer programming model. Aggregate and detailed level planning was presented under a rolling schedule. Wei et al. (2017) proposed a tactical PD planning model for a two stage production process. Two decomposition based heuristics; relax and fix and variable neighbourhood search were applied to solve the problem. The convergence of computational results was compared with branch and cut results obtained using CPLEX solver. Bajgiran et al. (2016) dealt with lumber supply chain tactical planning problem formulated as MIP model. To solve such complex problem, Lagrangian relaxation based heuristic algorithm is applied in which multiplier value is updated by subgradient optimization technique. The integrated model was compared with decoupled model which has shown profit improvement using integrated approach. Consideration of heterogeneous transportation with limited number of vehicle having a fixed loading capacity is presented by Feng et al. (2017). They addressed the problem of coordinated production and distribution planning and formulated a MILP model. The problem was solved using decomposition and Lagrangian relaxation based heuristics and results were compared with exact solution of CPLEX solver, which shown the efficiency of applied heuristic methods.

Apart from the single heuristics approach, few articles have presented hybrid solution approaches such as Melo and Wolsey (2012) proposed a MIP formulation for a two level production-distribution problem solved using hybrid heuristics based on relaxation induced neighbourhood search and local branching. Camacho-Vallejo et al. (2014) dealt with production and distribution planning problem considering three echelons; plants, distribution centre and retailers of supply chain. A bilevel mathematical problem is presented and solved using scatter search based heuristic algorithm. The computational results were compared with similar studies in literature and found to be efficient.

Table 4.1: Summary of relevant studies on MSIPDP problem using heuristics.

Author year	Multi-product	Multi-period	Production capacity	Setup		Vehicle fleet		Model	Solution
				Cost	Time	Heterogeneous transportation	Constrained Capacity		
Kuno & Utsunomiya (2000)			✓					MIP	Lagrangian relaxation heuristic
Wu & Golbasi (2004)	✓	✓	✓	✓	✓			MIP	Lagrangian decomposition heuristic
Park (2005)	✓	✓	✓	✓	✓			MIP	Two Phase heuristic
Lei et al. (2006)		✓	✓			✓	✓	MIP	Two Phase heuristic
Ekşioğlu et al. (2007)	✓	✓	✓	✓				MILP	Lagrangian decomposition heuristic
Kanyalkar & Adil (2011)	✓	✓	✓					MIP	Rolling schedule
Lidestam & Ronnvist (2011)	✓	✓	✓	✓				MILP	Lagrangian decomposition heuristic
Melo & Wolsey (2012)	✓	✓	✓	✓	✓		✓	MIP	Hybrid heuristic
Camacho-Vallejo et al. (2014)			✓					LP	Heuristic
Darvish et al. (2016)	✓	✓	✓	✓				ILP	Branch and Bound
Bajgiran et al. (2016)	✓	✓	✓			✓	✓	MIP	Lagrangian relaxation heuristic
Wei et al. (2017)	✓	✓	✓	✓	✓			MILP	Decomposition heuristic
Feng et al. (2017)		✓		✓		✓	✓	MILP & NLP	Lagrangian relaxation and decomposition heuristics
Current study	✓	✓	✓	✓	✓	✓	✓	MIP	Lagrangian relaxation heuristic

To summarise existing research work, a categorization is done on the basis of aspects of problem, model formulation and solution approach employed, as shown in Table 4.1. It can be observed from the table that most of the previous studies assumed homogeneous transportation with unlimited capacity of vehicles. In practical scenario, heterogeneous transportation is used because of flexibility and cost effectiveness (Feng et al. 2017). Current study considers all the mentioned problem aspects and employed Lagrangian relaxation approach to handle complexity of the resulting formulation.

4.3 Problem formulation

In this study, MSIPDP problem is addressed considering setup time/cost in production and fleet of heterogeneous transportation with constrained capacity in transportation. The description of problem, assumptions, notations and model formulation is already explained in Section 3.2. In the current study, backlogging situation is not considered. The problem is formulated as MILP model as follows:

$$Minimum\ total\ cost = \sum_i \sum_s \sum_t Pc_{i,s} x_{i,s,t} + \sum_i \sum_s \sum_t Sc_{i,s} y_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_g\, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l}\, Qt_{i,g,s,l,t} \quad (4.1)$$

$$Qs_{i,s,t} = Qs_{i,s,t-1} + x_{i,s,t} - \sum_g \sum_l Qt_{i,g,s,l,t} \quad \forall\, i, s, t \quad (4.2)$$

$$Qm_{i,l,t} = Qm_{i,l,t-1} + \sum_g \sum_s Qt_{i,g,s,l,t} - D_{i,l,t} \quad \forall\, i, l, t \quad (4.3)$$

$$x_{i,s,t} \le M.\, y_{i,s,t} \quad \forall\, i, s, t \quad (4.4)$$

$$\sum_i (x_{i,s,t} Pt_{i,s} + St_{i,s,t}\, y_{i,s,t}) \le Pcp_{s,t} \quad \forall\, s, t \quad (4.5)$$

$$\sum_i Qt_{i,g,s,l,t} \le Tc_g . Nt_{g,s,l,t} \quad \forall\, g, s, l, t \quad (4.6)$$

$$\sum_i \sum_t Qs_{i,s,t} \le Ss_s \quad \forall\, s \quad (4.7)$$

$$\sum_i \sum_t Qm_{i,l,t} \le Sm_l \quad \forall\, l \quad (4.8)$$

$$y_{i,s,t} \in \{1, 0\} \quad (4.9)$$

$$x_{i,s,t},\ Qs_{i,s,t},\ Qm_{i,l,t},\ Qt_{i,g,s,l,t},\ Nt_{g,s,l,t} \ge 0 \text{ and integer} \quad (4.10)$$

$$Qs_{i,s,0} = Qm_{i,l,0} = 0 \quad (4.11)$$

The complexity of the problem implies that the solution using standard solver will not be possible, especially with the large size of the problem. To handle this issue, a LR based approach is implemented to solve medium and large size problems in less computational time. Solution methodology of implementation of heuristics is descried in next Section.

4.4 Solution methodology

Lagrangian relaxation was first implemented by Held and Karp (1970, 1971) to solve large size optimization travelling salesman problems. The idea is to relax the set of hard constraints by adding them into objective function with a penalty, thus makes the problem easier to solve. The solution provides lower bound for the original problem in case of minimization problem. Upper bound of the problem is obtained by maintaining the feasibility of solution using a heuristic algorithm. The solution obtains when gap of lower bound and upper bound is reasonably small or the number of iterations reaches to a predefined number. Lagrangian dual aims to find the highest lower bound value for the MIP problem over all Lagrangian relaxations. LR methodology employed in this study is shown in Figure 4.1.

Two Lagrangian relaxation algorithms are employed in this study to compute lower bounds on the objective function values because of minimization problem. The hard constraints are added in the objective function with a given vector of non-negative multipliers called as Lagrangian multipliers. To iterate the value of multiplier and to obtain best lower bound value, subgradient optimization method is used, which is a most widely used method in the literature because it is easy to program and has performad well on many real life problems (Fisher 1981). The relaxed problems have infeasibility in most of the cases. The feasibility of the solutions is maintained by employing Lagrangian heuristics algorithms. Section 4.4.1 presents relaxation of production part and Section 4.4.2 for distribution part of the problem. Section 4.4.3 discuss about Lagrangian dual solution using subgradient optimization method. Section 4.4.4 discuss about Lagrangian heuristics used to maintain the feasibility of the relaxed problem.

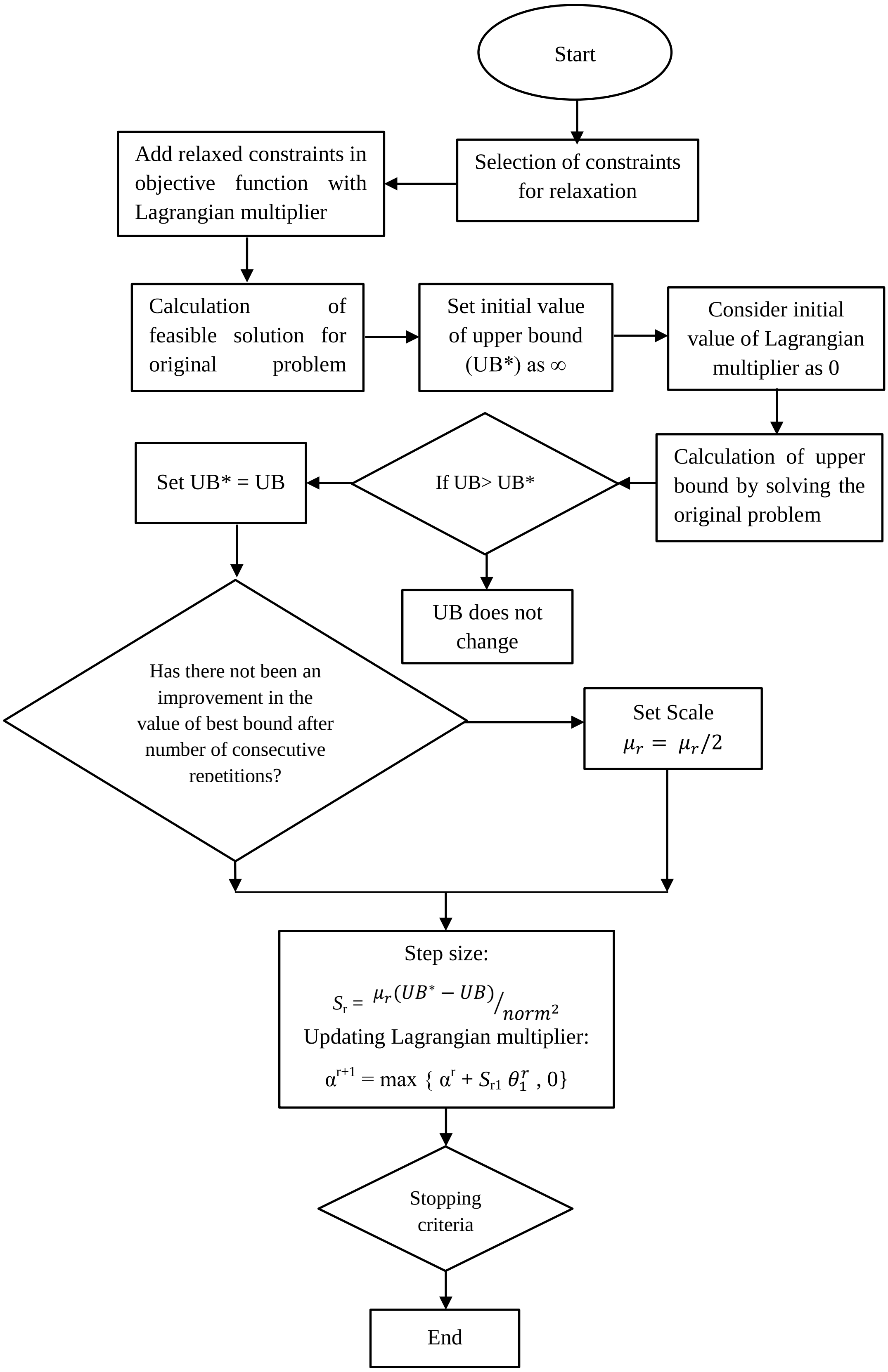


Figure 4.1: Lagrangian relaxation methodology of current study (adapted from Rafie-Majd 2018)

4.4.1 Lagrangian Relaxation for production

Production capacity constraint (4.5) is relaxed and added in the objective function, thus leading to the following relaxed problem:

Minimize $Z_{LR}(\lambda) = \sum_i \sum_s \sum_t Pc_{i,s} x_{i,s,t} + \sum_i \sum_s \sum_t Sc_{i,s} y_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_g \, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l} \, Qt_{i,g,s,l,t} + \alpha \, (X_{i,s,t} \, Pt_{i,s} + St_{i,s} Y_{i,s,t} - Pcp^{s,t})$

Constraint (4.2 – 4.11) and

$\alpha \geq 0.$ (4.12)

Where α is vector of Lagrange multiplier.

4.4.2 Lagrangian Relaxation for transportation

Transportation capacity constraint (4.6) is relaxed and added in the objective function, thus leading to the following relaxed problem:

Minimize $Z_{LR}(\lambda) = \sum_i \sum_s \sum_t Pc_{i,s} x_{i,s,t} + \sum_i \sum_s \sum_t Sc_{i,s} y_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_g \, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l} \, Qt_{i,g,s,l,t} + \beta \, (Qt_{p,g}^{m,l,t} - Tc_g . Nt_g^{m,l,t})$

Constraint (4.2 – 4.11) and

$\beta \geq 0.$ (4.13)

Where β is vector of Lagrange multiplier.

4.4.3 Lagrangian dual solution – Subgradient optimization

Subgradient optimization method is used to find vectors of Lagrangian multipliers that give the lower bound to near optimality. For a particular value of α and β, the feasible solution of LR(α) and LR(β) is optimal solution. There is no other way of proving optimality in subgradient method; because of this the method is terminated after some defined iteration limit. For detailed description about the method, refer Fisher (1981).

[AL1] max $Z_L(\alpha)$

s.to. $\alpha \geq 0$

and max Z_L (β)

s.to. $\beta \geq 0$

at iteration r, subgradient vectors θ^r is determined by

$$\theta_1^r = x_{i,s,t}{}^* Pt_{i,s} + St_{i,s,t}\, y_{i,s,t} - Pcp_{s,t}$$

$$\theta_2^r = Qt_{i,g,s,l,t}{}^* - Tc_g . Nt_{g,s,l,t}$$

where $x_{i,s,t}{}^*$ and $Qt_{i,g,s,l,t}{}^*$ are the optimal solutions of LR(α) and LR(β) obtained at iteration r.

Multiplier at iteration r, α^r, for the next generation multipliers α^{r+1}, is generated by

$$\alpha^{r+1} = \max\ \{ \alpha^r + S_{r1}\ \theta_1^r\ , 0\}$$

In the same manner, multiplier at iteration r, β^r, for the next generation multipliers β^{r+1}, is generated by

$$\beta^{r+1} = \max\ \{ \beta^r + S_{r2}\ \theta_2^r\ , 0\}$$

where S_{r1} and S_{r2} are positive scalar step sizes. This is obtained as-

$$S_{r1} = \mu_r (Z^* - Z_L(\alpha^r)) \Big/ ||\theta_1^r||^2$$

$$S_{r2} = \mu_r (Z^* - Z_L(\beta^r)) \Big/ ||\theta_2^r||^2$$

Where $0 < \mu_r \leq 2$, is a positive scalar, Z^* is an upper bound on optimal solution of $Z_L(\alpha)$ and Z_L (β). $||\theta_1^r||$ and $||\theta_2^r||$ denotes the Euclidean norm of vector θ^r. Z^* is usually obtained by applying a heuristic technique. The initial value of scalar is set as 2 and halved whenever the lower bound value fails to improve within a fixed number of iterations. There are multiple ways to stop this iteration process, when the gap between lower and upper bound value reaches to a predefined value, the number of specified iteration completed or when scalar value reaches to a specified close to zero value.

4.4.4 Lagrangian Heuristics

The relaxed problem may not satisfy the capacity constraints and thus leads to an infeasible solution. In order to provide the algorithm an upper bound, the infeasible solution needs to be converted into a feasible one. This Section presents two Lagrangian heuristics algorithms to solve production and distribution subproblems to find feasible solutions of the upper bound problem.

Top down heuristics

Solution methodology using lot shifting heuristic algorithm is as follows:

Step 1. Solve the uncapacitated PD planning problem with initial Lagrangian multiplier as zero. The solution of this problem will give production, inventory and transportation quantities at multiple sites and selling locations.

Step 2. The sum of production time and setup time gives value of production capacity consumption. If capacity consumption is less than the available production capacity, a feasible solution will be obtained and algorithm will stop, otherwise a capacity violation is exist and need to move to next step.

Step 3. Set the values for initial Lagrangian multiplier, step size and number of iterations.

Step 4. A lot shifting algorithm needs to be employed in which production quantities are shifted among periods. This is done into two phases, a backward phase and then a forward phase. In backward phase, products are transferred to previous time period to eliminate overtime capacity in pair of plant period of production capacity. Next in forward phase, transfer is done in subsequent periods.

Algorithm 1 below shows the pseudocode for the production capacity feasibility strategy.

Algorithm 1: Feasibility establishment of production capacity

---

```
Data: Approximate dual solution
Result: Either a heuristic feasible solution or an infeasible solution
repeat
  //Backward phase
  for t = T to 2 with excess capacity
    repeat
    identify the plant s with excess capacity in period t;
    repeat
    Update production variables by performing the transfer
    Until production capacity (s,t) does not represent excess capacity;
  if production variables is feasible then
    return production variables
  else
  //Forward phase
  for t = 1 to T-1
    repeat
      Identify the plant s with excess capacity in period t;
    repeat
    Update production variables by performing the transfer
    Until production capacity (s,t) does not represent excess capacity;
  If production variables is feasible then
    Return production variables;
  Else
Until the number of iteration has not been reached;
```

Step 5. Apply the sub-gradient method to update value of Lagrangian multiplier until the maximum number of iteration reached. If still solution not found then it is said that solution cannot be found.

Step 6. Check again for production capacity violation, if violation do not exist, a feasible solution will be obtained, otherwise go to step 4.

Step 7. The values of quantities are obtained and objective function value will be computed.

Bottom Up heuristics

Solution methodology using minimum transport cost algorithm

Step 1. Solve the uncapacitated production distribution planning problem with initial Lagrangian multiplier as zero. The solution of this problem will give production, inventory and transportation quantities at multiple sites and selling locations.

Step 2. Check for the transportation capacity. If the transportation capacity is less than the available capacity, a feasible solution will be obtained and algorithm will stop, otherwise a capacity violation is exist and need to move to next step.

Step 3. Set the values for initial Lagrangian multiplier, step size and number of iterations.

Step 4. A minimum transport cost algorithm needs to be employed in which products having minimum cost of transportation are employed. Sort the products in increasing order of transportation cost and then calculate optimal number of transportation quantities.

Step 5. Apply the sub-gradient method to update value of Lagrangian multiplier until the maximum number of iteration reached. If still solution not found then it is said that solution cannot be found.

