CHAPTER 1

INTRODUCTION

1.1 Background

Reliability of an item is defined as the probability that it will perform its intended function for a stated period of time in intended use environment.

In today's competitive world, reliability of a product is become a major issue for the design community and manufactures. Customers expect the product to be functionally reliable for its intended life period. But with the passage of time degradation process takes place in material properties and uncertain operating condition cause these product deliver inconsistent performance and sometime they become fail prematurely (Singh et al., 2010). These adverse condition force manufactures or designers to design highly reliable and durable product by considering the behavior with usage time and the uncertain operating condition.

To achieve this objective it's become a challenge for the design engineer to evolve various approaches and develop different tools to deal with the degradation of the product, variability and uncertainty at early design stage so as to improve the reliability and the overall product quality with the consistent performance. Understanding and modeling of the degradation process of any product or system can help to achieve its real life behavior under giving operating condition.

It is a great challenge for the design community to integrate varying customer requirements with the assured consistent product performance throughout its designed life period by making design to immune to factor like variability in material, uncertainty and the degradation right at the design stage to sustain in the competitive market. This demands the development of different techniques to consider the real life degradation behavior of the product. So there is need to develop the different method for reliability prediction.

Over the six decade progress different method developed for reliability prediction as statistical methods (exponential distribution, Weibull distribution, normal distribution, lognormal distribution , Bayesian statistical method), Reliability growth models (continuous and discrete), diagram based model for system analysis (failure mode and effect analysis, event tree analysis, fault tree analysis, reliability block diagram, decision tree analysis, root cause analysis), Analytical methods (Boolean algebra, Markov models, Monto Carlo simulation, optimization) and Physics-of-failure(degradation model) (Bhamare et al, 2007)

The traditional statistical methods do not provide any insight into failure mechanism and help improve product reliability during early product development stage. Problem with the classical statistical method is that they required a large data set to achieve statistically significant result. .

Degradation is the reduction in performance, reliability and life span of assets. Most assets degrade as they age or deteriorate due to some factors that termed as covariates. Hence, reliability declines when assets degrade or deteriorate. Assets fail when their level of degradation reaches a specified failure threshold.

1.2 Research motivation

Since degradation is complex and irreversible stochastic process, which ultimate lead to the failure of the product. The rate at which the degradation occurs is a function of the time, operating condition and quality of the material. So, to design the reliable products, it becomes necessary to understand the degradation process of the products by considering all sources of uncertainty and variability. Literature reveals that degradation data has been widely used for the reliability prediction as well as the reliability improvement.

Also, degradation in material strength of the components is one of the major reasons that contributed towards decreasing the reliability of any products with the time. Due to this, degradation in strength has direct implications on the degradation of the quality or performance characteristics of the products. Since many probabilistic model developed from time to time which takes into effect different condition. But there is effect of load interaction is not considered for any probabilistic model while it affect any component more while predicting the reliability of the product. So there is need to develop a nonlinear damage accumulation model which takes into effect load sequencing effect as well as load interaction effects.

1.3 Research objectives

The research gaps presented in the form of prime focus areas presented in the literature have been taken as the motivational aspect to undertake the proposed study. The proposed study aims to develop the method for the prediction of reliability.

The specific research objectives of the study undertaken as follow:

1. To develop a probabilistic degradation model to predict the degradation behavior of fatigue strength and reliability of mechanical component subjected to fatigue life while degradation behavior consider as nonlinear for single stress level.
2. To develop a probabilistic degradation model to predict the degradation behavior of fatigue strength and reliability of mechanical component subjected to fatigue life while degradation behavior consider as nonlinear for multi-stress level.
3. To develop a probabilistic degradation model to predict the degradation behavior of fatigue strength and reliability of mechanical component subjected to fatigue life while loading is in sequence.
4. To develop a probabilistic degradation model to predict the degradation behavior of fatigue strength and reliability of mechanical component subjected to fatigue life while load interaction effect take place.

1.4 Research approach

In order to fulfill the research objectives detailed in the previous section, the research study divide the work into two different stages by translating research objectives into research question.

Research question 1: is it possible to develop a probabilistic degradation model for predicting behavior of mechanical component while considering degradation behavior as non-linear.

Literature revels a good number of mechanical components are exposed to fatigue during their usage life. Fatigue is one of the major reasons for failure of the component. This has led to understanding of the mechanism behind the fatigue damage and the development of different type damage accumulation model for predicting the reliability and the useful life of the component. Earlier model are based on the deterministic nature of the process. But later on several researcher works on the probabilistic nature of the process (Liu and Mahadevan, 2007) and (Zhu et al., 2012). This fact has led to the development of an easy approach for nonlinear damage accumulation.

Research question 2: Is it possible to develop a model which takes into effect like load sequencing and load interaction?