Step 6. Check again for transportation capacity violation, if violation does not exist, a feasible solution will be obtained, otherwise goes to step 4.

Step 7. The values of quantities are obtained and objective function value will be computed.

Algorithm 2 below shows the pseudocode for the production capacity feasibility strategy.

Algorithm 2: Feasibility establishment of transportation capacity

---

```
Data: Approximate dual solution
Result: Either a heuristic feasible solution or an infeasible solution
repeat
  for t = 1 to T
     if Qt_{i,g,s,l,t} ≤ Tc_g.Nt_{g,s,l,t} transportation variable is feasible
        return
        update production and transportation variables;
     else
   //Feasibility establishment phase
     Sort the products in decreasing order of transportation cost
     Set P_s = 1....P and obtain optimal values of transportation quantity
     If transportation variables is feasible then
        Return production and transportation variables;
     Else
Until the number of iteration has not been reached;
```

4.5 Computational results

Using the real life data set of automobile manufacturing company, random data instances are generated that reflect the characteristics of the real life scenario as much as possible. Input data of all the parameters used in mathematical model are generated following the data provided by company and preliminary studies. A total 20 number of problem instances of small and large categories are generated using Uniform distribution following the data generation scheme of Park (2005). The sets of test problems are given in Table 4.2. Demand in each period is generated from interval [170, 250], processing time is generated from interval [5, 8]. Production cost is generated according to type of product from interval [13000, 17000]. Holding cost at site is between [20, 30] and at selling location is between

[30, 40], setup time is between [100, 120] and setup cost is between [60000, 80000]. Fixed transportation cost varies according to the capacity of vehicle, e.g. for a vehicle having 40 units loading capacity assigned fixed cost of 5000, another vehicle of 84 units loading capacity is having fixed cost of 10000. Variable transportation cost is decided on the basis of distance between manufacturing site and selling location on map. Production capacity is calculated following the generation scheme of Carvalho (2016), as follows:

$$Pcp_{s,t} = \left[\sum_{i=1}^{I}\sum_{s=1}^{S}\sum_{t=1}^{T}\frac{D_{i,l,t}\,Pt_{i,s} + St_{i,s}Y_{i,s,t}}{T}\right]$$

Storage capacity at site and at selling location is determined on the basis of total demand in a period as suggested by Park (2005).

$$Ss_s = \frac{\max_t \sum_l D_{i,l,t}}{0.5} \quad \text{and} \qquad Sm_l = \frac{\max_t \sum_l D_{i,l,t}}{0.8}$$

Table 4.2: Set of test problem

S.No.	Sets	Description
1	I	{3 - 15}
2	S	{3,4,5,6,7}
3	L	{3,5,7,9,11,13,15,18,20}
4	G	{2,3,4,5}
5	T	{3,4,6,9,12}

The combination of sets showing configuration and the complexity of problem instances in terms of number of variables and constraints are presented in Table 4.3.

Table 4.3: Configuration and complexity of problem instances

Problem no.	Number of nodes		Test problem specifications			No. of variable	No. of constraints
	S	***L***	***I***	***G***	***T***		
S1	3	3	3	2	3	168	342
S2	3	3	3	2	4	216	450
S3	3	5	4	3	3	316	875
S4	3	5	4	3	4	408	1156
S5	4	5	5	3	4	570	1825
S6	4	7	5	3	4	718	2451
S7	4	7	6	3	4	789	2874
S8	4	7	6	4	4	901	3658
L9	4	7	6	4	6	1289	5454
L10	5	9	7	4	6	2020	9746
L11	5	9	8	4	6	2148	10984
L12	5	9	8	4	9	3159	16420
L13	5	11	9	4	9	3886	22050
L14	6	11	10	4	9	4687	28916
L15	6	13	10	4	9	5321	33868
L16	6	13	11	5	9	6267	45398
L17	6	15	12	5	12	9633	75204
L18	7	15	13	5	12	11216	94102
L19	7	18	14	5	12	13395	120302
L20	7	20	15	5	12	15036	142185

*S –small size problems, L – Large size problems

Table 4.4: Comparison of lower bound values for small size problem instances

S.no.	Exact optimization		Top down heuristics (TDH)			Bottom up heuristics (BUH)		
	Obj. value	Runtime (sec)	Lower bound	Runtime (sec)	% gap	Lower bound	Runtime (sec)	% gap
S1	48508804	5.82	47668474	1.56	1.73	47954410	0.19	1.14
S2	64336781	10.55	63058688	19.25	1.99	62843169	0.50	2.32
S3	175936073	3.21	169894208	4.15	3.43	172320616	0.22	2.05
S4	230677251	7.13	225553648	9.58	2.22	226489200	1.34	1.82
S5	300290408	151.31	296150475	75.49	1.38	296295089	0.55	1.33
S6	424742906	277.28	423239051	166.03	0.35	422160842	2.1	0.61
S7	505846375	379.03	498533866	312.22	1.45	497981451	3.76	1.55
S8	502015538	400.23	493006190	298.41	1.79	495712267	2.50	1.26

Table 4.5: Computational results for large size results

S.no.	Top down heuristics (TDH)				Bottom up heuristics (BUH)				CPLEX Solution
	Lower bound	Upper bound	% gap	Runtime (sec)	Lower bound	Upper bound	% gap	Runtime (sec)	
L9	748448134	778386059	3.85	75.21	742050709	756891723	1.96	7.53	748449745
L10	1104448862	1159671305	4.76	150.45	1094761784	1111183211	1.48	10.58	1104472688
L11	1259339632	1309713217	3.85	199.69	1248344350	1263324482	1.19	27.28	1259330692
L12	1896064005	1952945925	2.91	252.36	1879429998	1911568251	1.68	15.24	1896061295
L13	2596402411	2726222532	4.76	350.12	2573592121	2580798179	0.28	9.82	2596414325
L14	2835866554	2920942551	2.91	354.66	2810168159	2876488128	2.31	45.12	2836019630
L15	3346755206	3463891638	3.38	450.71	3316441347	3378458800	1.84	36.17	3346839146
L16	3678735202	3811169669	3.47	454.75	3644843959	3696236259	1.39	40.47	3678711607
L17	6197634368	6445539743	3.85	534.2	6141513931	6299964990	2.52	50.25	6197061566
L18	6740760065	7044094268	4.31	644.75	6680081455	6804998978	1.84	65.74	6741167003
L19	8688343716	9105384214	4.58	541.28	8610196561	8713518920	1.19	66.96	8687874704
L20	10325525399	10759197466	4.03	737.24	10232469007	10329677463	0.94	115.47	10328371483

All the problem instances are run on a PC Intel Core i5 on a computer Intel Core i5 1.7 GHz with 4GB RAM. All the formulations and algorithms are implemented and solved using CPLEX solver provided via IBM ILOG CPLEX 12.7. Table 4.4 shows the total cost, solution time and % gap between optimal solution obtained by exact optimization, and lower bound values obtained by top down heuristics (TDH) and bottom up heuristics (BUH). The quality of solution is evaluated by calculating % gap as follows:

$$\% \, gap = \frac{Cplex - LB}{Cplex} \times 100$$

It can be seen from the table that both the heuristics give good results in terms of computation time. The overall average % gap from *TDH* is 1.79% and from *BUH* is 1.51%. The average % gap and computational time is less in case of *BUH* which shown superiority of relaxation of transportation constraints.

The generation of optimal solution for large size instances with solver takes large amount of time and usually has short of memory. Therefore, for large size problem instances, the performances of heuristics are evaluated based on % gap between the two heuristics approaches. The feasible solution for original problem are obtained using CPLEX solver in constrained CPU time of 1 hour. The computational results for this comparison are shown in Table 4.5. The qualities of solution of proposed heuristics are evaluated by calculating % gap as follows:

$$\% \, gap = \frac{UB - LB}{UB} \times 100$$

The average cost gap for large size problem instance using *TDH* is 3.89% and using *BUH* is 1.55%, which demonstrates that the two heuristics are efficient and comparable. The computation time is very less in comparison to direct solution using solver. Among the two heuristics, *BUH* is taking less time in obtaining a feasible solution and % gap is also less. The reason might be that constraint related to vehicle capacity is having more tightness as compared to production capacity constraint.

4.6 Chapter Summary

This chapter has analyzed a multi-product, multi-site integrated production and distribution planning problem. The problem is formulated as MILP model and solved directly using CPLEX solver. The solver provides optimal solutions for small sized instances but for medium or large size problems solver takes too much time or does not generate solution due to limitation of available memory. To solve large size problems, two Lagrangian relaxation based heuristic approaches are implemented that compute lower and upper bounds on the optimal solution value. Subgradient optimization method is used to find best lower bound value of Lagrangian dual problems. The computational performances of the algorithms are compared with exact optimization results which indicate that both heuristics are comparable and provide efficient results.

Chapter 5

Multi-Site Integrated Production and Distribution Planning: A Multi-Objective Approach

5.1 Introduction

In the real life situation, there is more than one criterion for a successful supply chain. Even if all the options for defining an optimal solution are considered in planning of the supply chain, it is unlikely that any one criterion defines the best optimal solution of the system. Instead, there will be a variety of acceptable trade-offs between multiple objectives. There is no single optimal solution for these multi-objective optimization problems, they creates a pareto-optimal solution which is a set of trade-off between all the conflicting objectives and their solutions. For decision making in supply chain, performance measures related with cost, customer service and responsiveness are of major importance. In this chapter, multiple objective functions are taken into consideration to represent it close to real life situation.

In the literature, minimisation of cost and maximisation of profit are the most commonly used objectives for PD planning problems. The components of commonly represented total cost are production, setup, inventory, and transportation cost. Another important measure of effective production and distribution planning is 'Responsiveness'. Responsiveness indicates quick changes to fulfil the customer needs and meeting of market demands. Cost and responsiveness are conflicting with each other. To have higher responsiveness, there is need for expedite distribution which leads to higher cost.

The ultimate goal of supply chain is to fulfil the demands of customer which makes customer service level as another important criterion. It measures on time satisfaction of customer demand. A low customer service level represents higher loss of sales or backorder level, which results in loss of profit (Liu and Papageorgiou, 2013). It is observed from the literature survey that very few articles have considered cost, responsiveness and customer service level simultaneously. In this study, these three objective functions are considered to formulate a multi-objective multi-site integrated production and distribution planning (MO-MSIPDP) model.

This chapter aims to achieve the second objective of this research work i.e. formulation of multi-objective mathematical model for MSIPDP problem. The problem handled in this study is illustrated by a case of an automobile industry located in India. The input parameter values are scaled down for proper demonstration of results and to maintain the confidentiality. The outcome of the mathematical model would determine optimum quantity of production, inventory level at manufacturing site and selling location, transported quantity from manufacturing site to selling location using each of the vehicle and unfulfilled demand or backordered quantity at each selling location at minimum level of all the three objective functions. The formulation and solution of production and distribution planning problem in multi-site manufacturing environment considering multiple conflicting objectives is presented in upcoming sections.

5.2 Problem description and Mathematical formulation

This section provides a general MILP formulation for MO-MSIPDP problem. The description of case along with supply chain network is presented in Section 3.2. The objective is to plan production and distribution quantities in order to satisfy the demand of products at each selling location at minimum total cost (includes production, setup, inventory holding, transportation and backorder cost), delivery time and backorder level. The assumption and notations are same as presented in Section 3.2.1 and 3.2.2.1 with following additional parameters and variables.

Additional Parameters:

$DT^{s,l}$ Transport time from manufacturing site *m* to selling location *l*

Additional Decision variables:

Al_1 Aspiration level of total cost goal

Al_2 Aspiration level of delivery time goal

Al_3 Aspiration level of backorder level goal

Auxiliary variables:

Al_1^+ Deviation of overachievement of Al_1

Al_1^- Deviation of underachievement of Al_1

Al_2^+ Deviation of overachievement of Al_2

Al_2^- Deviation of underachievement of Al_2

Al_3^+ Deviation of overachievement of Al_3

Al_3^- Deviation of underachievement of Al_3

Using above mentioned parameters and variables, initially a MILP model is formulated considering three conflicting objective functions. Later, a goal programming model is formulated to deal with all objective function simultaneously.

Objective functions:

The objective functions of the mathematical model are:

- Minimising total cost is the first objective function comprising production and inventory holding cost at manufacturing sites and selling locations, fixed and variable transportation cost between manufacturing sites and selling locations, setup cost at manufacturing sites and backorder cost for unfulfilled demand, given in the following expression.

$$\textit{Minimum total cost} = \sum_i \sum_s \sum_t Pc_{i,s} X_{i,s,t} + \sum_i \sum_s \sum_t Sc_{i,s} Y_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_{g,s,l}\, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l}\, Qt_{i,g,s,l,t} + \sum_i \sum_l \sum_t Bc_{i,l} Qb_{i,l,t} \quad (5.1)$$

- Minimization of total delivery time, defined as on time delivery of products transported from manufacturing sites to selling locations is taken as second objective function.

$$\textit{Minimum distribution time} = \sum_i \sum_g \sum_s \sum_l \sum_t (DT_{g,s,l} / TC_g)\, Qt_{i,g,s,l,t} \quad (5.2)$$

- Minimization of backorder level is considered as third objective function which depicts the customer satisfaction through minimization of the sum of maximum backlog quantities in all periods.

$$\textit{Minimum backorder level} = \sum_i \sum_l \sum_t Qb_{i,l,t} \quad (5.3)$$

Above three objective functions are conflicting in nature. Increasing the responsiveness decreases the cost efficiency because of large amount of inventory holding cost. To minimize backorder level, the quantity of transportation should be increased which will increase delivery time and distribution cost and vice versa. The constraints of above MILP model are same as the constraints described in Section 3.2.2.3.

5.3 Solution approach

Many solution approaches have developed for treating deterministic multi-objective optimization problems to get pareto-optimal solutions. Among them, goal programming, weighted sum and ε-constraint methods are most commonly implemented for converting multi-objective vector into scalar one. The scalar model can be solved by any programming solver or exact optimization techniques. Goal programming method developed by Charnes and Cooper (1957) is a well-known approach for solving multi-objective optimisation problems. A variance of goal programming method known as preemptive goal programming (PGP) is implemented in this study.

Preemptive goal programming (PGP) is a special case of goal programming (GP) method, in which goals or objective functions are optimised according to their priority level. In PGP, decision maker is able to set priorities of goals and provide target levels of achievement for each objective function. It looks for a solution that satisfies as many goals as possible according to their specified priority (Baykasoğlu, 2005). The approach tries to find a solution which decreases the deviation between achievement level and aspiration level of goals. Two auxiliary variables (positive and negative) represent the deviation from target value of goals. The goals of higher priority receive first attention and then lower priority goals are attended.

The mixed integer programming formulation of MSIPDP problem is transformed into PGP model formulation as follows:

$$Min\ P_1 Al_1^+ + P_2 Al_2^+ + P_3 Al_3^+$$

Subjected to,

$$\sum_i \sum_s \sum_t Pc_{i,s} X_{i,s,t} + \sum_i \sum_s \sum_t Sc_{i,s} Y_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_g\ Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l}\ Qt_{i,g,s,l,t} + \sum_i \sum_l \sum_t Bc_{i,l} Qb_{i,l,t} - Al_1^+ + Al_1^- = Al_1 \quad (5.4)$$

$$\sum_i \sum_g \sum_s \sum_l \sum_t (DT_{g,s,l} / TC_g)\ Qt_{i,g,s,l,t} - Al_2^+ + Al_2^- = Al_2 \quad (5.5)$$

$$\sum_i \sum_l \sum_t Qb_{i,l,t} - Al_3^+ + Al_3^- = Al_3 \quad (5.6)$$

$$Qs_{i,s,t} = Qs_{i,s,t-1} + x_{i,s,t} - \sum_g \sum_l Qt_{i,g,s,l,t} \quad \forall\ i,s,t \quad (5.7)$$

$$Qm_{i,l,t} - Qb_{i,l,t} = Qm_{i,l,t-1} - Qb_{i,l,t-1} + \sum_g \sum_s Qt_{i,g,s,l,t} - D_{i,l,t} \quad \forall\ i,l,t \quad (5.8)$$

$$x_{i,s,t} \leq M.y_{i,s,t} \quad \forall\ i,s,t \quad (5.9)$$

$$\sum_i (x_{i,s,t} Pt_{i,s} + St_{i,s,t}\ y_{i,s,t}) \leq Pcp_{s,t} \quad \forall\ s,t \quad (5.10)$$

$$\sum_i Qt_{i,g,s,l,t} \leq Tc_g . Nt_{g,s,l,t} \quad \forall\ g,s,l,t \quad (5.11)$$

$$Qb_{i,l,t} \leq \theta_i D_{i,l,t} \quad \forall\ i,l,t \quad (5.12)$$

$$\sum_i \sum_t Qs_{i,s,t} \leq Ss_s \quad \forall\ s \quad (5.13)$$

$$\sum_i \sum_t Qm_{i,l,t} \leq Sm_l \quad \forall\ l \quad (5.14)$$

$$y_{i,s,t} \in \{1, 0\} \quad (5.15)$$

$x_{i,s,t}$, $Qs_{i,s,t}$, $Qm_{i,l,t}$, $Qb_{i,l,t}$, $Qt_{i,g,s,l,t}$, $Nt_{g,s,l,t} \geq 0$ and integer (5.16)

$$Qs_{i,s,0} = Qm_{i,l,0} = Qm_{i,l,0} = 0 \quad (5.17)$$

$Al_1^+, Al_1^-, Al_2^+, Al_2^-, Al_3^+, Al_3^- \geq 0$ and integer (5.18)

P_1, P_2 and P_3 ($P_1 > P_2 > P_3$) represent the level of priority for goals specified by management with P_1 as the highest priority and P_3 as the lowest. The objective of preemptive model is to minimise the deviation from aspiration levels or goals. In this formulation, all the objective functions direction are minimisation, therefore, the undesirable deviation variable in all objective functions are overachievement of the goal or positive deviation variable. The deviation with highest priority must first be minimised to the extent possible and then next highest priority in order. The proposed PGP model can be solved by using any linear programming solver.