When there is fatigue loading on any component, then there is change of loading take place from time to time. So load interaction becomes an important factor for development of any model. Many researchers developed model for load sequencing for deterministic process. So there is a need to develop a model which takes into account both load sequencing and load interaction effect into consideration while damage accumulation is a nonlinear phenomenon.

1.5 Structure of thesis

The remainder of this thesis consists of 5 chapters. Chapter 2 presents a detailed review of the literature on the degradation.

Chapter 3 discusses about the methodology used for this research work in a systematic and step by step manner.

Chapter 4 details out the methodology and the approach used to develop a probabilistic damage accumulation model for predicting the behavior of the mechanical component subjected to fatigue.

Chapter 5 gives the numerical analysis of the proposed work.

Chapter 6 concludes the thesis with final discussion including that on the future scope of the research work undertaken.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Degradation is the reduction in performance, reliability and life span of assets. Most assets degrade as they age or deteriorate due to some factors that termed as covariates. Hence, reliability declines when assets degrade or deteriorate. Assets fail when their level of degradation reaches a specified failure threshold.

Degradation model in reliability analysis can be classified according to the given figure below:

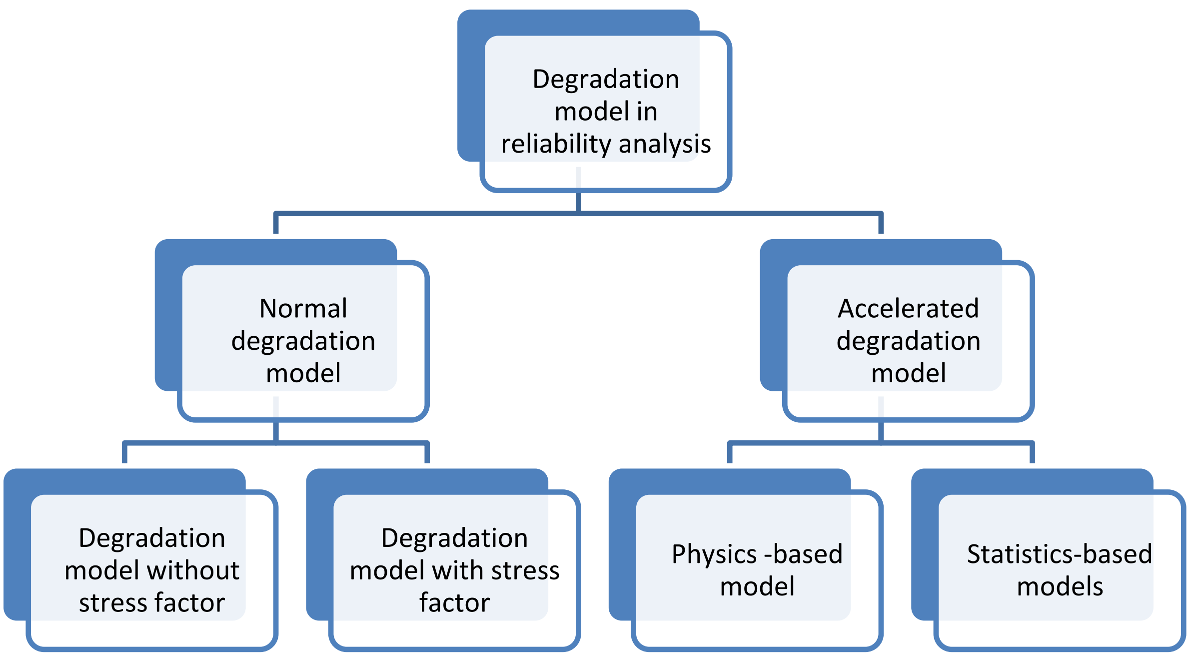

Figure 2.1: Degradation models in reliability analysis (Gorjian et al.,2006)

2.2 Degradation modeling

If a product or component is unable to perform its intended function then it is termed as the failure of product. Degradation is an irreversible phenomenon which is the cause of failure of any product. Degradation of the product occurs with the usage of product and result decline in the performance of the product. The failure is specified when the degradation reached or cross the pre-specified level (called as threshold level) (Coit et al., 2005). In degradation modeling system degradation is measure with the some performance parameters. Depends upon the type of product or component, there are different possible degradation measure that can be defined to capture the failure of any product.

(Augustine et al., 2011) for the failure modes that are identified as the occurrence of pre-specified level of degradation; a degradation models essentially gives their time to failure distribution. Degradation behavior and its effect on the performance of the product can be understood by considering the monotonically decreasing degradation path of a population of similarly degrading parts as shown in the figure 2.2.