5.4 Illustration of the problem

Input parameter values are generated in the similar fashion as described in Section 3.3.1. Input value of an additional parameter i.e. delivery time is generated as U (2-14) hours. The proposed mathematical model was written and solved using CPLEX solver provided via IBM ILOG CPLEX 12.7 on a PC Intel Core i5 1.7 GHz and 4GB RAM. The branch and cut algorithm of CPLEX terminates only if an optimal solution of the problem is found.

The target value or aspiration levels of objectives can be obtained by consulting the company's management or can be computed by solving single objective models. In this study, the aspiration level is computed by solving single objective models individually. The computed target values are {91477427; 545.7; 158}. The preemptive goal programming model consists of 282 constraints and 480 variables out of which 27 are binary, 447 are integer and 6 are deviation variables. The highest priority goal is minimisation of overachievement of total cost followed by delivery time and backorder level. Using the target values, the PGP model is formulated as discussed in Section 5.3. Objective function values after computation of the preemptive goal programming model are {91477427; 704.5; 1289}. Optimum values of decision variables are shown in Table 5.1, to Table 5.4.

Table 5.1: Solution obtained for production quantity at manufacturing site

Product	Manufacturing site	Time period		
		1	2	3
1	1	307	307	307
	2	310	296	0
	3	308	280	84
2	1	304	304	304
	2	314	314	314
	3	313	313	0
3	1	315	315	315
	2	305	304	0
	3	198	187	0

Table 5.2: Solution obtained for backorder quantity at selling location

Product	Selling location	Time period		
		1	2	3
1	1	2	0	111
	2	0	0	104
	3	15	0	101
	4	3	0	110
2	1	0	4	0
	2	22	57	115
	3	0	0	104
	4	53	0	96
3	1	0	0	112
	2	0	0	92
	3	0	0	72
	4	0	0	104

Table 5.1 shows the optimum level of production quantity of product *i*, manufactured at site *s* in time period *t*. For example $x_{1,3,2}$ = 280, means that 280 units of product 1 should be produced by third manufacturer in time period 2. Other values in the table have same meaning.

Table 5.2 shows the optimum level of backorder quantity of product *i*, at selling location *l* in time period *t*. For example $Qb_{1,3,1}$ = 15, means that the product 1 is short by 15 units at third selling location in first time period. Other values in the table have same meaning.

Table 5.3: Solution obtained for transportation quantity from manufacturing site to selling location

Vehicle	Product	Manufacturing site	Selling location	Time period 1	2	3
1	**1**	**1**	1	0	0	0
			2	0	0	20
			3	0	22	92
			4	0	0	0
		2	1	0	0	0
			2	173	120	0
			3	53	0	8
			4	0	0	0
		3	1	0	0	0
			2	0	0	0
			3	71	33	0
			4	0	0	0
	2	**1**	1	0	0	0
			2	0	0	13
			3	75	106	46
			4	0	0	25
		2	1	0	0	0
			2	0	0	0
			3	74	105	72
			4	0	0	80
		3	1	0	0	0
			2	0	0	0
			3	126	47	0
			4	187	266	0
	3	**1**	1	0	160	108
			2	0	0	87
			3	85	146	62
			4	0	0	53

Vehicle	Product	Manufacturing site	Selling location	Time period 1	2	3
		2	1	0	0	0
			2	67	200	0
			3	142	55	0
			4	0	0	0
		3	1	0	0	0
			2	0	0	0
			3	3	0	0
			4	12	14	0
2	**1**	**1**	1	243	203	111
			2	64	82	84
			3	0	0	0
			4	0	0	0
		2	1	0	0	0
			2	0	0	0
			3	84	168	0
			4	0	0	0
		3	1	0	0	0
			2	0	0	0
			3	0	0	0
			4	237	247	84
	2	**1**	1	229	198	220
			2	0	0	0
			3	0	0	0
			4	0	0	0
		2	1	0	0	0
			2	240	209	162
			3	0	0	0
			4	0	0	0
		3	1	0	0	0
			2	0	0	0
			3	0	0	0
			4	0	0	0
	3	**1**	1	200	19	5
			2	20	0	0
			3	0	0	0
			4	0	0	0
		2	1	0	0	0
			2	96	43	6
			3	0	0	0
			4	0	0	0
		3	1	0	0	0
			2	0	0	0
			3	0	0	0
			4	183	173	0

Table 5.3 shows the optimum level of transportation quantity of product *i*, transport by vehicle type *g*, from manufacturing site *s* to selling location *l* in time period *t*. For example $Qt_{2,1,1,3,2}$ = 106, means that the 106 units of product 2 should be transported by vehicle type 1 from manufacturing site 1 to selling location 3 in time period 2. Other values in the table have same meaning.

Table 5.4: Solution obtained for number of vehicles of both type used for transportation

Vehicles	**Manufacturing site**	**Selling location**	**Time Period 1**	**2**	**3**
1	**1**	1	0	4	3
		2	0	0	3
		3	4	7	5
		4	0	0	2
	2	1	0	0	0
		2	6	8	0
		3	7	4	2
		4	0	0	2
	3	1	0	0	0
		2	0	0	0
		3	5	2	0
		4	5	7	0
2	**1**	1	8	5	4
		2	1	1	1
		3	0	0	0
		4	0	0	0
	2	1	0	0	0
		2	4	3	2
		3	1	2	0
		4	0	0	0
	3	1	0	0	0
		2	0	0	0
		3	0	0	0
		4	5	5	1

Table 5.4 shows the optimum level of number of vehicles of type g needed to transport from manufacturing site s to selling location l in time period t. For instance $Nt_{1,3,4,2}$ = 7, means that 7 vehicles of type 1 should be used to transported quantity from manufacturing site 3 to selling location 4 in time period 2. Other values in the table have same meaning.

Minimisation of the overachievement of total cost is at highest priority. The positive deviation variable Al_1^+ is 0, which means that the goal is achieved. The next goal on priority is minimisation of total delivery time. The positive deviation variable is 158.8, which means that this goal is overachieved. The same is happening with third priority goal of minimisation of backlog level having an overachievement of 1131. The reason for the high amount of backorder level goal is that the minimisation of total cost goal is at highest priority and because of that there is less production and market demand is less fulfilled.

Table 5.5: Effect of priority level on objective function values

Run	Objective function (Priority)			Deviation					
	Total Cost	**Total delivery time**	**Backorder level**	Al_1^+	Al_1^-	Al_2^+	Al_2^-	Al_3^+	Al_3^-
1	91477427 (P1)	704.5 (P2)	1289 (P3)	0	0	158.8	0	1131	0
2	91477427 (P1)	822.9 (P3)	1131 (P2)	0	0	277.2	0	973	0
3	91521348 (P2)	701.2 (P1)	1277 (P3)	43920	0	155.5	0	1119	0
4	91845343 (P3)	554.8 (P1)	1397 (P2)	367916	0	9	0	1239	0
5	91895177 (P2)	886.6 (P3)	586 (P1)	417750	0	340.9	0	428	0
6	92430140 (P3)	904.9 (P2)	158 (P1)	952713	0	359.2	0	0	0

The feasible solutions obtained at different priority settings shows the difference in objective function and deviation values. It can be seen from Table 5.5 that in first two scenarios minimizing total cost objective in on highest priority and the results obtained are having no deviation from target value. The target value of total delivery time goal is achieved when backorder level is on second priority but when total cost is on second priority the goal is not achieved. The reason might be transportation quantity value, when the transportation quantity is less; there is more backorder as can be seen in run 4. The same can be observed for other objective values on different priority settings. These values show the difference of solution from the target values generally decided by management team. After analysing the results it can be stated that focusing or providing high priority to one objective may not necessary cause an improvement in another objectives value even if they are having same direction.

5.5 Chapter Summary

This chapter has dealt with the multi-objective production and distribution planning problem in a multi-site manufacturing environment. Three conflicting objectives optimised simultaneously are minimisation of total cost, total delivery time and backorder level. Three important aspect of production and distribution function as setup cost, transportation capacity in terms of vehicle loading capacity and backorder are considered in an integrated manner.

The mathematical model is solved using preemptive goal programming method. For illustration, the proposed model has been implemented to an Indian automobile company manufacturing two-wheelers. The computational results illustrate the performance of the model and provide an estimate to management for different priority levels of objective functions. This study provides a quantitative tool for management or decision maker to analyse trade-off and priority consideration between multiple conflicting objectives.

Chapter 6

Fuzzy Multi-Objective Optimizaion of Multi-Site Integrated Production and Distribution Planning

6.1 Introduction

In the previous chapter, a multi-objective mathematical model for MSIPDP problem in deterministic environment was formulated and solved using premptive goal programming method. According to Arikan and Güngör (2007), in real life situation, decision making have two properties, one is conflicting objectives and other is uncertain parameters. The purpose of this chapter is to achieve the third objective of this research work by presenting an optimization model, incorporating these two properties of decision making.

It can be observed from the literature that there exist a good amount of articles dealing with impreciseness in PD planning problem, considering different members of supply chain and aspects of problem. Very few articles are available in the literature considering impreciseness of objective functions along with ambiguity of parameters. The practical IPDP problem in supply chain often has trade-off among multiple conflicting objectives which need to be simultaneously optimised by the decision maker (DM). These objective functions are often fuzzy or imprecise due to several possible factors such as variation in human performance, changing environmental conditions, and unavailability or improper information (Liang, 2007). Also, the parameters such as demand, capacity and various associated costs are also usually changing. To incorporate and handle the impreciseness in objective functions as well as in parameters, this study develops a fuzzy multi-objective mixed integer linear programming (FMO-MILP) model considering multiple products and multiple time periods and demonstrates the same on a real life industrial problem.

The advantages of fuzzy multi-objective optimization compared to other techniques are as follows:

1. The benefit of applying fuzzy set theory is that it allows imprecise aspiration of the decision maker to be quantified (Hannan 1981). The DM may simultaneously consider various conflicting imprecise objectives for which aspiration level decided by experience of DM.
2. The objective function can be of different directions i.e. minimization or maximization. To handle these multiple objectives simultaneously, there is a need to put them on a common scale. Using fuzzy multi-objective programming these multiple objectives can be represented by degree of membership.
3. Application of weighted additive, preemptive goal programming, α – cut approach makes it possible to control objective function values so that aspiration level of decision maker can be achieved.

The purpose of this chapter is to make some contribution on MSIPDP problem in an uncertain environment of fuzziness and obtain pareto-optimal solution of multi-objective problem formulation. This chapter is structured in six Sections: current Section providing introduction to the research problem. Sections 6.2 describe and formulate the FMOMILP model for the two echelon supply chain network problem. In Section 6.3, solution methodology of the approach to solve the problem is discussed. Section 6.4 illustrates the application of the proposed model to a case company and Section 6.5 ends with contribution and chapter summary.

6.2 Problem formulation

6.2.1 Problem description

This Section describes the formulation of fuzzy MO-MILP model. The description of network with figure is provided in Section 3.2. The demand and production capacity parameters are considered as imprecise. The objective is to plan production and distribution quantities in order to satisfy the demand of products at each selling location at minimum total cost, delivery time and backorder level.

Following assumptions are taken to convert the real life problem in to the fuzzy mathematical model:

1. It is assumed that all the products produced and transported are defect free; hence the products are produced on the basis of exact order quantity.
2. There are no quantity based discounts.
3. The associated storage capacities of each manufacturing site and selling location are known.
4. The objective functions are fuzzy and have imprecise aspiration level.
5. The demands of all the products originating from fixed number of selling locations and production capacity of each manufacturing site are imprecise.
6. A piecewise linear membership function is used to represent the fuzzy objective functions and triangular membership function is used for imprecise parameters.
7. The minimum aggregation operator is used for combining the objective functions.

Demand and production capacity parameters are considered as imprecise and distribution time is the additional parameter, rest of the notations are same as provided in 3.2.2.1.

Parameters:

$\widetilde{D}_{i,l,t}$	Fuzzy demand of product *i* at selling location *l* in period *t*
$\widetilde{Pcp}_{s,t}$	Fuzzy production capacity of manufacturing site *s* in terms of time in any period
$DT_{g,s,l}$	Distribution time by vehicle *g* from manufacturer *s* to selling location *l*

6.2.2 Fuzzy multi-objective mixed integer linear programming model

In the present global competitive market, it is imperative for firms to focus on multiple objectives simultaneously for effective production and distribution planning. It is seen from the relevant literature that objectives related to profit maximization, cost minimization, customer service level and responsiveness are mostly considered as multiple conflicting objectives. Liang (2007) considered three objectives: minimization of total transportation cost, total number of defective items, and total delivery time. Other typical objective functions considered in earlier studies include; maximize profit (Torabi and Moghaddam 2012; Chen et al. 2003; Selim et al. 2008), minimize total production and distribution cost (Roghanian et al. 2007; Liang 2008a; Liang 2008b; Gholamian et al. 2015), number of rejected or defective items (Liang 2008b; Torabi and Hassini 2009), safe inventory level (Chen et al. 2003; Chen and Lee 2004) and customer service level in terms of ratio of sales to demand and backorder (Chen and Lee 2004), delivery time or late deliveries (Liang 2008a; Liang 2008b; Torabi and Hassini 2009), and backorder level (Selim et al. 2008; Gholamian et al. 2015; Jolai et al. 2011).

It is observed from the literature that in terms of cost, maximizing total profit and minimizing total production and distribution cost are representative objectives. Another important aspect is maximization of customer service level which is represented by delivery time and backorder level objectives. In this study, these three important and conflicting objectives are considered in formulation of the mathematical model.

In the real world problem, there are so many parameters whose values are assigned by experts or decision makers. The decision makers do not know the value of these uncertain parameters precisely and therefore the value assigned on the basis of knowledge of experts is considered as fuzzy. To handle the impreciseness in objective function and parameter, fuzzy programming approach is implemented in this study. The symbol '$\cong$' in equation (1)-(3) is fuzzy version of '=', represent the fuzzification of the aspiration levels. It shows transformation of a non-fuzzy set into a fuzzy set which is approximately equal to it.

The three objective function considered for the formulation of fuzzy mathematical model are minimization of total cost, delivery time and backorder level; description of these conflicting objective functions and constraints are given in deterministic model of Section 5.3.1.

Minimize total cost $\tilde{Z}_1 \cong \sum_i \sum_s \sum_t Pc_{i,s} X_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_{g,s,l} Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l} Qt_{i,g,s,l,t} + \sum_i \sum_s \sum_t Sc_{i,s} Y_{i,s,t} + \sum_i \sum_l \sum_t Bc_{i,l} Qb_{i,l,t}$ (6.1)

Minimize total delivery time $\tilde{Z}_2 \cong \sum_i \sum_g \sum_s \sum_l \sum_t (DT_{g,s,l} / TC_g)\, Qt_{i,g,s,l,t}$ (6.2)

Minimize backorder level $\tilde{Z}_3 \cong \sum_t \max \sum_p \sum_l Qb_{i,l,t}$ (6.3)

The constraints of the proposed mathematical models are:

$Qs_{i,s,t} = Qs_{i,s,t-1} + x_{i,s,t} - \sum_g \sum_l Qt_{i,g,s,l,t}$ $\quad \forall\, i, s, t$ (6.4)

$Qm_{i,l,t-1} - Qb_{i,l,t-1} - Qm_{i,l,t} + Qb_{i,l,t} + \sum_g \sum_s Qt_{i,g,s,l,t} = \tilde{D}_{i,l,t}$ $\quad \forall\, i, l, t$ (6.5)

$x_{i,s,t} \le A.Y_{i,s,t}$ $\quad \forall\, i, s, t$ (6.6)

$\sum_i (x_{i,s,t} Pt_{i,s} + St_{i,s,t}\, y_{i,s,t}) \le \widetilde{Pcp}_{s,t}$ $\quad \forall\, s, t$ (6.7)

$\sum_i Qt_{i,g,s,l,t} \le Tc_g.Nt_{g,s,l,t}$ $\quad \forall\, g, s, l, t$ (6.8)

$Qb_{i,l,t} \le \theta_i \tilde{D}_{i,l,t}$ $\quad \forall\, i, l, t$ (6.9)

$\sum_i \sum_t Qs_{i,s,t} \le Ss_s$ $\quad \forall\, s$ (6.10)

$\sum_i \sum_t Qm_{i,l,t} \le Sm_l$ $\quad \forall\, l$ (6.11)

$y_{i,s,t} \in \{1, 0\}$ (6.12)

$x_{i,s,t}, Qs_{i,s,t}, Qm_{i,l,t}, Qt_{i,g,s,l,t}, Qb_{i,l,t}, Nt_{g,s,l,t} \ge 0$ (6.13)

$Qs_{i,s,0} = Qm_{i,l,0} = Qb_{i,l,0} = 0$ (6.14)

6.3 Solution Methodology

To solve the proposed fuzzy mathematical model, first imprecise parameters are defuzzified using triangular fuzzy numbers and then the solution methodology of fuzzy programming method using piecewise linear membership function is implemented.

6.3.1 Modelling the imprecise data and constraints

A possibility distribution represents the degree of occurrence of an event with imprecise data (Khalili-Damghani et al. 2014). In this study, triangular possibility distribution is assumed to adopt by DM to represent the imprecise data. This study follows Lai and Hwang's (1992) weighted average method to convert triangular fuzzy parameters into auxiliary crisp representatives. If minimum acceptance possibility, β, is given, the auxiliary crisp equality constraint for equation (6.5), (6.7) and (6.9) can be represented as follows:

$$Qm_{i,l,t-1} - Qb_{i,l,t-1} - Qm_{i,l,t} + Qb_{i,l,t} + \sum_g \sum_s Qt_{i,g,s,l,t} = w_1 D^p_{i,l,t,\beta} + w_2 D^m_{i,l,t,\beta} + w_3 D^o_{i,l,t,\beta} \quad \forall\, i,l,t \qquad (6.15)$$

$$X_{i,s,t}\, Pt_{i,s} + St_{i,s} Y_{i,s,t} \leq w_1 Pcp^p_{s,t,\beta} + w_2 Pcp^m_{s,t,\beta} + w_3 Pcp^o_{s,t,\beta} \;\forall\, i,s,t \qquad (6.16)$$

$$Qb_{i,l,t} \leq \theta_i (w_1 D^p_{i,l,t,\beta} + w_2 D^m_{i,l,t,\beta} + w_3 D^o_{i,l,t,\beta}) \quad \forall\, i,l,t \qquad (6.17)$$

Where w_1, w_2, w_3 represents the corresponding weights of lower bound value, model value and upper bound value of the imprecise parameters, respectively. Assuming all the possibility distribution as symmetric and applying α cut concept of fuzzy set theory, the values can be calculated as:

$$D^m_{i,l,t,\beta} = D^m_{i,l,t}$$

$$D^p_{i,l,t,\beta} = \left(D^m_{i,l,t} - D^p_{i,l,t}\right) \times \beta + D^p_{i,l,t}$$

$$D^o_{i,l,t,\beta} = D^o_{i,l,t} - \left(D^o_{i,l,t} - D^m_{i,l,t}\right) \times \beta$$

The value of corresponding weights and acceptance level can be taken subjectively on the basis of DM's knowledge and experience. In this study, after following the literature (Liang 2008a), the weights are taken as w_1 = w_3 = 1/6, w_2 = 4/6, and acceptance level β = 0.5 for constraint (6.15), (6.16) and (6.17).