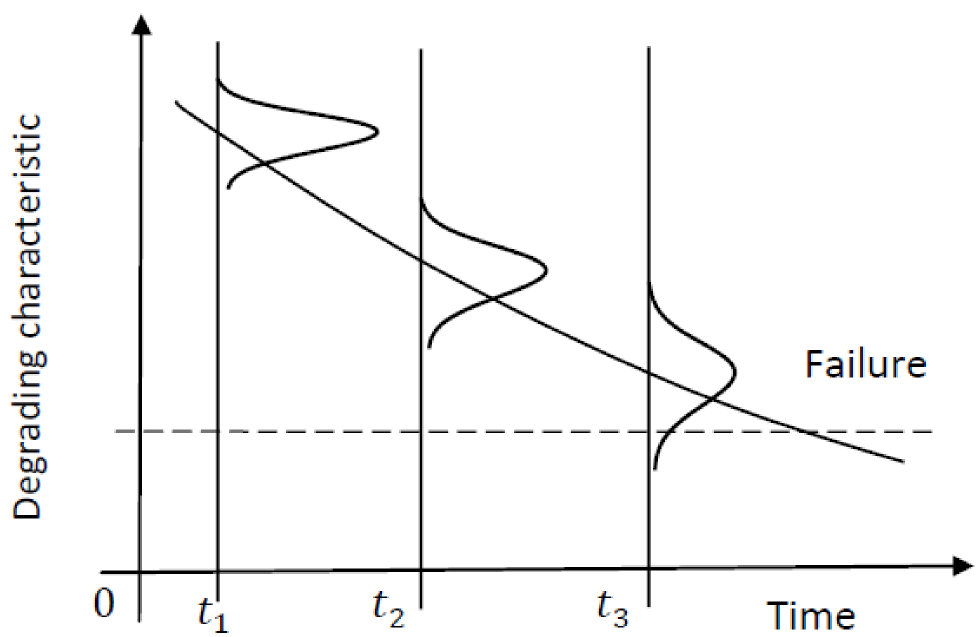

Figure 2.2: degradation behavior

Degradation with the time is probabilistic in nature and it can be represented with the mean and variance characteristics (Coit et al., 2005). This may not represents of all failure mechanism characteristics by decreasing degradation path; monotonically increasing path also observed. But this consideration also important that in both the cases as the time increases variability of the degrading characteristic increases. This type of deterioration and variability in material properties and design parameters cause the system performance to deteriorate considerably. So by capturing the probabilistic nature of degrading characteristics, a significant improvement in the reliability of the products during optimization can be achieved.

Literature shows that degradation modeling has been used as a more effective way for the prediction of reliability of any products. The successes of the reliability prediction methods and the potential utility of the degradation modeling in reliability improvement motivated designer to develop more reliable product based on the degradation behavior of product. (Coit et al. 2005) have developed a methodology to correlate field life with the observed degradation for electronic module and formulated a conditional probability function to give changing mean and variance for normally distributed shear strength of the electronic module. The clear understanding of the failure mechanism at an early design stage can help designers to improve the design by eliminating root cause of failure..

2.2.1 Normal degradation models

Normal degradation model are used to estimate reliability of any product or component at normal operating condition.

Normal degradation model can be classified into two type(normal degradation with stress factor and normal degradation without stress factor).while estimating the reliability of any component and the force(stress which resist the force) acting on the component is not taking into consideration then these models are called normal degradation model without stress factor. In these models' reliability is estimates at a fixed stress level and if stress is considered then these called normal degradation model with stress factor.

2.2.1.1 Normal degradation without stress factor

These can be classified into different types (general degradation path model, linear/nonlinear regression model, random process model, time series model).

2.2.1.1.1 General degradation path model

(Jiang et al., 2010) focus on the inter-item behavior of component by combining general path model and condition time dependent increment process model, consider shape of distributed model is more flexible compare to gamma process. (Peng and Tseng, 2009) considered unit to unit variation with the time dependent structure and also describe that the effect of model mis-specification is not critical on for large sample when estimate mean time to failure. (Jiajie and Kam-Chuen, 2012) three method employed for degradation data(approximation ,analytical and two stage method) to estimate the MTTF, confidence interval, and reliability function. (Xiao-Sheng and Donghua, 2014) reliability estimation is done by two stage method by adding Brownian parameter in the general degradation path model.

2.2.1.1.2 Random process model

(Wang and Huang, 2012) for fuzzy data SAP (saddle point approximation) is extended for reliability prediction (Jiang and Feng, 2012) for soft and hard failure reliability model is defined by shifting failure threshold. (Zhang and Liao, 2014) on the basis of destructive degradation data, two delayed-degradation model developed for reliability prediction of a product with an exponentially distributed degradation initiation time.

2.2.1.1.3 Linear/nonlinear regression model

(Wiesel and Eldar, 2008) here analyze the performance by using the Cramer-Rao bound (CRB) on the mean square error and also estimate an unknown parameter vector in linear regression model with random Gaussian uncertainty in the mixing matrix. (Sridhar and Chan, 2009) parametric statistical model best fit is found by using lognormal, Weibull, Gamma, Exponential according to the requirement. (Yuan and Pandey,2009) since Uncertainties affect the degradation data by-random effect, temporal uncertainty or serial correlation, and measurement errors, due to this mixed-effect model used.

2.2.1.1.4 Time series model:

(Arulampalam and Maskell, 2002) a state-space approach is useful to handle multivariate data and nonlinear/non-Gaussian processes Particle filters generalize the traditional Kalman-filtering approaches. (Heng and Zhang, 2009) .This approach to modern maintenance practice promises to reduce downtime, spares inventory, maintenance costs, and safety hazard. (Yip and Fan, 2014) Both GRNN and Box–Jenkins time series models can describe the behavior and predict the maintenance costs of different equipment categories and fleets with an acceptable level of accuracy.

(GDPM) By using general degradation path model we can easily interpret the inter-item behavior of any product. also we can easily predict the unit to unit variation in any component. this model id directly related to statistical analysis of degradation data. By general degradation path Parameter estimation of the general nonlinear mixed-effects model is computationally simple compared with maximum likelihood estimation method.

(RPM) When ever failure rate depends upon a no of parameter or environmental condition random process model is very useful for reliability estimation. Since in the present condition a no of forces etc acts on a component so a different no of degradation data occurs sometimes at a point. When-ever the degradation path is not available the random process model gives a significant evaluation of the reliability.

(LRM) If the sample size or the no of degradation data point available is less then this method give good reliability estimation. This method is more flexible compared to random process model because there is no requirement for multiple observations at each fixed time point. Formulation by using regression is become simpler.

(TSM) This model is practical in applications where critical operational conditions are required such as system maintenance, tool-replacement, and human/machine performance assessment. by it includes on-line multivariate monitoring and forecasting of selected performance measures and conditional performance reliability estimates. For a dynamic environment individual performance estimation is reliable by using this method.

2.2.1.2 Physics of failure models (normal degradation with stress factor)

The conventional approach of reliability prediction is based upon mainly physics of failure model. These methods focus mainly on the reason behind the failure of the product. The uses of these models at an early design stage are considered as highly essential for elevating reliability of the product (Snock et al., 2003). The knowledge of these models is incorporated into the design process to make the product more resistant to well known failure and consequently elevate the reliability (Pecht and Dasgupta, 1996). Some of the well known models discussed below:

2.2.1.2.1 Stress-strength interference modeling

Stress-strength interference modeling is mainly used to failure analysis and reliability prediction of structural and mechanical component that are subjected to different type of stresses (Kapur and Lamberson, 1977; Rao, 1992). Engineering material used for manufacturing of product have different types of flaws, this is the reason behind statistical distribution used to describe the stress and strength. Basically in stress-strength interference modelling the stress (load) is considered that the activity that promotes or activate deterioration and strength is one that resist this deterioration. Both stress and stress is treated as a random variable, and in any loading set-up a failure is identified when the induced stress exceed the strength of the component (Augustine, 2011).

In SSI modeling, it is important to know the PDFs of both the stress S and strength Q i.e. f(S) and f(Q) respectively . The probability of failure is given by the interference of area of both the curve as shown in the figure 2.3

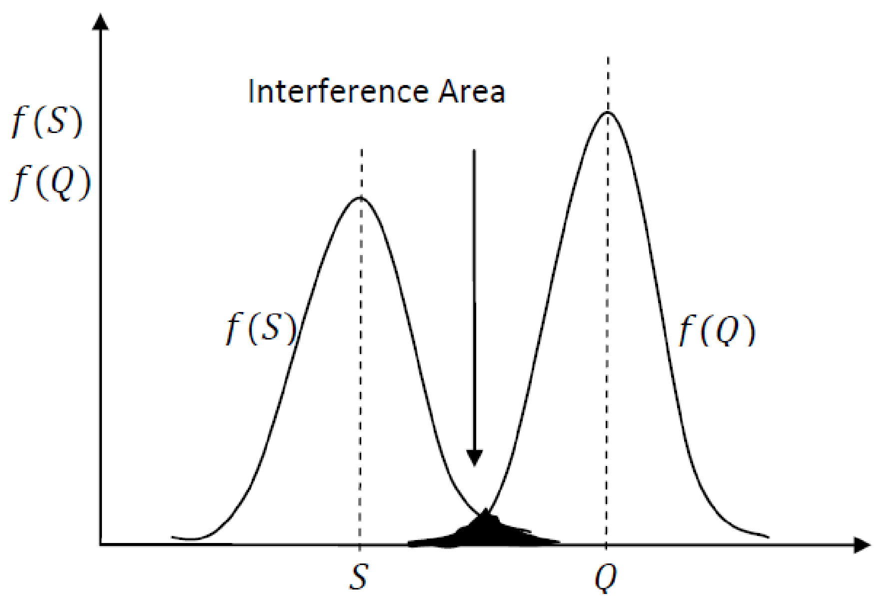

Figure 2.3: stress-strength interference diagram

In the conventional SSI modeling, the PDF of strength is treated as independent of time (means constant with the time). But in physical condition, the strength of the component is continuously affected by corrosion, wear, fatigue etc. this gives the limiting condition on use of the conventional SSI model. To overcome the limitation of conventional SSI models, research have proposed many time dependent (stress depends upon the time) SSI models.