6.3.2 Solution procedure

The solution methodology of fuzzy programming method using piecewise linear membership function suggested by Hannan (1981) and maximizing solution strategy for the minimum operator suggested by Zimmermann (1978) are followed in this study. Use of piecewise linear membership function is advantageous because it quantifies the DM's preference to produce a computationally controllable membership function. Since all the three objective functions might not be achieved simultaneously, the membership function values provided by DM is used to define the achievement level of each objective function. An auxiliary variable λ ($0 \leq \lambda \leq 1$) is derived which represent the DM's satisfaction level and a fuzzy min operator, to integrate the fuzzy set to transform the fuzzy multi-objective model into crisp model. The steps for this conversion are described as follows:

- Step 1. For each objective function z_n(n = 1, 2, 3), specify a membership function or grade of membership $f_n(z_n)$ from the DM.
- Step 2. Plot the piecewise linear membership function curves.
- Step 3. Convert each membership function $f_n(z_n)$ into the fuzzy linear equations.
- Step 4. Define the positive and negative deviational variables d_{ne}^{+} and d_{ne}^{-}. These variables show the deviation of objective function value from target value.
- Step 5. Maximize the minimum membership function value by introducing the auxiliary variable λ ($0 \leq \lambda \leq 1$) and convert the fuzzy multi-objective model into an equivalent crisp linear programming form.

Step 6. Solve the crisp linear programming problem and if the DM wants to improve the solution obtained in terms of satisfaction level and objective function values, modify the membership function values until a satisfactory solution is obtained.

6.4 An illustration of the proposed model

Description of the case considered in this study and input data values are same as provided in Section 5.4. The fuzzy parameter values for imprecise demand and production capacity are generated using symmetric triangular fuzzy numbers.

6.4.1 Solution and Analysis of results

Using triangular fuzzy numbers and piecewise linear membership function, imprecise MSIPDP problem is converted into FMOMIP problem. The FMOMIP problem is solved using solution approach described in Section 3.3. The solution obtained using following steps:

Step 1. Determine the initial optimal solutions by solving the problem for individual objective function. The computational results obtained after solving the individual objective function are Z_1 = 90839629, Z_2 = 543.7 and Z_3 = 116. Using these initial optimal solutions, the FMOMIP model is formulated. The optimal solutions of each objective function can be taken as aspiration level of associated fuzzy objective. Table 6.1 shows the piecewise linear membership function values.

Table 6.1: Piecewise linear membership function values

z_1	>12,00,00,000	12,00,00,000	11,00,00,000	10,00,00,000	900,00,000	<900,00,000
$f_1(z_1)$	0	0	0.5	0.8	1	1
z_2	>1400	1400	1100	800	500	<500
$f_2(z_2)$	0	0	0.5	0.8	1	1
z_3	>3200	3200	2200	1200	200	<200
$f_3(z_3)$	0	0	0.5	0.8	1	1

Step 2. Represent both imprecise coefficients ($\widetilde{D}_{i,l,t}, \widetilde{Pcp}_s$) used in right hand side of constraints using triangular possibility distribution. Using the minimum acceptable possibility, β, and the weighted average method of the fuzzy ranking concepts, formulate crisp equivalent constraints using equation (6.15), (6.16) and (6.17).

Step 3. Using membership function data shown in Table 6.1, plot the piecewise linear membership function curve. The curves plotted are shown in Figure 6.1, 6.2 and 6.3. The curve helps in approximating membership values for intermediate points.

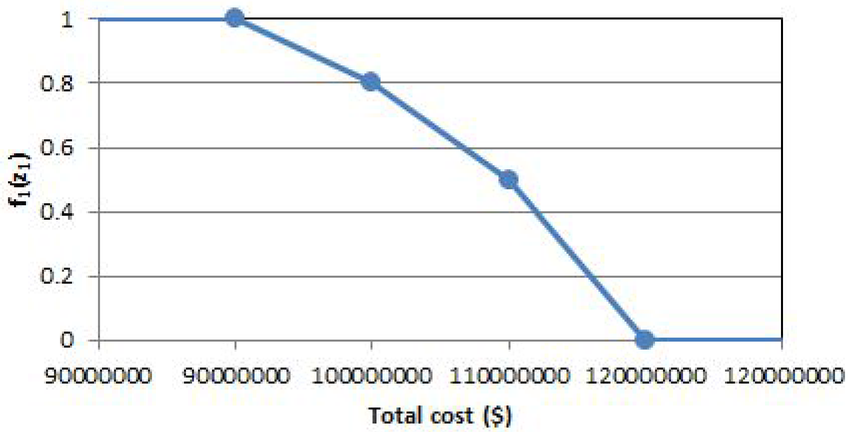


Figure 6.1: Piecewise linear membership function curve (z_1,$f_1(z_1)$)

Figure 6.2: Piecewise linear membership function curve (z_2,$f_2(z_2)$)

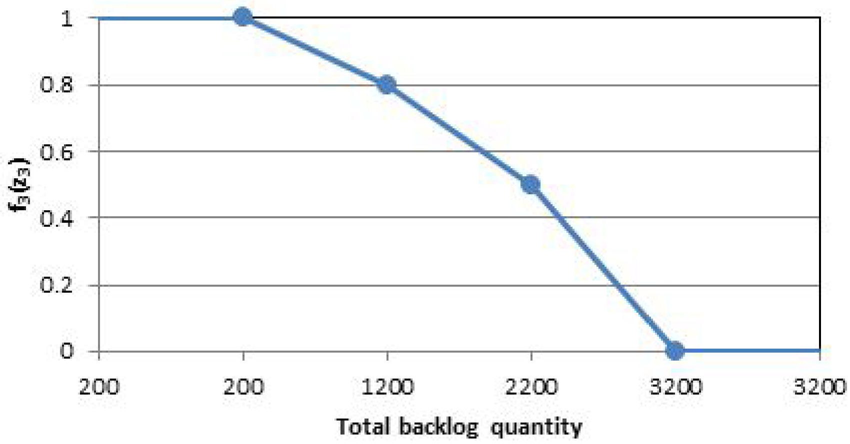


Figure 6.3: Piecewise linear membership function curve (z_3,$f_3(z_3)$)

Step 4. Convert each membership functions $f_n(z_n)$ into fuzzy linear equations:

$f_1(z_1)$ = -0.00000001 ($z_1 -$ 110000000) – 0.000000005 ($z_1 -$ 100000000) – 0.000000035z_1 + 4.4

$f_2(z_2)$ = -0.000333 (z_2-1100) – 0.000167 (z_2-800) – 0.00117z_2 + 1.83

$f_3(z_3)$ = -0.0001 (z_3-2200) – 0.00005 (z_3-1200) – 0.00035z_3 + 1.32

Step 5. Introduce deviational variables in positive and negative direction. The deviation variable shows underachievement or overachievement of objective function value form its target or aspiration level. Considering deviation variables, convert each membership function $f_n(z_n)$ into the piecewise linear equations.

$f_1(z_1) = -0.00000001\ (d_{11}^{+} - d_{11}^{-}) - 0.000000005\ (d_{12}^{+} - d_{12}^{-}) - 0.000000035$ $\{\sum_i \sum_s \sum_t Pc_{i,s} X_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} +$ $\sum_g \sum_s \sum_l \sum_t Ft_{g,s,l}\, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l}\, Qt_{i,g,s,l,t} + \sum_i \sum_s \sum_t Sc_{i,s} Y_{i,s,t}$ $+ \sum_i \sum_l \sum_t Bc_{i,l} Qb_{i,l,t}\ \} + 4.4$

$f_2(z_2) = -0.000333\ (d_{21}^{+} - d_{21}^{-}) - 0.000167\ (d_{22}^{+} - d_{22}^{-}) - 0.00117$ $\{\sum_i \sum_g \sum_s \sum_l \sum_t (DT_{g,s,l} / TC_g)\, Qt_{i,g,s,l,t}\} + 1.83$

$f_3(z_3) = -0.0001\ (d_{31}^{+} - d_{31}^{-}) - 0.00005\ (d_{32}^{+} - d_{32}^{-}) - 0.00035\ \{\sum_t \sum_p \sum_l Qb_{i,l,t}\}$ $+ 1.32$

Step 6. Maximize the minimum membership function value by introducing the auxiliary variable λ ($0 \leq \lambda \leq 1$) and transform the FMOMIP problems into an equivalent crisp linear programming form. The resulting equivalent crisp linear programming form can be formulated as follows:

Max λ

Subject to:

$\lambda \leq -0.00000001\ (d_{11}^{+} - d_{11}^{-}) - 0.000000005\ (d_{12}^{+} - d_{12}^{-}) - 0.000000035\ \{\sum_i \sum_s \sum_t Pc_{i,s} X_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_{g,s,l}\, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l}\, Qt_{i,g,s,l,t} + \sum_i \sum_s \sum_t Sc_{i,s} Y_{i,s,t} + \sum_i \sum_l \sum_t Bc_{i,l} Qb_{i,l,t}\} + 4.4$

$\lambda \leq -0.000333\ (d_{21}^{+} - d_{21}^{-}) - 0.000167\ (d_{22}^{+} - d_{22}^{-}) - 0.00117\ \{\sum_i \sum_g \sum_s \sum_l \sum_t (DT_{g,s,l} / TC_g)\, Qt_{i,g,s,l,t}\} + 1.83$

$\lambda \leq -0.0001\ (d_{31}^{+} - d_{31}^{-}) - 0.00005\ (d_{32}^{+} - d_{32}^{-}) - 0.00035\ \{\sum_t \sum_p \sum_l Qb_{i,l,t}\} + 1.32$

$\sum_i \sum_s \sum_t Pc_{i,s} X_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_{g,s,l}\, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l}\, Qt_{i,g,s,l,t} + \sum_i \sum_s \sum_t Sc_{i,s} Y_{i,s,t} + \sum_i \sum_l \sum_t Bc_{i,l} Qb_{i,l,t} + d_{11}^{+} - d_{11}^{-} = 110000000$

$\sum_i \sum_s \sum_t Pc_{i,s} X_{i,s,t} + \sum_i \sum_s \sum_t Hs_{i,s} Qs_{i,s,t} + \sum_i \sum_s \sum_t Hm_{i,l} Qm_{i,l,t} + \sum_g \sum_s \sum_l \sum_t Ft_{g,s,l}\, Nt_{g,s,l,t} + \sum_i \sum_g \sum_s \sum_l \sum_t Vt_{i,g,s,l}\, Qt_{i,g,s,l,t} + \sum_i \sum_s \sum_t Sc_{i,s} Y_{i,s,t} + \sum_i \sum_l \sum_t Bc_{i,l} Qb_{i,l,t} + d_{12}^{+} - d_{12}^{-} = 100000000$

$\sum_i \sum_g \sum_s \sum_l \sum_t (DT_{g,s,l} / TC_g)\, Qt_{i,g,s,l,t} + d_{21}^{+} - d_{21}^{-} = 1100$

$\sum_i \sum_g \sum_s \sum_l \sum_t (DT_{g,s,l} / TC_g)\, Qt_{i,g,s,l,t} + d_{22}^{+} - d_{22}^{-} = 800$

$\sum_t \sum_p \sum_l Qb_{i,l,t} + d_{31}^{+} - d_{31}^{-} = 2200$

$\sum_t \sum_p \sum_l Qb_{i,l,t} + d_{32}^{+} - d_{32}^{-} = 1200$

$0 \leq \lambda \leq 1$

Equations (6.4), (6.6), (6.8), (6.10) to (6.17).

$d_{11}^{+}, d_{11}^{-}, d_{12}^{+}, d_{12}^{-}, d_{22}^{+}, d_{22}^{-}, d_{21}^{+}, d_{21}^{-}, d_{31}^{+}, d_{31}^{-}, d_{32}^{+}, d_{32}^{-} \quad \geq 0$

According to the solution procedure explained in Section 6.4.2, the fuzzy mathematical model is transformed into crisp form. The crisp formulation is in the form of single objective MILP model which is solved using CPLEX 12.7 solver provided via IBM ILOG CPLEX on a computer Intel Core i5 1.7 GHz with 4GB RAM.

Table 6.2 Solutions obtained by different methods

Item	**MIP-1**	**MIP-2**	**MIP-3**	**FMOMIP**
Objective functions	**Min.** Z_1	**Min.** Z_2	**Min.** Z_3	**Max. λ**
Z_1	90839629*	108918582	94651039	93414170
Z_2	807.1	543.7*	1111.9	604.8
Z_3	1223	2866	116*	576
λ (%)	100%	100%	100%	92.48

* represent the optimal solution of individual single objective MILP model.

Computational results of individual single-objective MILP models and crisp linear programming model are shown in Table 6.2. The proposed fuzzy multi-objective optimization approach simultaneously minimize total cost, delivery time and backorder level and yields an efficient compromised solution. If the DM is not satisfied with the obtained results, the membership function values would be revised.

6.4.2 Sensitivity analysis

After obtaining results, it is important to perform sensitivity analysis for analyzing the effect of change in aspirational levels of objective function values on DM's satisfaction level. The sensitivity of decision parameters is analyzed based on three scenarios with numerical examples to implement the FMOMIP model. By changing one of the objective function values and keeping rest of the two objectives unchanged, three different scenarios are created. A two-step 5% scale variation in both increase (Run 4 and Run 5) and decrease (Run 1 and Run 2) for each scenario is designated and Run 3 is taken as a test base. Sensitivity analysis results obtained after implementing the above three scenarios are shown in Table 6.3.

Table 6.3: Sensitivity analysis results

Scenarios	**Objectives**	**Run 1 (-10%)**	**Run 2 (-5%)**	**Run 3 (0%)**	**Run 4 (5%)**	**Run 5 (10%)**
Scenario 1	z_1	90851314	91225003	93414170	93414170	93414170
Change in z_1	z_2	819	666.5	604.8	604.8	604.8
Objective function	z_3	1223	782	576	576	576
	λ	0.7802	0.8835	0.9248	0.9248	0.9248
Scenario 2	z_1	92580152	92769734	93414170	92886502	92292722
Change in z_2	z_2	590	597.1	604.8	613.7	620.6
Objective function	z_3	716	648	576	493	429
	λ	0.8968	0.9102	0.9248	0.9414	0.9541
Scenario 3	z_1	93202336	92877253	93414170	93434916	93384814
Change in z_3	z_2	609.5	607	604.8	602.4	600.5
Objective function	z_3	532	556	576	599	617
	λ	0.9216	0.9234	0.9248	0.9264	0.9277

Sensitivity analysis results indicate the effect of specific degree of membership function on satisfaction level and output solutions. As shown in Table 6.3, the three scenarios are displaying conflicts and trade-offs among dependent objective functions. It is showing effect of change in membership function value of objective functions on output solution and satisfaction level. Run 3 is the test base and considered as reference point to depict the change in respective values. In scenario 1, change in membership function value of total cost objective function on positive side doesn’t change any output value but on the negative side delivery time and backorder level values are increases while total cost value and overall satisfaction level decreases. The reason might be reduction in production and transportation quantity.

In scenario 2, change of membership function value of delivery time objective on output values is analyzed. The increment in the value results in increased value of delivery time and satisfaction level and simultaneously decreased values of total cost and backorder level. Effect of change of membership function value on backorder level is shown in scenario 3. All three scenarios reflect that increasing the membership function value increases the output value of that objective function and overall satisfaction level, also decreases the other objective function values. This depicts DM’s flexibility in specifying the membership function values for each objective function to obtain satisfactory result.

6.5 Chapter summary

This study addresses the IPDP problem for a two echelon supply chain. Production and distribution aspects in the multi-site environment are discussed and a multi-objective mixed integer linear programming model is formulated. Further uncertainty and impreciseness in the problem is considered and fuzzy multi-objective optimisation approach is implemented, which simultaneously optimize three objectives; total cost, delivery time and backorder level. The solution approach uses piecewise linear membership function of Hannan (1981) and fuzzy min operator approach of Zimmerman (1976). The fuzzy model was transformed into crisp linear form and solved using CPLEX solver. For illustration, the proposed model has been implemented to an Indian automobile company manufacturing two-wheelers. To test the efficiency and effect of change in membership function value on satisfaction level and objective function values, sensitivity analysis is performed. The implementation result shows the applicability of the proposed model to handle fuzziness in practical environments and flexibility of DM for analysing the trade-offs among objective function values to get a satisfactory result.

This chapter contributes to the literature by incorporating ambiguity of parameters along with vagueness in the objective functions and implementing fuzzy solution approach to handle the impreciseness. The analytical results and sensitivity analysis depicts a systematic procedure that allows a DM to modify the related parameters until a desired satisfaction level is achieved. The illustration of mathematical model and solution approach using a real life case may be helpful in handling conflicting multi-objective PD planning problems in multi echelon supply chain. The methodology adopted here can be applied or extended on any other situation of IPDP in real world supply chain.