Wen and Chen (1989) and Melchers (1992) have proposed stress-strength time dependent model (considering load is dependent on time). Hooke (1987), Zibdeh and Heller (1989), and Boehm and Lewis (1992) they have treated as time dependent stochastic processes. Some of the stress-strength interference modeling approaches considered the strength degradation due to ageing under stochastic loading condition (Lewis and Chen, 1994; Xue and Yang, 1997). (Huang and Askin, 2004) A numerical recurrence formula is described based on the Gauss-Legendre quadrature formula to calculate multiple integrations of a random variable vector.

(Eryılmaz et al., 2008) we provide minimum variance unbiased estimation of system reliability when the stress and strength distributions are exponential with unknown scale parameters.

(SH. Lv and Liu, 2009) They have developed a reliability calculation model for gear is developed with multiple failure modes. (Huang and An, 2009) They have developed a Universal generating function of unilaterally dependent discrete variables is developed, which is employed to describe the characteristics of discrete stress and strength. (Xue and Li, 2012) They considering temperature as the stress, degradation process gamma simulate the model and give the simulated storage life.

Liao et al., (1995) divided these reliability models into two groups:

(1) Static statistical models
(2) Dynamic statistical models

Where in dynamic statistical models consider the damage accumulating dynamically with the time. Basically these dynamic statistical models are developed similar to the classical SSI reliability models with the certain assumptions.

SSI model is traditionally used in structural engineering; however by better understanding about strength and stress, it can be applied in many other engineering disciplines for reliability analysis, as in the case of random stress dispersion and the place where wear-out, fatigue, and crack growth with static or dynamic loading forces occurs. Also the sensitivity analysis of reliability estimation is done by using it.

2.2.1.2.2 Damage accumulation modeling

During services, engineering products, structures and component are subjected to varying and constant type loading. Due to this type of loading causes a continuous irreversible damage accumulation in the structure or components. This type of failure is termed as fatigue failure. Fatigue failure is one of the most important persistent problems in the engineering, particularly in the rotating and reciprocating machine components and in large structure (bridge and buildings) (Dasgupta, 1993). Damage accumulation start with the development of microscopic cracks at the location of material defects or flaws and propagates with applied stresses, and ultimate result complete failure of the failure. The traditional damage accumulation model as Miner's rule (Miner, 1945), Paris relationship (Place et al., 1999) and the Coffin- Manson relation (Coffin, 1954) are based on the above mentioned all theories.

(Hwang and Hang, 1986) they have give a review on cumulative damage accumulation models and for the multi-stress fatigue life prediction. First damage accumulation theory was proposed by Palmgren-Miner's rule called as a linear damage rule. This rule shows damage as the ratio of no of cycles of usages to the no of cycle to failure.

$$D= n/N_f$$

Where D is the damage accumulation, n is the no of usage cycles and N_f is the no of cycle to fatigue failure. This models considered damage as a linear phenomenon, and due to the simplicity of this model, this is widely acceptable throughout the world. This model fails to predict the effect of loading sequences. As a matter of fact the sequence of loading has a significant effect on the strength of any material. When the sum of damage reaches to unity it is assumed that failure is occurred. While the experimental study shows that damage sum to failure is more than unity for the low-high loading (stress) condition and damage sum is less than unity for high-low loading (stress) condition. Since in real life behavior damage line follow a nonlinear curve so many researchers have tried to change the Miner's rule of damage accumulation, so as to incorporate nonlinearity to predict multi-stress level fatigue life more precisely. Leve (1960) proposed a nonlinear damage accumulation models as:

$$D = (n/N_f)^C$$

Where C is greater than one and is dependent on the stress level. Similar kinds of stress dependent models were also proposed by Macro and Starkey (1954). Marin (1954) proposed a cumulative damage theory based on the consideration of the relations between damages as a function of the cycle ratio and change in the S-N curve due to damage accumulation. He formulated the model as

$$S^m N = C$$

Where m and C are constants. It has been know that Marin's criterion reduce to linear damage accumulation as a special case of linear damage.