Chapter 7

Multi Objective Optimizaion of Multi-Site Integrated Production and Distribution Planning in Neutrosophic Environment

7.1 Introduction

Chapter 6 has presented the implementation of fuzzy optimization approach to handle uncertainty in multi-objective MSIPDP problem. With the advancement in new mathematical tools and solution methods, several approaches are presented to handle uncertainty. The concept of Fuzzy set theory introduced by Zadeh (1965) is the most popular approach used to deal with problems having imprecise and vague information. The drawback of fuzzy approach is that decisions based on available information are not good enough to represent the level of accuracy. Atanassov (1986) advanced fuzzy sets to intuitionistic fuzzy sets which incorporate both belongingness and non-belongingness of a membership function. In 1999, the concept of Neutrosophy was introduced by Florentin Smarandache as a branch of philosophy to deal with "the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra". The philosophy laid foundation for new mathematical theories which are neutrosophic sets, neutrosophic logic, neutrosophic probability and neutrosophic statistics.

The problem considered in this study is an IPDP problem for a two echelon supply chain network comprising of multiple manufacturers serving multiple selling locations. The practical IPDP problem in supply chain contains of indeterminate or inconsistent information due to various reasons like imperfection in data, poor demand forecasting and incomplete information or awareness about the problem. To handle this situation, this study develops a neutrosophic programming model considering multiple products and multiple time periods, demonstrating the same on a real-life industrial problem.

The aim of this chapter is to obtain a compromised solution for MO-MSIPDP problem in neutrosophic environment. The organization of this chapter is as follows. Section 7.2 presents preliminaries of the fuzzy and neutrosophic programming approaches. In Section 7.3, solution methodology of intuitionistic fuzzy and neutrosophic approach is discussed. Section 7.4 illustrates the application of the proposed model to a case company. Section 7.5 ends with chapter summary.

7.2 Preliminaries

This Section provides information about background knowledge and concepts related to Intuitionistic fuzzy sets and Neutrosophic sets applied in this study.

Intuitionistic fuzzy set

In fuzzy logic, the set of truth values {0,1} are replaced by real number interval [0,1], which shows degree of truth where 0 represent false and 1 represent truth value.

Definition 1: Let a set *E* be fixed. An intuitionistic fuzzy set *A* in *E* can be defined as

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in E\},$$

Where the functions $\mu_A : E \rightarrow [0,1]$ and $v_A : E \rightarrow [0,1]$ defines the degree of membership and the degree of non-membership of the element $x \in E$ and holds the following condition:

$$0 \leq \mu_A(x) + v_A(x) \leq 1$$

Each ordinary fuzzy set has the form

$$\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\}$$

Definition 2: The degree of non-determinacy or uncertainty of the element $x \in E$ to the intuitionistic fuzzy set *A* expressed as:

$$1 - \mu_A(x) - v_A(x)$$

Definition 3: (α, β) level intervals or cuts

A set of (α, β) cut generated by IFS A, where α and β are fixed number in $(\alpha, \beta) \in [0,1]$ and $(\alpha + \beta) \in [0,1]$ can be defined as:

$$A_{\alpha,\beta} = \begin{cases} \left(x, \mu_A(x), v_A(x)\right) : x \in E \\ \mu_A(x) \geq \alpha,\ v_A(x) \leq \beta,\ \ \alpha, \beta \in [0,1] \end{cases}$$

Neutrosophic Sets

Definition 4: Let X be a universal set having elements denoted by *x*. The neutrosophic set B on the universal set X is categorized into three membership functions as the true $T_B(x)$, indeterminate $I_B(x)$ and false $F_B(x)$ contained in a subset of $]^{-}0,\ 1^{+}[$.

$$T_B(x) \rightarrow\ ]^{-}0,\ 1^{+}[$$

$$I_B(x) \rightarrow\ ]^{-}0,\ 1^{+}[$$

$$F_B(x) \rightarrow\ ]^{-}0,\ 1^{+}[$$

These three membership function values are independent to each other and hence there is no restriction on sum of $T_B(x), I_B(x)$ and $F_B(x)$, so

$$^{-}0 \leq T_B(x) + I_B(x) + F_B(x) \leq 3^{+}$$

Definition 5: Single valued neutrosophic sets

Let X is a universal set having elements denoted by *x*. The neutrosophic set B on the universal set X is an objective having the form as

$$\mathrm{B} = \{\langle X, T_B(x), I_B(x), F_B(x) \rangle | x \in X\}$$

Where

$$T_B(x) \rightarrow [0,1],\ I_B(x) \rightarrow [0,1] \text{ and } F_B(x) \rightarrow [0,1] \text{ with}$$

$$0 \leq T_B(x) + I_B(x) + F_B(x) \leq 3$$

7.3 Solution methodology of Neutrosophic programming

Neutrosophic sets are used to solve the proposed MO-MSIPDP model descried in Section 5.3.1, with indeterminate or inconsistent information. The solution methodology of neutrosophic programming method using concept of Bellman and Zadeh (1970) is followed in this study. The step by step solution methodology is stated below.

Step 1. Determine the initial optimal solutions by solving the individual objective function subjected to all the constraints.

Step 2. Using initial solutions, calculate the value of upper and lower bound for each objective function for neutrosophic environment

Step 3. Define membership functions of truth, indeterminacy and falsity using upper and lower bound values.

Step 4. Construct neutrosophic programming model using membership functions and formulate simplified model using auxiliary parameters.

Step 5. Solve the neutrosophic programming model and if decision maker wants to improve the solution, modify the values of *s* and *t* until satisfactory solution is obtained.

The proposed neutrosophic programming approach aims to maximize truth degree and falsity degree, minimizes indeterminacy degree simultaneously. To convert the multi-objective model into neutrosophic programming model, the concept of Bellman and Zadeh (1970) have been followed. According to the concept, fuzzy decision is conjunction of fuzzy objective functions and fuzzy constraints. Applying the concept into neutrosophic sets, neutrosophic decision can be considered as conjunction of neutrosophic objective function and neutrosophic constraints, which can be written mathematically as:

$$D_N = \left(\bigcap_{k=1}^{K} O_k\right)\left(\bigcap_{i=1}^{n} C_i\right) = \left(x, T_D(x), I_D(x), F_D(x)\right)$$

Where $T_D(x)$ is the truth membership function, $I_D(x)$ is the indeterminacy membership function and $F_D(x)$ is the falsity membership function, which can be represented as:

$$T_D(x) = \min\begin{pmatrix} T_{O1}(x), T_{O2}(x) \dots T_{OK}(x) \\ T_{C1}(x), T_{C2}(x) \dots T_{Cn}(x), \end{pmatrix} \; for\ all\ x\ \in X$$

$$I_D(x) = \min\begin{pmatrix} I_{O1}(x), I_{O2}(x) \dots I_{OK}(x) \\ I_{C1}(x), I_{C2}(x) \dots I_{Cn}(x), \end{pmatrix} \; for\ all\ x\ \in X$$

$$F_D(x) = \min\begin{pmatrix} F_{O1}(x), F_{O2}(x) \dots F_{OK}(x) \\ F_{C1}(x), F_{C2}(x) \dots F_{Cn}(x), \end{pmatrix} \; for\ all\ x\ \in X$$

The next step is to find upper and lower bounds for each objective function for formulation of membership functions. The bounds are calculated as follows:

For truth membership function $\quad U_k^T = U_k, \quad L_k^T = L_k$

For indeterminacy membership function $\quad U_k^I = L_k^T + s_k(U_k^T - L_k^T), \; L_k^I = L_k^T$

For falsity membership function $\quad U_k^F = U_k^T, \; L_k^F = L_k^T + t_k(U_k^T - L_k^T)$

Where s_k and t_k are predetermined real numbers in (0,1). U_k and L_k are the upper and lower values for each objective function calculated by optimizing each objective function one at a time represented mathematically as

$U_k = \max\{f_k(x)\}_{k=1}^K \qquad L_k = \min\{f_k(x)\}_{k=1}^K$

The membership function can be determined using above bounds as follows:

$$T_k\big(f_k(Z_k)\big) = \begin{cases} 1 & if\ f_k(Z_k) < L_k^T \\ 1 - \dfrac{f_k(Z_k) - L_k^T}{U_k^T - L_k^T} & if\ L_k^T \le f_k(Z_k) \le U_k^T \\ 0 & if\ f_k(Z_k) > U_k^T \end{cases}$$

$$I_k\big(f_k(Z_k)\big) = \begin{cases} 1 & if\ f_k(Z_k) > L_k^I \\ 1 - \dfrac{f_k(Z_k) - L_k^I}{U_k^I - L_k^I} & if\ L_k^I \le f_k(Z_k) \le U_k^I \\ 0 & if\ f_k(Z_k) > U_k^I \end{cases}$$

$$F_k\big(f_k(Z_k)\big) = \begin{cases} 1 & if\ f_k(Z_k) > U_k^F \\ 1 - \dfrac{U_k^F - f_k(Z_k)}{U_k^F - L_k^F} & if\ L_k^F \le f_k(Z_k) \le U_k^F \\ 0 & if\ f_k(Z_k) < L_k^F \end{cases}$$

The neutrosophic programming model can be obtained as follows:

$$\max\ min_{k=1,2,..K}\ T_k\big(f_k(Z_k)\big)$$

$$\min\ max_{k=1,2,..K}\ I_k\big(f_k(Z_k)\big)$$

$$\max\ min_{k=1,2,..K}\ F_k\big(f_k(Z_k)\big)$$

Subject to all constraints from (5.7) to (5.17).

Using auxiliary parameters, above formulation can be converted in to following:

$$Max\ \alpha$$

$$Max\ \gamma$$

$$Min\ \beta$$

$$T_k\big(f_k(Z_k)\big) \ge \alpha\,, \quad I_k\big(f_k(Z_k)\big) \ge \gamma,\ F_k\big(f_k(Z_k)\big) \le \beta$$

$$\alpha \ge \gamma, \quad \alpha \ge \beta, \quad \alpha + \gamma + \beta \le 3, \quad \alpha, \beta, \gamma \in [0,1]$$

Constraints (5.7) to (5.17).

The simplified model can be represented as follows-

$$Max\ \alpha - \beta + \gamma$$

$$f_k(Z_k) + (U_K^T - L_K^T)\alpha \le U_K^T$$

$$f_k(Z_k) + (U_K^I - L_K^I)\gamma \le U_K^I$$

$$f_k(Z_k) - (U_K^F - L_K^F)\beta \le L_K^F$$

$$\alpha \ge \gamma, \quad \alpha \ge \beta, \quad \alpha + \gamma + \beta \le 3, \quad \alpha, \beta, \gamma \in [0,1]$$

Constraints (5.7) to (5.17).

Solution of the above neutrosophic model will provide the values of all three auxiliary parameters along with the values of different variables of the problem.

7.4 Illustration of the proposed model

To demonstrate procedure of neutrosophic programming, illustrative example of an automotive manufacturing company is considered. The description of which is provided in Section 3.2. The notations and mathematical formulation is the same as provided in 3.2.2.1.

7.4.1 Solution procedure

Neutrosophic Solution

Step 1. Solve the MILP model to obtain initial optimal solutions by individually solving the problem for each objective function. The computational results obtained after solving the individual objective function are Z_1 = 91403462, Z_2 = 543.67 and Z_3 = 191.

Step 2. Determine the upper and lower bounds for each objective function

$U_1 = \max f_1(z_1)$ = 134713192 $\quad$ $L_1 = \min f_1(z_1)$ = 91403462

$U_2 = \max f_2(z_2)$ = 1816.66 $\quad$ $L_2 = \min f_2(z_2)$ = 543.67

$U_3 = \max f_3(z_3)$ = 3843 $\quad$ $L_3 = \min f_3(z_3)$ = 191

Step 3. Calculate the bounds for neutrosophic environment

a. Truth values

$U_1^T = U_1 = 134713192$ $\quad$ $L_1^T = L_1 = 91403462$

$U_2^T = U_2 = 1816.66$ $\quad$ $L_2^T = L_2 = 543.67$

$U_3^T = U_3 = 3843$ $L_3^T = L_3 = 191$

b. Falsity values

$U_1^F = U_1^T =$ 134713192

$U_2^F = U_2^T = 1816.66$

$U_3^F = U_3^T = 3843$

$L_1^F = L_1^T + t_1(U_1^T - L_1^T)$ = 91403462 + t_1(134713192 - 91403462)

= 91403462 + 43309730t_1

$L_2^F = L_2^T + t_2(U_2^T - L_2^T)$ = 543.67 + t_2 (1816.66 – 543.67)

= 543.67 + 1272.99t_2

$L_3^F = L_3^T + t_3(U_3^T - L_3^T)$ = 191 + t_3 (3843 - 191) = 191 + 3652t_3

c. Indeterminacy values

$U_1^I = L_1^T + s_1(U_1^T - L_1^T)$ = 91403462 + s_1(134713192-91403462)

= 91403462 + 43309730s_1

$U_2^I = L_2^T + s_2(U_2^T - L_2^T)$ = 543.67 + s_2 (1816.66 – 543.67)

= 543.67 + 1272.99s_2

$U_3^I = L_3^T + s_3(U_3^T - L_3^T)$ = 191 + s_3 (3843 - 191) = 191 + 3652s_3

$L_1^I = L_1^T = 91403462$

$L_2^I = L_2^T = 543.67$

$L_3^I = L_3^T = 191$

Step 4. Using above bounds, calculate the membership function value

a. Truth membership

$$T_1(f_1(Z_1)) = \begin{cases} 1 & if\ f_1(Z_1) < 91403462 \\ 1 - \dfrac{f_1(Z_1) - 91403462}{43309730} & if\ 91403462 \le f_1(Z_1) \le 134713192 \\ 0 & if\ f_1(Z_1) > 134713192 \end{cases}$$

$$T_2(f_2(Z_2)) = \begin{cases} 1 & if\ f_2(Z_2) < 543.67 \\ 1 - \dfrac{f_2(Z_2) - 543.67}{1272.99} & if\ 543.67 \le f_2(Z_2) \le 1816.66 \\ 0 & if\ f_2(Z_2) > 1816.66 \end{cases}$$

$$T_2(f_3(Z_3)) = \begin{cases} 1 & if\ f_3(Z_3) < 191 \\ 1 - \dfrac{f_3(Z_3) - 191}{3652} & if\ 191 \le f_3(Z_3) \le 3843 \\ 0 & if\ f_3(Z_3) > 3843 \end{cases}$$

b. Falsity membership

$$F_1(f_1(Z_1)) = \begin{cases} 1 & if\ f_1(Z_1) > 134713192 \\ 1 - \dfrac{134713192 - f_1(Z_1)}{43309730(1 - t_1)} & if\ 91403462 + 43309730t_1 \le f_1(Z_1) \le 134713192 \\ 0 & if\ f_1(Z_1) < 91403462 + 43309730t_1 \end{cases}$$

$$F_2(f_2(Z_2)) = \begin{cases} 1 & if\ f_2(Z_2) > 1816.66 \\ 1 - \dfrac{1816.66 - f_2(Z_2)}{1272.99(1 - t_2)} & if\ 543.67 + 1272.99t_2 \le f_2(Z_2) \le 1816.66 \\ 0 & if\ f_2(Z_2) < 543.67 + 1272.99t_2 \end{cases}$$

$$F_3(f_3(Z_3)) = \begin{cases} 1 & if\ f_3(Z_3) > 3843 \\ 1 - \dfrac{3843 - f_3(Z_3)}{3652(1 - t_3)} & if\ 191 + 3652t_3 \le f_3(Z_3) \le 3843 \\ 0 & if\ f_3(Z_3) < 191 + 3652t_3 \end{cases}$$

c. Indeterminacy membership

$$I_1\big(f_1(Z_1)\big) = \begin{cases} 1 & if\ f_1(Z_1) > 91403462 \\ 1 - \dfrac{f_1(Z_1) - 91403462}{43309730(1+s_1)} & if\ 91403462\ \le f_1(Z_1) \le 91403462\ +\ 43309730s_1 \\ 0 & if\ f_1(Z_1) > 91403462\ +\ 43309730s_1 \end{cases}$$

$$I_2\big(f_2(Z_2)\big) = \begin{cases} 1 & if\ f_2(Z_2) > 543.67 \\ 1 - \dfrac{f_2(Z_2) - 543.67}{1272.99(1+s_2)} & if\ 543.67\ \le f_2(Z_2) \le 543.67\ +\ 1272.99s_2 \\ 0 & if\ f_2(Z_2) > 543.67\ +\ 1272.99s_2 \end{cases}$$

$$I_3\big(f_3(Z_3)\big) = \begin{cases} 1 & if\ f_3(Z_3) > 191 \\ 1 - \dfrac{f_3(Z_3) - 191}{3652(1+s_3)} & if\ 191\ \le f_3(Z_3) \le 191\ +\ 3652s_3 \\ 0 & if\ f_3(Z_3) > 191\ +\ 3652s_3 \end{cases}$$

Step 5. Using auxiliary parameters, formulate the neutrosophic model as following-

$$Max\ \alpha - \beta + \gamma$$

$$f_1(Z_1) + (43309730)\alpha \ \le\ 134713192$$

$$f_2(Z_2) + (1272.99)\alpha \ \le 1816.66$$

$$f_3(Z_3) + (3652)\alpha \ \le\ 3843$$

$$f_1(Z_1) + (43309730s)\gamma \ \le 91403462 + 43309730s$$

$$f_2(Z_2) - (1272.99s)\gamma \ \le\ 543.67 + 1272.99s$$

$$f_3(Z_3) - (3652s)\gamma \ \le 191 + 3652s$$

$$f_1(Z_1) + (43309730(1-t))\beta \ \le\ 91403462 + 43309730t$$

$$f_2(Z_2) + (1272.99(1-t))\beta \ \le\ 543.67 + 1272.99t$$

$$f_3(Z_3) + (3652(1-t))\beta \ \le\ 191 + 3652t$$

$$\alpha \geq \gamma, \qquad \alpha \geq \beta, \qquad \alpha + \gamma + \beta \leq 3, \qquad \alpha, \beta, \gamma \in [0,1]$$

Constraints (5.7) to (5.17)

After deciding the values of *s* and *t*, solve the above model to obtain compromised results in the neutrosophic environment.

IFP Solution

To demonstrate the performance of neutrosophic model, IFP model is formulated and solved as described in following steps.

Step 1. Solve the initial MILP model by individually solving the problem for each objective function. The computational results obtained after solving the individual objective function are Z_1 = 91403462, Z_2 = 543.67 and Z_3 = 191. These values are representing lower bound for each objective function.