The main limitation of the traditional damage accumulation is that they have treated damage accumulation as a linear phenomenon or in a deterministic fashion and by simple arrive to a rule or relation that represents the physical significance of the damage accumulation phenomenon. But in real life scenario damage accumulation is considered as a nonlinear (stochastic) phenomenon. This nonlinearity arises due to the random nature of the loading process as well as the stochasticity of fatigue resistance of material itself. Considering the facts various approaches have been proposed to model the probabilistic nature of the damage accumulation. (Sethuraman and Young, 1986) they have developed a cumulative damage threshold crossing model. Their model considered as a product consists of multiple component and each component of the product is subjected to continuous degradation as with usage as the time passes. The failure is said to occur when the total damage to any one or more of the component exceeds a predefined threshold value.

Also, Liao et al., (1995) have proposed a cumulative fatigue dynamic interference model with the some assumption as cumulative damage follow either a normal or a lognormal distribution. (Nagode and Fajidia, 1998) they have proved a conditional PDF of the no of cycle to failure at any stress level can be modeled by the normal distribution based on the DeMoivre-Laplace principal. (Shen et al., 2000) have proposed a probabilistic damage accumulation model which considers a random distribution of stress amplitude, and also accounts for randomness of fatigue resistance of the material by introducing a random variable for single cycle fatigue damage. (Liu and Mahadevan, 2007) they have tried to merge a non-linear fatigue damage accumulation rule with a probabilistic S-N curve representation techniques for modeling fatigue life under the variable amplitude loading.

Albrecht (1983) and White and Ayyub (1987) they have proposed a method to graphically transform PDF of no of usage cycle to PDF of stress. They have used this transformed PDF in probabilistic design of the bridges and the marine welded joints. The detailed expression for the transformation of PDF to capture the damage accumulation is detailed in the next paragraph. (Figure 2.4)

Albrecht (1983) and White and Ayyub (1987) they have proposed this procedure for the lognormal distribution of fatigue life. In the above figure the S-N curve shows the randomness in fatigue failure life or usage cycles (N_f) caused by the applied stress. In case of two functionally variables; by knowing the PDF of one random variable, PDF of other random variable can be obtained (White and Ayyub, 1997; Benjamin and Cornell, 1970).

In the case of fatigue loading the degradation in the fatigue strength is caused by the usage cycles. So, by knowing the PDF of the fatigue life or usage cycle (N_f) , PDF of degrading fatigue strength can be deduced. It can be achieved by transforming the PDF of fatigue life or usage cycle (N_f) to the PDF of degrading fatigue strength (S). Although the distribution of the strength will retain the same form as of life distribution after this transformation (i.e. lognormal), but their standard deviation (S.D) might not be the same (White and Ayyub, 1997; Benjamin and Cornell, 1970). To obtain the mathematical relation b/w the standard deviation before and after the transformation, a procedure based on the finding of Albrecht (1983) can be adopted.

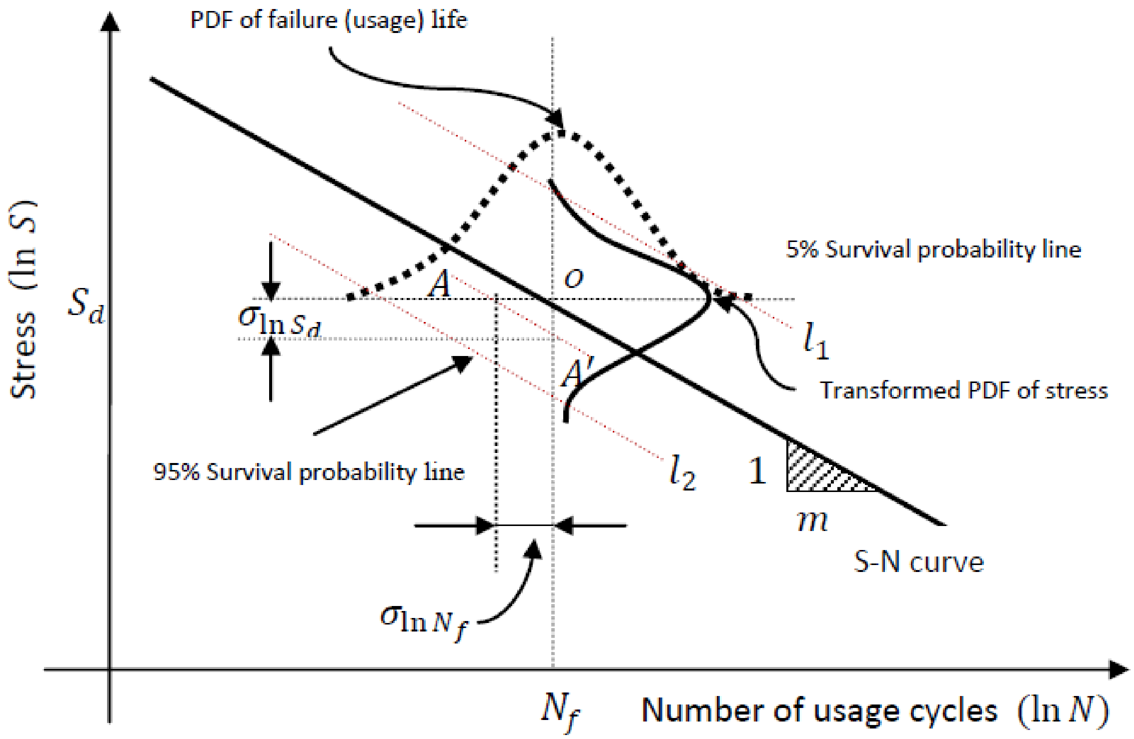

Figure 2.4: transformation of PDF of usage cycles to PDF of stress(adopted from Albrecht, 1983)

According to Albrecht (1983), any line parallel to the resistance line in the S-N curve will always pass through same survival probability point of both the PDFs of fatigue strength in terms of strength in terms of stress as well as that of the fatigue life or usage cycles. Figure 2.4 shows that this concept diagrammatically, wherein PDF of fatigue life is depicted with the dotted curve and the PDF of the degrading fatigue strength is depicted with the dark line. Considering two lines l_1 and l_2 drawn parallel to the S-N curve and passing through the 5% and 95% survival probability respectively for both PDFs. Similarly, a line A-A' is drawn parallel to the S-N curve such that it passes through the standard deviation (1σ) points of both PDFs. The line A-A' is parallel to the S-N curve line, and so its slope is also 1/m. Point O is the point about which PDF is transformed. From this diagrammatic stet up, using a simple geometric interpretations; a relation between a standard deviation before and after transformation can be derived. The slope of the line $A - A'$ can be obtained as:

$$\frac{OA'}{OA} = 1/m$$

But the distance $OA' = \sigma_{lnS_d}$ and $OA = \sigma_{lnN_f}$. Using these relations, the following equation is to be obtained:

$$\frac{\sigma_{lnS_d}}{\sigma_{lnN_f}} = 1/m$$

Where m is the slope of the S-N curve; and σ_{lnS_d} and σ_{lnN_f} are the standard deviation of PDFs of the strength in terms of stress and the usage cycle of failure, respectively. It is clear from the equation that during the transformation, the PDF of strength (stress) is scaled down by a factor of $1/m$ as compared to the PDF of the usage cycle to the failure. The idea of deriving the unknown PDF has been utilized in this work to propose a simple probabilistic damage accumulation approach. The details of this approach are given in chapter 3.

It is a known fact that the degradation modeling can help to capture the behavior of the given product. These degradation models and the knowledge generated from these models can help to capture the behavior of a product at an early design stage. A number of various research paper published on damage accumulation modeling for various parameter discussed below.

Stoyanov and Mackag(2004) have proposed that the Finite Element slice model of a Plastic Ball Grid Array (PBGA) package and suitable energy based damage models for crack length predictions are used for aerospace component. Hernandez-Mangas and Pelaz (2005) have proposed a new statistical damage accumulation based on the modified Kin-chin Pease model which takes into account the abrupt regime of the crystal amorphous transition It works with different temperatures and dose-rates and also models the transition temperature. (Tryon and Dey(2006) have proposed Monti-Carlo simulation used for reliability prediction to proceed three level. First, dislocations accumulate causing a crack to nucleate at a micro structural, size, second, the crack propagates microscopically and lies within relatively few grains. (Liu and Mahadevan (2007) combine a nonlinear fatigue damage accumulation rule and a stochastic S-N curve for variable level amplitude.

Pelaz and Marques (2007) they have describe the, Atomistic model for amorphization based on the accumulation of bond defects captures the sensitivity of defect accumulation to implant parameters, such as wafer temperature or flux. Gupta and Rychlik (2007) they have proposed that due to load effect fatigue damage accumulation is nonlinear so approximation and bound for the mean rain-flow fatigue damage can be developed. Zhu and Huang(2009) have proposed on the basis of frequency modified Manson–Coffin equation and Ostergren's model, a new model for high temperature low cycle fatigue (HTLCF) is developed for the separation of approximate strain energy and real strain energy absorbed during the damage process. Ai-ling and Wen-Hua, (2010) they have proposed for estimation of fatigue life of crankshaft by ANSYS high stress zone is found then by using rain-flow method cycle counting is obtained for stress-time, together with SHOP, fatigue load spectra of key parts are compiled. Giancane and Nobile(2010) they have proposed for fatigue life of the notched component with the help of S-N curve continuum damage mechanism model predict the sequence effect and to simulate a more realistic loading condition test with various loading blocks were carried on and in particular high -low, low-high and random block were applied to the specimens considered.