Step 2. Determine the upper and lower bounds by maximizing and minimizing each objective function

$U_1 = \max f_1(z_1)$ = 134713192 $\qquad L_1 = \min f_1(z_1)$ = 91403462

$U_2 = \max f_2(z_2)$ = 1816.66 $\qquad L_2 = \min f_2(z_2)$ = 543.67

$U_3 = \max f_3(z_3)$ = 3843 $\qquad L_3 = \min f_3(z_3)$ = 191

Step 3. Calculate the bounds for IF environment

a) Membership function values

$U_k^{\mu} = \max f_k(Z_k) \qquad L_k^{\mu} = \min f_k(Z_k)$

$U_1^{\mu} = U_1 = 134713192 \qquad L_1^{\mu} = L_1 = 91403462$

$U_2^{\mu} = U_2 = 1816.66 \qquad L_2^{\mu} = L_2 = 543.67$

$U_3^{\mu} = U_3 = 3843 \qquad L_3^{\mu} = L_3 = 191$

b) Non-membership function value

$U_k^\nu = U_k^\mu$ $\quad$ $L_k^\nu = L_k^\mu + \lambda(U_k^\mu - L_k^\mu)$

$U_1^\nu = 134713192$ $\quad$ $L_k^\nu = 91403462 + \lambda(43309730)$

$U_k^\nu = 1816.66$ $\quad$ $L_k^\nu = 543.67 + \lambda(1272.99)$

$U_k^\nu = 3843$ $\quad$ $L_k^\nu = 191 + \lambda(3652)$

Step 4. Consider membership and non-membership function as following linear equation

Membership function

$$\mu_k(f_k(Z_k)) = \begin{cases} 1 & if\ f_1(Z_1) < L_k^\mu \\ \dfrac{U_k^\mu - f_k(Z_k)}{U_k^\mu - L_k^\mu} & if\ L_k^\mu \le f_1(Z_1) \le U_k^\mu \\ 0 & if\ f_1(Z_1) > U_k^\mu \end{cases}$$

$$\mu_1(f_1(Z_1)) = \begin{cases} 1 & if\ f_1(Z_1) < 91403462 \\ \dfrac{134713192 - f_1(Z_1)}{43309730} & if\ 91403462 \le f_1(Z_1) \le 134713192 \\ 0 & if\ f_1(Z_1) > 134713192 \end{cases}$$

$$\mu_2(f_2(Z_2)) = \begin{cases} 1 & if\ f_1(Z_1) < 543.67 \\ \dfrac{1816.66 - f_2(Z_2)}{1272.99} & if\ 543.67 \le f_1(Z_1) \le 1816.66 \\ 0 & if\ f_1(Z_1) > 1816.66 \end{cases}$$

$$\mu_3(f_3(Z_3)) = \begin{cases} 1 & if\ f_1(Z_1) < 191 \\ \dfrac{3843 - f_3(Z_3)}{3652} & if\ 191 \le f_1(Z_1) \le 3843 \\ 0 & if\ f_1(Z_1) > 3843 \end{cases}$$

Non-membership function

$$\nu_k(f_k(Z_k)) = \begin{cases} 1 & if\ f_1(Z_1) > U_k^\nu \\ \dfrac{f_k(Z_k) - L_k^\nu}{U_k^\nu - L_k^\nu} & if\ L_k^\nu \le f_1(Z_1) \le U_k^\nu \\ 0 & if\ f_1(Z_1) < L_k^\nu \end{cases}$$

$$\nu_1\big(f_1(Z_1)\big) = \begin{cases} 1 & if\ f_1(Z_1) > 134713192) \\ \frac{f_1(Z_1) - 91403462 - \lambda(43309730)}{43309730(1-\lambda)} & if\ 91403462 + \lambda(43309730) \le f_1(Z_1) \le 134713192) \\ 0 & if\ f_1(Z_1) < 91403462 \end{cases}$$

$$\nu_2\big(f_2(Z_2)\big) = \begin{cases} 1 & if\ f_2(Z_2) > 1816.66 - \lambda(1272.99) \\ \frac{f_2(Z_2) - 543.67 - \lambda(1272.99)}{1272.99(1-\lambda)} & if\ 543.67 + \lambda(1272.99) \le f_2(Z_2) \le 1816.66 \\ 0 & if\ f_2(Z_2) < 543.67 \end{cases}$$

$$\nu_3\big(f_3(Z_3)\big) = \begin{cases} 1 & if\ f_3(Z_3) > 3843 - \lambda(3652) \\ \frac{f_3(Z_3) - 191 + \lambda(3652)}{3652(1-\lambda)} & if\ 191 + \lambda(3652) \le f_3(Z_3) \le 3843 \\ 0 & if\ f_3(Z_3) < 191 \end{cases}$$

Step 5. Formulate the IFP model using auxiliary parameters as follows-

$$Max\ (\alpha - \beta)$$

$$134713192 - f_1(Z_1) \ge 43309730 * \alpha$$

$$1816.66 - f_2(Z_2) \ge 1272.99 * \alpha$$

$$3843 - f_3(Z_3) \ge 3652 * \alpha$$

$$f_1(Z_1) - 91403462 - \lambda(43309730) \le 43309730(1-\lambda) * \beta$$

$$f_2(Z_2) - 543.67 - \lambda(1272.99) \le 1272.99(1-\lambda) * \beta$$

$$f_3(Z_3) - 191 + \lambda(3652) \le 3652(1-\lambda) * \beta$$

$$\alpha \ge \beta, \quad \alpha + \beta \le 1, \quad \beta \ge 0, \quad \alpha, \beta \in [0,1]$$

Constraints (5.7) to (5.17)

7.5 Results and Discussion

This Section provides analytical results obtained after solving the proposed mathematical models. The proposed neutrosophic model is analysed using illustrative example of a real life problem. The mathematical models are solved using CPLEX solver provided via IBM ILOG CPLEX 12.7 on a computer Intel Core i5 1.7 GHz with 4GB RAM. The values of ideal anti-ideal solution for each objective function are represented in Table 7.1. These values are representing a region of satisfaction and are helpful for obtaining the lower and upper bound for each objective function.

Table 7.1: Ideal and anti-ideal solution for each objective function

Item	**MIP-1**	**MIP-2**	**MIP-3**	**MIP-1**	**MIP-2**	**MIP-3**
Objective functions	**Min.** $\boldsymbol{Z_1}$	**Min.** $\boldsymbol{Z_2}$	**Min.** $\boldsymbol{Z_3}$	**Max.** $\boldsymbol{Z_1}$	**Max.** $\boldsymbol{Z_2}$	**Max.** $\boldsymbol{Z_3}$
Z_1	91403462*	110213067	94776095	134713192*	108519226	121792415
Z_2	793.66	543.67*	1207.12	1097.69	1816.66*	1039.06
Z_3	1307	2982	191*	1149	907	3843*

Table 7.2: Comparison of solutions obtained using different approaches

Solution Technique	**Total Cost** $\boldsymbol{Z_1}$	**Delivery time** $\boldsymbol{Z_2}$	**Backlog level** $\boldsymbol{Z_3}$	**Auxiliary parameters**
Deterministic programming	91403462	543.67	191	-
Neutrosophic programming	93718317	627	430	α = 0.934539 β = 0 γ = 0.836349
Intuitionistic fuzzy programming	93739937	627	430	α = 0.934539 β = 0

According to the steps given in Section 7.3, initial MO-MIP model is transformed into neutrosophic and IFP model. IFP model is formulated and solved for comparison purpose and to demonstrate performance of the proposed neutrosophic model. Table 7.2 shows the comparison of solutions obtained by deterministic, neutrosophic programming and IF programming. After following the relevant literature, values of t and s parameter is taken as 0.3 and 0.4. In the intuitionistic fuzzy programming, value of λ is taken as 0.6.

It can be seen from the Table 7.2 that solutions obtained by neutrosophic programming approach has better results for total cost objective function. The values obtained by neutrosophic programming model are near to the ideal solution of the problem. The value of auxiliary parameter of truth membership function is same as the membership function parameter of IF programming. Based on these values of objective functions, the DM or system analyst can make a proper PD plan for the actual multi-site manufacturing scenario.

7.6 Chapter Summary

This study addressed the MSIPDP problem in neutrosophic environment. A multi-objective mathematical model is developed and converted to neutrosophic compromised model. The proposed neutrosophic programming model is solved and discussed using an illustrative example of automotive industry. The performance of neutrosophic programming model is compared with intuitionistic fuzzy programming model and effect of parameters related with indeterminate and false membership functions on auxiliary parameter values are analyzed. Results obtained after solving both the models shows that neutrosophic approach provides better compromised solution in comparison to IFP approach. The formulation of compromised model and solution of this study can be helpful in development of effective neutrosophic solution approach for solving other real life industrial problems.

Chapter 8
Conclusion

8.1 Introduction

This final chapter presents the conclusion of the thesis. The chapter discuss about the contribution of present research work in the field of multi-site production and distribution planning considering several aspects. The managerial implications and limitations of current research work along with potential areas of future research are also presented.

Section 8.2 presents managerial implication, Section 8.3 provide discussion and concluding remarks of this study and Section 8.4 ends with limitations of current research work and suggestions for future research opportunities.

8.2 Managerial implications

In this thesis, novel mathematical models are formulated for MSIPDP problem of a two echelon supply chain. A real life case of an automobile company is considered to demonstrate the performance of the proposed mathematical models. The problem includes most of the characteristics of the industry such as consideration of setup, backordering and heterogeneous fleet of vehicles with distinct capacities.

There are a number of managerial implications in the different phases of the proposed research work. Use of an integrated approach in business decision making will be less reluctant because it makes the problem more understandable. The proposed integrated models and tools can be implemented in the system architecture of organizations by linking information between several modules. ERP system can be used to input and manage relevant database. This architecture will provide flexibility to decision makers and could be used with other information tools of the company.

The proposed mathematical model and solution reflects the actual scenario up to a sufficient degree of reality and can be helpful to decision makers to effectively plan PD activities in a multi-site manufacturing scenario. The heuristic approach implemented in this study could be helpful to solve real life complex problems. Also, the computational results of multi-objective models provide an estimate to management for different priority levels of objective functions. This research provides a quantitative tool for management or decision maker to analyse trade-off and priority consideration between multiple conflicting objectives. The network considered in this research is illustrated through real life scenario of manufacturing organization which provides understanding and practicality to supply chain managers.

8.3 Concluding remarks and Discussion

This research presented the integration of production and distribution functions that were used to make decisions in two echelon supply chain network. The aim was to propose a methodology that can be helpful for integration of PD planning functions and develop such kind of tools that effectively solve this coordinated problem. Traditionally, these functions were handled by managers in a hierarchal manner, which meets the expectations of decision makers but does not produce a cost optimal solution. Nowadays, with the advancement of technologies for information sharing, it is possible to employ decision tools which can solve problems in an integrated view. Considering these facts and following research gaps in the literature, research objectives are articulated and structured methodology is proposed.

Initially, a single objective mixed integer linear programming model was formulated in Chapter 3 that was used as the foundation for the mathematical models developed in later chapters. The model was focused on tactical level decisions of supply chain i.e. production and distribution planning. The model includes several aspects of production and distribution such as consideration of setup cost/time, backordering and heterogeneous transportation with constrained vehicle capacity. Exact optimization is used to solve the model which generated the need to employ a heuristic approach. Lagrangian relaxation based heuristics approach was implemented in Chapter 4. Two heuristics were proposed, one for production and another for distribution. To maintain the feasibility of solutions, top down and bottom up heuristics were applied. The computational results were compared with exact optimization results which show better performance of heuristics in terms of computational time.

Multi-criteria decision making model was developed in Chapter 5 considering three criteria: cost, distribution time and customer service level. Preemptive goal programming method was used to solve the proposed mathematical model. Sensitivity analysis was also conducted to analyze the effect of priority level on objective function values. The analysis was helpful for decision makers to assign priorities to different criteria's. Chapter 6 and 7 optimizes the multi-objective mathematical model under uncertain environment. Possibilistic programming approaches i.e. Fuzzy, Intuitionistic fuzzy and Neutrosophic programming approaches are used to handle the ambiguity in parameters and impreciseness in objective functions.

The present research work contributes to the literature in following ways:

- The study presented here is based on critical review of literature on PD planning problem in multi-site manufacturing scenario.
- The proposed mathematical model formulation is novel in the sense that it incorporates major aspects of production and distribution functions representing real life situations.
- This study has presented solution of large number of problem instances with practical problem sizes (up to 7 manufacturing sites, 15 product categories, 5 heterogeneous vehicles, 20 selling locations and 12 time periods) to demonstrate most generic results.
- Solutions of all the mathematical models formulated in this study are illustrated with real life case of an Indian automotive manufacturing organization, which shows the practical applicability of the proposed models and solution approaches.

- This study extended the mathematical model formulation by considering multiple criteria's and analysed the problem in this multi-objective framework with sensitivity analysis.
- There are several studies available on optimization and analysis of multi-objective model under uncertain environments using stochastic programming but use of possibilistic programming approaches is not implemented much. This study dealt with the impreciseness in objective functions as well as in input parameters.

8.4 Research Limits and Future scope

8.4.1 Limitations

This study has tried to cover major aspects, concepts and procedures by presenting some unique mathematical models which were representing the potential application in the manufacturing organizations. Even though, there are some limitations of present work, which are identified and presented in this section as follows.

The problem considered in this study was focused on only two echelons of the supply chain. The methodology needs to be applied at more levels and regarding different entities of the supply chain. The research methodology of current study is illustrated using case of an automotive industry and has been helpful in providing interesting results. In order to generalize the method and analytical results, the approach should be applied to other type of industries. The relaxation approaches implemented in this study is applied on production and distribution part separately, in order to obtain more integrated results, the approach should be applied on linking constraints of the problem. The logic of benefit through integration of several aspects is represented by mathematical procedures, and it should be empirically analysed and validated.

8.4.2 Future Scope

This research work has proposed several mathematical models and solution approaches for dealing with different problems in PD planning of two echelon supply chain. To illustrate practical scenario, it has attempted to address problem of consideration of major aspects, multiple criteria as well as uncertainties that are of significant importance to the organization and fill the gaps in the literature. Considering the complete supply chain incorporating all the production, distribution and transportation aspects can be investigated in mathematical modelling. To solve such as complex problem, hyrid heuristics and metaheuristics algorithms can be applied in future.

It was shown in this study that integration of different decision making aspects could be beneficial, but it seems difficult to implement due to restructure of internal processes of the organizations. Future work can be directed towards conducting an empirical study focusing on analysing factors and barriers for implementation of integration of different decision making functions.

In this study, different integrated mathematical models have been introduced that were used to generate results for decision making. Current approach is only based on model integration. It will be interesting to analyse combination of model as well as method integration.

The input data values for different problem instances were collected from the case company and generated through uniform distribution. These input data values were representing a specific situation; a possible scope for future research is to take generic data in the form of ratios and solve the model. By changing the ratios, more results can be drawn and graphically presented to see the trends and impact of change.

Considering the effect of uncertainty, possibilistic programming approaches were implemented in this study. Another segment for dealing with uncertainty i.e. stochastic programming can be applied to handle uncertainty in objective function and parameters. Current research can also be extended by considering fuzzy objectives, decision parameters and constraints.

The suggestions for future research work can be summarized as follows.

- Consideration of all the members of the supply chain for model as well as method integration.
- Empirical studies focusing enablers and barriers to implementation of integration.
- Use of heuristics and metaheuristics approaches for solving complex deterministic as well as uncertain problems.
- Use of hybrid solution approaches such as mathematical modelling and simulation, Analytical hierarchy process with genetic algorithm.
- Formulation of mathematical models based on continuous time representation for representation of real life scenario especially in process system industries.
- Implementation of stochastic programming approach to deal with uncertainty.
- Considering environmental factors and sustainability in production and distribution planning models like waste reduction in production phase and gas emission in distribution phase can be a good area of research.

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ANNEXURE A: CPLEX Codes

Integrated model:

```
/********************************************
 * OPL 12.7.1.0 Model
 * Author: Choudhary
 * Creation Date: 26-Oct-2016 at 5:59:46 pm
 ********************************************/
using CPLEX;
//sets
int numberofproducts =...;
range products = 1..numberofproducts;
int numberofsites =...;
range sites = 1..numberofsites;
int numberofmarkets =...;
range markets = 1..numberofmarkets;
int numberofperiods =...;
range periods = 1..numberofperiods;
int numberofvehicles =...;
range vehicles =1..numberofvehicles;
//parameters
int demand[products,markets,periods] =...;
float setuptime [products,sites] =...;
float processingtime [products,sites] =...;
int productioncost[products,sites] =...;
int holdingcosts [products,sites] =...;
```

```
int holdingcostm [products,markets] =...;

int vtranscost [products,vehicles,sites,markets] =...;

int ftranscost [vehicles] =...;

int setupcost [products,sites] =...;

int backordercost [products,markets] =...;

int maxstoragem [markets] =...;

int maxstorages [sites] =...;

int maxtransport [vehicles] =...;

int productioncapacity[sites,periods]=...;

float theta [products] =...;

int M = 1000000;
```

//decision variables

```
dvar int+ pquantity[products,sites,periods];

dvar int+ iquantitys[products,sites,0..numberofperiods];

dvar int+ iquantitym[products,markets,0..numberofperiods];

dvar int+ tquantity[products,vehicles,sites,markets,periods];

dvar int+ bquantity[products,markets,0..numberofperiods];

dvar boolean bvsetup[products,sites,periods];

dvar int+ ntransport[vehicles,sites,markets,periods];
```

//objective functions

```
dexpr float production = (sum(p in products, m in sites, t in periods)
productioncost[p,m]*pquantity[p,m,t]);

dexpr float holdings =   (sum(p in products, m in sites, t in periods)
holdingcosts[p,m]* iquantitys[p,m,t]);

dexpr float holdingm = (sum(p in products, l in markets, t in periods)
holdingcostm[p,l]*iquantitym[p,l,t]);

dexpr int setup =  (sum(p in products, m in sites, t in periods)
setupcost[p,m]*bvsetup[p,m,t]);
```

```
dexpr float transportation = (sum(p in products, g in vehicles, m in sites, l in markets, t in periods) vtranscost[p,g,m,l]*tquantity[p,g,m,l,t]);

dexpr float fixedtrans = (sum(g in vehicles, m in sites, l in markets, t in periods) ftranscost[g]*ntransport[g,m,l,t]);

dexpr float backorder = (sum(p in products,l in markets, t in periods) backordercost[p,l]*bquantity[p,l,t]);

dexpr float totalcost = (production + holdings + holdingm + transportation + setup + fixedtrans + backorder);

  minimize totalcost;

subject to {

        forall (p in products, m in sites)

        ctStartInvs:    iquantitys[p,m,0] == 0;

        forall (p in products, l in markets){

        ctStartInvm:   iquantitym[p,l,0] == 0;

        ctstartbquan: bquantity[p,l,0] ==0;

        }

forall (p in products, m in sites, t in periods)

//inventory balance equation at site

ct10: iquantitys[p,m,t] == iquantitys[p,m,t-1] + pquantity[p,m,t] - sum ( g in vehicles,l in markets) tquantity[p,g,m,l,t];

forall (p in products, l in markets, t in periods)

ct11: iquantitym[p,l,t-1] - iquantitym[p,l,t] + bquantity[p,l,t] - bquantity[p,l,t-1] + sum(g in vehicles,m in sites) tquantity[p,g,m,l,t] == demand[p,l,t];

forall (p in products, m in sites, t in periods)

 //production capacity constraint

 ct13: pquantity[p,m,t] <= M*bvsetup[p,m,t];

forall (m in sites, t in periods)

  //production time constraint
```

```
ct14: sum (p in products)((pquantity[p,m,t]*processingtime[p,m]) +
(setuptime[p,m]*bvsetup[p,m,t])) <= productioncapacity[m,t];

forall (g in vehicles,m in sites, l in markets, t in periods)

// transportation constraint

ct15: sum (p in products)tquantity[p,g,m,l,t] <=
maxtransport[g]*ntransport[g,m,l,t];

forall ( l in markets)

//storage capacity constraint at market

ct16: sum (p in products,t in periods)iquantitym[p,l,t] <= maxstoragem[l];

forall (m in sites)

//storage at site

ct17: sum(p in products, t in periods) iquantitys[p,m,t] <= maxstorages[m];

forall (p in products, l in markets, t in periods)

//backoredered demand

ct18: bquantity[p,l,t] <= theta[p]*demand[p,l,t];

};


execute DISPLAY {

writeln("production cost =",production);

writeln("setup cost =",setup);

writeln("holding cost at site =",holdings);

writeln("holding cost at market =",holdingm);

writeln("backorder cost =",backorder);

writeln("fixed transportation cost =",fixedtrans);

writeln("transportation cost =",transportation);

writeln("totalcost =",totalcost);

};
```

Relaxed model:

```
/********************************************
 * OPL 12.7.1.0 Model
 * Author: Choudhary
 * Creation Date: 27-Jul-2018 at 2:56:02 pm
 *********************************************/
using CPLEX;
//sets
int numberofproducts =...;
range products = 1..numberofproducts;
int numberofsites =...;
range sites = 1..numberofsites;
int numberofmarkets =...;
range markets = 1..numberofmarkets;
int numberofperiods =...;
range periods = 1..numberofperiods;
int numberofvehicles =...;
range vehicles =1..numberofvehicles;
```

//parameters

```
int demand[products,markets,periods] =...;
float setuptime [products,sites] =...;
float processingtime [products,sites] =...;
int productioncost[products,sites] =...;
int holdingcosts [products,sites] =...;
int holdingcostm [products,markets] =...;
int vtranscost [products,vehicles,sites,markets] =...;
```

```
int ftranscost [vehicles] =...;

int setupcost [products,sites] =...;

int maxstoragem [markets] =...;

int maxstorages [sites] =...;

int maxtransport [vehicles] =...;

int productioncapacity[sites,periods]=...;

int M = 1000000;
```

//decision variables

```
dvar float+ pquantity[products,sites,periods];

dvar float+ iquantitys[products,sites,0..numberofperiods];

dvar float+ iquantitym[products,markets,0..numberofperiods];

dvar float+ tquantity[products,vehicles,sites,markets,periods];

dvar boolean bvsetup[products,sites,periods];

dvar float+ ntransport[vehicles,sites,markets,periods];
```

//objective functions

```
dexpr float production = (sum(p in products, m in sites, t in
periods)productioncost[p,m]*pquantity[p,m,t]);

dexpr float holdings =   (sum(p in products, m in sites, t in
periods)holdingcosts[p,m]* iquantitys[p,m,t]);

dexpr float holdingm = (sum(p in products, l in markets, t in
periods)holdingcostm[p,l]*iquantitym[p,l,t]);

dexpr int setup =  (sum(p in products, m in sites, t in
periods)setupcost[p,m]*bvsetup[p,m,t]);

dexpr float transportation =

  (sum(p in products, g in vehicles, m in sites, l in markets, t in
periods)vtranscost[p,g,m,l]*tquantity[p,g,m,l,t]);

dexpr float fixedtrans =
```

```
  (sum(g in vehicles, m in sites, l in markets, t in periods)
ftranscost[g]*ntransport[g,m,l,t]);

dexpr float totalcost = (production + holdings + holdingm + transportation + setup
+ fixedtrans);

  minimize totalcost;

subject to {

        forall(p in products, m in sites)

        ctStartInvs:     iquantitys[p,m,0] == 0;

        forall(p in products, l in markets){

        ctStartInvm:    iquantitym[p,l,0] == 0;

        }

forall (p in products, m in sites, t in periods)

//inventory balance equation at site

ct10: iquantitys[p,m,t] == iquantitys[p,m,t-1] + pquantity[p,m,t] - sum ( g in
vehicles,l in markets) tquantity[p,g,m,l,t];

forall (p in products, l in markets, t in periods)

ct11: iquantitym[p,l,t] == iquantitym[p,l,t-1] - demand[p,l,t] + sum(g in
vehicles,m in sites) tquantity[p,g,m,l,t];

forall (p in products, m in sites, t in periods)

 //production capacity constraint

 ct13: pquantity[p,m,t] <= M*bvsetup[p,m,t];

forall (m in sites, t in periods)

  //production time constraint

ct14: sum (p in products)((pquantity[p,m,t]*processingtime[p,m]) +
(setuptime[p,m]*bvsetup[p,m,t])) <= productioncapacity[m,t];

forall (g in vehicles,m in sites, l in markets, t in periods)

// transportation constraint
```

```
ct15: sum (p in products)tquantity[p,g,m,l,t] <=
maxtransport[g]*ntransport[g,m,l,t];

forall ( l in markets)

//storage capacity constraint at market

ct16: sum (p in products,t in periods)iquantitym[p,l,t] <= maxstoragem[l];

forall ( m in sites)

//storage at site

ct17: sum(p in products,t in periods) iquantitys[p,m,t] <= maxstorages[m];

};
```

Lower bound model:

```
/*********************************************
 * OPL 12.7.1.0 Model
 * Author: Choudhary
 * Creation Date: 27-Jul-2018 at 2:48:15 pm
 *********************************************/
using CPLEX;
//sets
int numberofproducts =...;
range products = 1..numberofproducts;
int numberofsites =...;
range sites = 1..numberofsites;
int numberofmarkets =...;
range markets = 1..numberofmarkets;
int numberofperiods =...;
range periods = 1..numberofperiods;
int numberofvehicles =...;
range vehicles = 1..numberofvehicles;
//parameters
int demand[products,markets,periods] =...;
float setuptime [products,sites] =...;
float processingtime [products,sites] =...;
int productioncost[products,sites] =...;
int holdingcosts [products,sites] =...;
int holdingcostm [products,markets] =...;
int vtranscost [products,vehicles,sites,markets] =...;
```

int ftranscost [vehicles] =...;

int setupcost [products,sites] =...;

int maxstoragem [markets] =...;

int maxstorages [sites] =...;

int maxtransport [vehicles] =...;

int productioncapacity[sites,periods]=...;

int M = 1000000;

float lambda[products] =...;

//decision variables

dvar int+ pquantity[products,sites,periods];

dvar int+ iquantitys[products,sites,0..numberofperiods];

dvar int+ iquantitym[products,markets,0..numberofperiods];

dvar int+ tquantity[products,vehicles,sites,markets,periods];

dvar boolean bvsetup[products,sites,periods];

dvar int+ ntransport[vehicles,sites,markets,periods];

//objective functions

dexpr float lagrangian_obj = (sum(p in products, m in sites, t in periods) productioncost[p,m]*pquantity[p,m,t]) +

(sum(p in products, m in sites, t in periods) holdingcosts[p,m]*iquantitys[p,m,t]) +

(sum(p in products, l in markets, t in periods) holdingcostm[p,l]*iquantitym[p,l,t]) + (sum(p in products, g in vehicles, m in sites, l in markets, t in periods) vtranscost[p,g,m,l]*tquantity[p,g,m,l,t]) + (sum(p in products, m in sites, t in periods) setupcost[p,m]*bvsetup[p,m,t]) + (sum(g in vehicles, m in sites, l in markets, t in periods) ftranscost[g]*ntransport[g,m,l,t]) + (sum(p in products, m in sites, t in periods) (lambda[p] * ((productioncapacity[m,t] - pquantity[p,m,t]*processingtime[p,m]) + (setuptime[p,m]*bvsetup[p,m,t]))));

minimize lagrangian_obj;

subject to {

forall(p in products, m in sites){

```
ctStartInvs:      iquantitys[p,m,0] == 0;           }

forall(p in products, l in markets){

ctStartInvm:    iquantitym[p,l,0] == 0;           }

forall (p in products, m in sites, t in periods)

//inventory balance equation at site

ct10: iquantitys[p,m,t] == iquantitys[p,m,t-1] + pquantity[p,m,t] - sum ( g in
vehicles,l in markets) tquantity[p,g,m,l,t];

forall (p in products, l in markets, t in periods)

ct11: iquantitym[p,l,t] == iquantitym[p,l,t-1] - demand[p,l,t] + sum(g in
vehicles,m in sites) tquantity[p,g,m,l,t];

forall (p in products, m in sites, t in periods)

 //production capacity constraint

 ct13: pquantity[p,m,t] <= M *bvsetup[p,m,t];

forall (g in vehicles,m in sites, l in markets, t in periods)

// transportation constraint

ct15: sum (p in products)tquantity[p,g,m,l,t] <=
maxtransport[g]*ntransport[g,m,l,t];

forall ( l in markets)

//storage capacity constraint at market

ct16: sum (p in products, t in periods)iquantitym[p,l,t] <= maxstoragem[l];

forall ( m in sites)

//storage at site

ct17: sum(p in products, t in periods) iquantitys[p,m,t] <= maxstorages[m];

};

execute DISPLAY {

writeln("lagrangian_obj = ", lagrangian_obj);

};
```

Upper bound

```
/*********************************************
 * OPL 12.7.1.0 Model
 * Author: Choudhary
 * Creation Date: 27-Jul-2018 at 2:48:30 pm
 *********************************************/
using CPLEX;
```

//sets

```
int numberofproducts =...;
range products = 1..numberofproducts;
int numberofsites =...;
range sites = 1..numberofsites;
int numberofmarkets =...;
range markets = 1..numberofmarkets;
int numberofperiods =...;
range periods = 1..numberofperiods;
int numberofvehicles =...;
range vehicles = 1..numberofvehicles;
```

//parameters

```
int demand[products,markets,periods] =...;
float setuptime [products,sites] =...;
float processingtime [products,sites] =...;
int productioncost[products,sites] =...;
int holdingcosts [products,sites] =...;
int holdingcostm [products,markets] =...;
int vtranscost [products,vehicles,sites,markets] =...;
```

int ftranscost [vehicles] =...;

int setupcost [products,sites] =...;

int maxstoragem [markets] =...;

int maxstorages [sites] =...;

int maxtransport [vehicles] =...;

float productioncapacity[sites,periods]=...;

int M = 1000000;

//this data is calculated in the script using result of previous solve

int Spquantity[products,sites,periods] =...;

int Sbvsetup[products,sites,periods] =...;

//decision variables

dvar int+ iquantitys[products,sites,0..numberofperiods];

dvar int+ iquantitym[products,markets,0..numberofperiods];

dvar int+ tquantity[products,vehicles,sites,markets,periods];

dvar int+ ntransport[vehicles,sites,markets,periods];

//objective functions

dexpr float production = (sum(p in products, m in sites, t in periods)productioncost[p,m]*Spquantity[p,m,t]);

dexpr float holdings = (sum(p in products, m in sites, t in periods) holdingcosts[p,m]* iquantitys[p,m,t]);

dexpr float holdingm = (sum(p in products, l in markets, t in periods)holdingcostm[p,l]*iquantitym[p,l,t]);

dexpr float transportation = (sum(p in products, g in vehicles, m in sites, l in markets, t in periods)vtranscost[p,g,m,l]*tquantity[p,g,m,l,t]);

dexpr float setup = (sum(p in products, m in sites, t in periods)setupcost[p,m]*Sbvsetup[p,m,t]);

dexpr float fixedtrans = (sum(g in vehicles, m in sites, l in markets, t in periods)ftranscost[g]*ntransport[g,m,l,t]);

```
dexpr float totalcost = (production + holdings + holdingm + transportation + setup
+ fixedtrans);

  minimize totalcost;

subject to {

        forall(p in products, m in sites){

        ctStartInvs:    iquantitys[p,m,0] == 0;        }

        forall(p in products, l in markets){

        ctStartInvm:    iquantitym[p,l,0] == 0;        }

forall (p in products, m in sites, t in periods)

//inventory balance equation at site

ct10: iquantitys[p,m,t] == iquantitys[p,m,t-1] + Spquantity[p,m,t] - sum ( g in
vehicles,l in markets) tquantity[p,g,m,l,t];

forall (p in products, l in markets, t in periods)

ct11: iquantitym[p,l,t] == iquantitym[p,l,t-1] - demand[p,l,t] + sum(g in
vehicles,m in sites) tquantity[p,g,m,l,t];

forall (p in products, m in sites, t in periods)

 //production capacity constraint

 ct13: Spquantity[p,m,t] <= M*Sbvsetup[p,m,t];

forall (p in products, m in sites, t in periods)

  //production time constraint

ct14: (Spquantity[p,m,t]*processingtime[p,m]) +
(setuptime[p,m]*Sbvsetup[p,m,t]) <= productioncapacity[m,t];

forall (g in vehicles,m in sites, l in markets, t in periods)

// transportation constraint

ct15: sum (p in products) tquantity[p,g,m,l,t] <=
maxtransport[g]*ntransport[g,m,l,t];
```

```
forall ( l in markets)

//storage capacity constraint at market

ct16: sum (p in products,t in periods) iquantitym[p,l,t] <= maxstoragem[l];

forall ( m in sites)

//storage at site

ct17: sum(p in products,t in periods) iquantitys[p,m,t] <= maxstorages[m];

};
```

Lagrangian Relaxation for Production problem

```
/*********************************************
 * OPL 12.7.1.0 Model
 * Author: Choudhary
 * Creation Date: 27-Jul-2018 at 2:45:18 pm
 *********************************************/
//sets
int numberofproducts =...;
range products = 1..numberofproducts;
int numberofsites =...;
range sites = 1..numberofsites;
int numberofmarkets =...;
range markets = 1..numberofmarkets;
int numberofperiods =...;
range periods = 1..numberofperiods;
int numberofvehicles =...;
range vehicles = 1..numberofvehicles;
//parameters
int demand[products,markets,periods] =...;
float setuptime [products,sites] =...;
float processingtime [products,sites] =...;
int productioncost[products,sites] =...;
int holdingcosts [products,sites] =...;
int holdingcostm [products,markets] =...;
int vtranscost [products,vehicles,sites,markets] =...;
int ftranscost [vehicles] =...;
```

```
int setupcost [products,sites] =...;
int maxstoragem [markets] =...;
int maxstorages [sites] =...;
int maxtransport [vehicles] =...;
int productioncapacity[sites,periods]=...;
int M = 1000000;
main {
 function maxArray(arr) {
  var max;
  if (arr.size <= 0)
   max = undefined;
  else {
   max = arr[1];
   for (var p=2;p<=arr.size;p++)
    if (arr[p]>max)
     max = arr[p];
  }
  return max;
 }
 thisOplModel.settings.mainEndEnabled = true;
 thisOplModel.generate();
 var data = thisOplModel.dataElements;
 writeln("--- LP Relaxation ---");
 var m1Source = new IloOplModelSource("relax.mod");
 var m1Cplex = new IloCplex();
 var m1Def = new IloOplModelDefinition(m1Source);
```

```
var m1Opl = new IloOplModel(m1Def,m1Cplex);
m1Opl.addDataSource(data);
m1Opl.generate();
if (m1Cplex.solve()) {
  var LB = m1Cplex.getObjValue();
}
m1Opl.end();
m1Def.end();
m1Cplex.end();
m1Source.end();

var m2Source = new IloOplModelSource("Lowerbound.mod");
var m2Cplex = new IloCplex();
var m2Def = new IloOplModelDefinition(m2Source);
// model used to retrieve data common at each iteration
var m2_init = new IloOplModel(m2Def,m2Cplex);
m2_init.addDataSource(data);
var datalambda = new IloOplDataSource("lambda.dat");
m2_init.addDataSource(datalambda);
var data2 = m2_init.dataElements;
m2_init.generate();
 var m3Source = new IloOplModelSource("Upperbound.mod");
var m3Cplex = new IloCplex();
var m3Def = new IloOplModelDefinition(m3Source);
var m3_init = new IloOplModel(m3Def,m3Cplex);
m3_init.addDataSource(data);
```

```
var dataSpquantity = new IloOplDataSource("Spquantity.dat");
var dataSbvsetup = new IloOplDataSource("Sbvsetup.dat");
m3_init.addDataSource(dataSpquantity);
m3_init.addDataSource(dataSbvsetup);
m3_init.generate();
var data3 = m3_init.dataElements;
  // begin the Lagrangian calculation here
writeln();
writeln(" beginning the Lagrangian calculation here... ");
// maximum number of iteration we want to run the loop
var iter_limit = 10;
// initialize arrays and variables used in the loop that follows
var same = 0;
var same_limit = 3;
var slack = new Array(thisOplModel.products);
var temp = new Array(thisOplModel.products);
var lambda = new Array(thisOplModel.products);
var some = 0;
var excess = new Array(thisOplModel.sites);
var UB = 0;
UB += 56165000;
for (var p in thisOplModel.products) {
  slack[p] = 0.0;
  temp[p] = 0.0;
  lambda[p] = 0.0;
```

```
}
for (var s in thisOplModel.sites) {
for (var t in thisOplModel.periods) {
 excess [s][t] = 0.0;
}}

var scale = 1.0;
var norm = 0.0;
var step = 0.0;
//arrays to store the UB, LB, scale and step values at each iteration
var LBlog = new Array(iter_limit);
var UBlog = new Array(iter_limit);
var scalelog = new Array(iter_limit);
var steplog = new Array(iter_limit);
// executes LowerBound and UpperBound model
for(var k=1; k<=iter_limit;k++) {
  LBlog[k] = 0.0;
  UBlog[k] = 0.0;
  scalelog[k] = 0.0;
  steplog[k] = 0.0;
  writeln();
  writeln(" ITERATION:  " , k );
  var m2 = new IloOplModel(m2Def,m2Cplex);
  for (p in thisOplModel.products){
    data2.lambda[p] = lambda[p];
  }
```

```
m2.addDataSource(data2);

m2.generate();

var Lagrangian;

if (m2Cplex.solve()) {

  Lagrangian = m2Cplex.getObjValue();

}

norm = 0;

for(p in thisOplModel.products) {

  slack[p] = 0;

  for (var m in thisOplModel.sites) {

 for (var t=1;t<=thisOplModel.numberofperiods;t++){

 slack[p] += ((thisOplModel.processingtime[p][m]) * (m2.pquantity[p][m][t])) + ((thisOplModel.setuptime[p][m]) * (m2.bvsetup[p][m][t]));

 slack[p] -= thisOplModel.productioncapacity[m][t];}}

  norm += Opl.pow(slack[p],2);}


writeln("lower bound obj value: ", Lagrangian);

if (Lagrangian > LB + 0.000001) {

  LB = Lagrangian;

  same = 0;

}

else {

  same ++;

}

if (same == same_limit) {

 scale = scale/2;
```

```
 same = 0;
 }
step = scale * ((UB - Lagrangian) / norm);
 for (m in thisOplModel.sites) {
 for (t in thisOplModel.periods){
 excess = (m2.productioncapacity[m][t]) -
((m2.pquantity[p][m][t]*m2.processingtime[p][m])+(m2.setuptime[p][m] *
m2.bvsetup[p][m][t]));
 }}
 var sum1 = 0;
 var sum2 = 0;
for (p in thisOplModel.products){
for (m in thisOplModel.sites) {
for (t in thisOplModel.periods){
sum1 += (m2.pquantity[p][m][t]* thisOplModel.processingtime[p][m]) +
(thisOplModel.setuptime[p][m] * m2.bvsetup[p][m][t]);
}}}
 for (m in thisOplModel.sites) {
 for (t in thisOplModel.periods){
 sum2 -= m2.productioncapacity[m][t];
 }}

 if (sum1 <= sum2){
 // solve the model to get the Upper Bound
 var m3 = new IloOplModel(m3Def,m3Cplex);
 // get the dvar values of the model just solved
  // to use in Upper Bound model
```

```
for (p in thisOplModel.products){
  for (m in thisOplModel.sites){
  for (t in thisOplModel.periods){
  data3.Spquantity[p][m][t] = m2.pquantity[p][m][t];
  data3.Sbvsetup[p][m][t] = m2.bvsetup[p][m][t];
      }}}
  m3.addDataSource(data3);
  m3.generate();
  if (m3Cplex.solve()) {
    writeln("upper bound model value 1: ", m3Cplex.getObjValue());
    if (m3Cplex.getObjValue() < UB)
      UB = m3Cplex.getObjValue();
  }
  m3.end();
  }
if (sum1 >= sum2)  {
  var m3 = new IloOplModel(m3Def,m3Cplex);
for (p in thisOplModel.products){
  for (m in thisOplModel.sites){
  for (t=3; t<=thisOplModel.numberofperiods;t--){
  data3.Spquantity [p][m][t] = m2.pquantity[p][m][t]+
(excess[m][t]/m2.processingtime[p][m]);
  data3.Sbvsetup[p][m][t] = m2.bvsetup[p][m][t];
}}}
  for (p in thisOplModel.products){
  for (m in thisOplModel.sites){
```

```
    for (t=1; t<=thisOplModel.numberofperiods;t++){
    data3.Spquantity [p][m][t] = m2.pquantity[p][m][t]+
(excess[m][t]/m2.processingtime[p][m]);
    data3.Sbvsetup[p][m][t] = m2.bvsetup[p][m][t];
  }}}
    m3.addDataSource(data3);
    m3.generate();

    if (m3Cplex.solve()) {
      writeln("upper bound model value 2: ", m3Cplex.getObjValue());
      if (m3Cplex.getObjValue() < UB)
        UB = m3Cplex.getObjValue();
    }
    m3.end();
    }
  // update mult to pass it as input data to LowerBound model in next iteration
  for(p in thisOplModel.products) {
    temp[p] = lambda[p];
    if (temp[p] - (step * slack[p]) > 0 )
      lambda[p] = temp[p] - (step * slack[p]) ;
    else
      lambda[p] = 0;
  }
  LBlog[k] = LB;
  UBlog[k] = UB;
  scalelog[k] = scale;
```

```
  steplog[k] = step;
  m2.end();
  }
 //end of main "for loop"
 datalambda.end();
 m2_init.end();
 dataSpquantity.end();
dataSbvsetup.end();
 m3_init.end();
 writeln("---------------------");
 writeln();
 write("LBlog = ");
 for (p=1;p<=iter_limit;p++)
   writeln(LBlog[p]);
 writeln();
 writeln("UBlob = ");
 for (p=1;p<=iter_limit;p++)
   writeln(UBlog[p]);
 writeln();
 writeln("scalelog = ");
 for (p=1;p<=iter_limit;p++)
   writeln(scalelog[p]);
 writeln();
 writeln("steplog = ");
 for (p=1;p<=iter_limit;p++)
   writeln(steplog[p]);
```

```
  writeln();
writeln("lambdalog = ");
for (p=1;p<=iter_limit;p++)
  writeln(lambda[p]);
m3Def.end();
m3Cplex.end();
m3Source.end();
m2Def.end();
m2Cplex.end();
m2Source.end();
  }
```

Main LR for transport

```
/*********************************************
 * OPL 12.7.1.0 Model
 * Author: Choudhary
 * Creation Date: 01-Nov-2018 at 5:42:11 pm
 *********************************************/
//sets
int numberofproducts =...;
range products = 1..numberofproducts;
int numberofsites =...;
range sites = 1..numberofsites;
int numberofmarkets =...;
range markets = 1..numberofmarkets;
int numberofperiods =...;
range periods = 1..numberofperiods;
int numberofvehicles =...;
range vehicles = 1..numberofvehicles;
//parameters
int demand[products,markets,periods] =...;
float setuptime [products,sites] =...;
float processingtime [products,sites] =...;
int productioncost[products,sites] =...;
int holdingcosts [products,sites] =...;
int holdingcostm [products,markets] =...;
int vtranscost [products,vehicles,sites,markets] =...;
int ftranscost [vehicles] =...;
```

```
int setupcost [products,sites] =...;
int maxstoragem [markets] =...;
int maxstorages [sites] =...;
int maxtransport [vehicles] =...;
int productioncapacity[sites,periods]=...;
int M = 1000000;
tuple sproducts{
      key int numberofproducts;
      int ftranscost;
}
main {
 function maxArray(arr) {
  var max;
  if (arr.size <= 0)
   max = undefined;
  else {
   max = arr[1];
   for (var t=2;t<=arr.size;t++)
    if (arr[t]>max)
     max = arr[t];
  }
  return max;
 }
 thisOplModel.settings.mainEndEnabled = true;
 thisOplModel.generate();
 var data = thisOplModel.dataElements;
```

```
writeln("--- LP Relaxation ---");
var m1Source = new IloOplModelSource("relaxTrans.mod");
var m1Cplex = new IloCplex();
var m1Def = new IloOplModelDefinition(m1Source);
var m1Opl = new IloOplModel(m1Def,m1Cplex);
m1Opl.addDataSource(data);
m1Opl.generate();
if (m1Cplex.solve()) {
  var LB = m1Cplex.getObjValue();
}
m1Opl.end();
m1Def.end();
m1Cplex.end();
m1Source.end();
var m2Source = new IloOplModelSource("LowerboundTrans.mod");
var m2Cplex = new IloCplex();
var m2Def = new IloOplModelDefinition(m2Source);
// model used to retrieve data common at each iteration
var m2_init = new IloOplModel(m2Def,m2Cplex);
m2_init.addDataSource(data);
var databeta = new IloOplDataSource("beta.dat");
m2_init.addDataSource(databeta);
var data2 = m2_init.dataElements;
m2_init.generate();
```

```
var m3Source = new IloOplModelSource("UpperboundTrans.mod");
var m3Cplex = new IloCplex();
var m3Def = new IloOplModelDefinition(m3Source);
var m3_init = new IloOplModel(m3Def,m3Cplex);
m3_init.addDataSource(data);
var dataRtquantity = new IloOplDataSource("Rtquantity.dat");
var dataRntransport = new IloOplDataSource("Rntransport.dat");
m3_init.addDataSource(dataRtquantity);
m3_init.addDataSource(dataRntransport);
m3_init.generate();
var data3 = m3_init.dataElements;
// begin the Lagrangian calculation here
writeln();
writeln(" beginning the lagrangian calculation here... ");
// maximum number of iteration we want to run the loop
var iter_limit = 10;
// initialize arrays and variables used in the loop that follows
var same = 0;
var same_limit = 3;
var slack = new Array(thisOplModel.periods);
var temp = new Array(thisOplModel.periods);
var beta = new Array(thisOplModel.periods);
var UB = 0;
UB += 56165000;
for (var t in thisOplModel.periods) {
  slack[t] = 0.0;
```

```
  temp[t] = 0.0;
  beta[t] = 0.0;
}
var scale = 1.0;
var norm = 0.0;
var step = 0.0;
//arrays to store the UB, LB, scale and step values at each iteration
var LBlog = new Array(iter_limit);
var UBlog = new Array(iter_limit);
var scalelog = new Array(iter_limit);
var steplog = new Array(iter_limit);
// executes LowerBound and UpperBound model
for(var k=1; k<=iter_limit;k++) {
  LBlog[k] = 0.0;
  UBlog[k] = 0.0;
  scalelog[k] = 0.0;
  steplog[k] = 0.0;
  writeln();
  writeln(" ITERATION:  " , k );
  var m2 = new IloOplModel(m2Def,m2Cplex);
  for (t in thisOplModel.periods){
    data2.beta[t] = beta[t];
  }
  m2.addDataSource(data2);
  m2.generate();
  var Lagrangian;
```

```
    if (m2Cplex.solve()) {
      Lagrangian = m2Cplex.getObjValue();
    }
    norm = 0;
    for(t in thisOplModel.periods) {
      slack[t] = 0;
    for (var g in thisOplModel.vehicles) {
    for (var m in thisOplModel.sites) {
    for (var l in thisOplModel.markets) {
    slack[t] -= ((m2.maxtransport[g]) * (m2.ntransport[g][m][l][t]));
    for (var p in thisOplModel.products){
     slack[t] += m2.tquantity[p][g][m][l][t];
     }}}}}
      for (t in thisOplModel.periods) {
      norm += Opl.pow(slack[t],2);
}

    writeln("lower bound obj value: ", Lagrangian);
    if (Lagrangian > LB + 0.000001) {
      LB = Lagrangian;
      same = 0;
    }
    else {
      same ++;
    }
    if (same == same_limit) {
```

```
  scale = scale/2;
  same = 0;
 }
step = scale * ((UB - Lagrangian) / norm);
 var sum1 = 0;
 var sum2 = 0;
for (p in thisOplModel.products){
for (g in thisOplModel.vehicles) {
for (m in thisOplModel.sites) {
for (l in thisOplModel.markets) {
for (t in thisOplModel.periods){
sum1 += (m2.tquantity[p][g][m][l][t]);
}}}}}
 for (g in thisOplModel.vehicles) {
 for (m in thisOplModel.sites) {
 for (l in thisOplModel.markets) {
 for (t in thisOplModel.periods){
 sum2 += (m2.maxtransport[g]*m2.ntransport[g][m][l][t]);
 }}}}
  if (sum1 >= sum2) {
   var m3 = new IloOplModel(m3Def,m3Cplex);
  sorted {sproducts} sp = {<1,2,3> | <1,2,3 in > ftranscost};
  for (sp in thisOplModel.products){
   for (m in thisOplModel.sites){
   for (t=1; t<=thisOplModel.numberofperiods;t++){
   data3.Rtquantity[p][g][m][l][t] = m2.tquantity[p][g][m][l][t];
```

```
}}}}
else if (sum1 <= sum2){
var m3 = new IloOplModel(m3Def,m3Cplex);
  for (g in thisOplModel.vehicles){
  for (m in thisOplModel.sites){
  for (t=1; t<=thisOplModel.numberofperiods;t++){
  data3.Rntransport[g][m][l][t] = m2.ntransport[g][m][l][t];
  for (p in thisOplModel.products){
  data3.Rtquantity[p][g][m][l][t] = m2.tquantity[p][g][m][l][t];
  }}}}
  m3.addDataSource(data3);
  m3.generate();


  if (m3Cplex.solve()) {
    writeln("upper bound model value: ", m3Cplex.getObjValue());
    if (m3Cplex.getObjValue() < UB)
      UB = m3Cplex.getObjValue();
  }
  m3.end();
  }
// update mult to pass it as input data to LowerBound model in next iteration
for(t in thisOplModel.periods) {
  temp[t] = beta[t];
  if (temp[t] - (step * slack[t]) > 0 )
    beta[t] = temp[t] - (step * slack[t]) ;
  else
```

```
    beta[t] = 0;
  }
  LBlog[k] = LB;
  UBlog[k] = UB;
  scalelog[k] = scale;
  steplog[k] = step;

  m2.end();
  }
//end of main "for loop"
databeta.end();
m2_init.end();
dataRtquantity.end();
dataRntransport.end();
m3_init.end();
writeln("---------------------");
writeln();
write("LBlog = ");
for (p=1;p<=iter_limit;p++)
  writeln(LBlog[p]);
writeln();
writeln("UBlob = ");
for (p=1;p<=iter_limit;p++)
  writeln(UBlog[p]);
writeln();
writeln("scalelog = ");
```

```
 for (p=1;p<=iter_limit;p++)
   writeln(scalelog[p]);
 writeln();
 writeln("steplog = ");
 for (p=1;p<=iter_limit;p++)
   writeln(steplog[p]);
 m3Def.end();
 m3Cplex.end();
 m3Source.end();
 m2Def.end();
 m2Cplex.end();
 m2Source.end();
}
```

ANNEXURE B: List of Publications

Papers Published or Accepted in International Journals:

1. **Badhotiya, G. K.**, Soni, G. & Mittal, M. L. (2018). Fuzzy multi-objective optimization for multi-site integrated production and distribution planning in two echelon supply chain. *The International Journal of Advanced Manufacturing Technology*. http://dx.doi.org/10.1007/s00170-018-3204-2 In Press [**SCI, SCIE, SCOPUS – 2.6**]

2. **Badhotiya, G. K.**, Soni, G., & Mittal, M. L. (2018). Multi-site integrated production and distribution planning: a multi-objective approach. *International Journal of Product Development*, 22(6), 488-501. [**SCOPUS**]

3. **Badhotiya, G. K.**, Soni, G., & Mittal, M. L. (2018). An analysis of mathematical models for multi-site production and distribution planning. *International Journal of Intelligent Enterprise*, 5(4), 309-332. [**SCOPUS**]

Papers submitted or working for international journals:

1. Badhotiya, G. K., Soni, G., & Mittal, M. L. Multi-Objective Optimization of Multi-Site Integrated Production and Distribution Planning in Neutrosophic Environment. *Computers in Industry* [**SCIE, SCOPUS – 2.8**] (Communicated)

2. Badhotiya, G. K., Soni, G., & Mittal, M. L. Lagrangian heuristics for Multi-Site Integrated Production and Distribution Planning. (To be communicated)

Papers published in proceedings of peer reviewed international conferences:

1. Badhotiya G.K., Prakash S., Soni G., Mittal M.L., (2016). A Review on Planning and Scheduling in Multi-Plant Manufacturing, *International Conference on Emerging Trends in Mechanical Engineering* (ICETiME-2016), 23-24, September, IFHE Hyderabad. (ISBN - 9788131427941).

2. Badhotiya G.K., Soni G., Mittal M.L., "A preemptive goal programming model for multi-site production and distribution planning", *International Conference on Recent Advances & Applications in Computer Engineering* (RAACE - 2017), 23-25 November, 2017, proceedings in *Asset Analytics Series - Springer* Book, (Performance of Integrated Systems and Its Software Engineering Applications)

***SCI*:** Science Citation Index (Web of Science- Thomson Reuters)

***SCIE*:** Science Citation Index Expanded (Web of Science- Thomson Reuters)

***SSCI*:** Social Science Citation Index (Web of Science- Thomson Reuters)

***Scopus*:** Scopus Index (Elsevier)

ANNEXURE C: Biographical Profile of the Candidate

Gaurav Kumar Badhotiya was born on 20th May, 1990 in Ladnun, Dist. Nagaur, Rajasthan, India. He received his Polytechnic degree in Mechanical Engineering from Government Polytechnic College, Bikaner (Rajasthan), India, Bachelor of Technology (B.Tech.) degree in Production & Industrial Engineering from UCE, Kota, India, and Master of Technology (M.Tech.) degree in Manufacturing System Engineering from MNIT Jaipur (Rajasthan), India. He is currently pursuing Ph.D. in Industrial Engineering from Malaviya National Institute of Technology Jaipur (Rajasthan), India. He has worked as Research associate in MNREGA project of Government of Rajasthan, India for six months duration after completing his Post-graduation. His research interests are inclined towards production planning, supply chain management and optimisation techniques. He is a member of Indian Institution of Industrial Engineering, and Production and Operations Management Society (USA).