Zhu and Huang (2011) a ductility exhaustion theory, the generalized energy-based damage parameter, a new viscosity-based life prediction model is used for the mean strain/stress effects in the low cycle fatigue regime.Zhu and Huang(2011)they have proposed a damage accumulation model based on the Miner rule to investigate the damage induced by stresses below the fatigue limit and study the load sequence effects.(for two stress level or multilevel stress)

Liao and Yang (2012) they have proposed Manson-Coffin law and an energy-based damage parameter, a general energy-based model is used to predict fatigue life under both HCF and LCF conditions for high temperature structural material For the components under LCF, plastic deformation is the main deformation that occurred in the materials. Moreover, the loading stress is greater than the ultimate tensile strength of material, which leads to a nonlinear relationship between the stress and strain. Yuan and Li (2012) have proposed a new model based on Nonlinear Continuum Damage Mechanics which take into account the damage evolution of material under different loading levels and the effects of loading sequence on fatigue life, mean stress in its damage evolution with fewer parameters. Shao and Cao(2012) they have proposed a model for low-cycle fatigue, random response surface method is adopted to fit the life distribution function, probability fatigue accumulation damage theory and local stress and strain method are combined to obtain the high-cycle and low-cycle stochastic fatigue reliability. Liu and Gong(2012) they have proposed a low cycle fatigue damage accumulation model, based on the CDM and within the framework of irreversible thermodynamic theory, is proposed through rigorous mathematical derivation.

Tucker and Chan (2012) they have proposed a max entropy fracture model to describe how fracture initiate and progress due to stress by using a single damage accumulation parameter to relate the probability of fracture to accumulated en tropic dissipation.

Zhu and Huang (2012) developed a probabilistic technique is developed to remove the shortcoming of Miner's rule by combining nonlinear damage accumulation model, a probabilistic *S-N* curve and one-to-one probability density functions transformation technique. Wei and Fei(2012) The thermal fatigue life of QFN is calculated based on the maximum strain range and the Coffin-Manson equation ANSYS used to develop a quarter of model about QFN, which is subjected to the thermal recycling. Zhu and Huang (2012) they proposed that Corten–Dolan exponent d is found by practical approach (its depends not only the material, but also upon the load spectrums) life prediction capability improve compare to conventional method where d is constant.

Zhu and Huang (2012) have proposed that the ductility exhaustion related only to the plastic strain and creep strain caused by tensile stress under stress-controlled conditions a new low cycle fatigue–creep life prediction model that is consistent with the fatigue–creep damage mechanism and sensitive to the fatigue damage process is describe to develop viscosity-based approaches for general use in isothermal and thermo-mechanical loading. Zhu and Huang(2012) have proposed model for life prediction of turbine disk , creep and mean stress/strain effect in the low cycle fatigue regime by using energy based theory. Huang and Gong (2012) they have proposed a model based on the linear damage rule, the fatigue life of an aircraft engine was estimated by considering the load spectrum difference factor.

Zhu and Huang(2012) have proposed a model for low cycle fatigue life prediction using an energy-based damage parameter a Bays theorem.

Zengah and Aid (2013) for fatigue life assessment by (algorithms based on numerical methods of cycle counting and the other group uses spectral analysis of stochastic processes) rain flow used for cycle counting and damage accumulation according to the assumed hypothesis are the main operations. Gao and Zhu (2013) have proposed a modified non-linear fatigue damage accumulation model under two-level loading on the basis of damage curve approach to consider the load interaction effects. Lv and Gao(2013) they have proposed a non linear Corten-Dolan model is used to take into the effect of damage accumulation load interaction effect. Zhang and Cui (2013) they have proposed a model for creep life prediction of turbine blade is (for both uniaxial state of stress or multi-axial state stress) based on the macroscopic phenomenological analysis simulate the macro creep failure phenomenon of materials, and combine with finite element method to calculate and analysis the creep life of structure containing complex stress state. Raghunathan and Chakraborty(2013) have proposed the accumulated degradation of SiGe HBTs under time-dependent mixed-mode stress using a new physics-based TCAD degradation model that simulates hot carrier generation and propagation to oxide interfaces, resulting in trap formation. Sun and Hu(2014) have proposed a model that low cycle fatigue life prediction of steam turbine is on the basis of a continuous damage variable to describe the local distribution of micro defects.

Rathod and Yadav (2011) have proposed a linear damage accumulation model of Palgren-Miner's a probabilistic S-N curve ,and an approach for a one to one transformation of probability density function used for probabilistic modeling for single and multi stress level. Here it is considered that the fatigue life curve follows a normal distribution curve and limitation is that it is for the linear degradation while in the real life practice degradation follows a non linear curve. Gao and Huang(2014) use a modified nonlinear fatigue damage accumulation model based on damage curve approach to consider the load interaction effects for two loading condition. Zhu and Hunag (2012) they have proposed a nonlinear fatigue damage accumulation model while degradation follow a log-normal distribution. So there is need to develop an model for the reliability prediction which takes into account both load sequencing effect and also load interaction effect and also a common model which can be applied in any situation.

This need can be fulfilled by using a Weibull distribution which has two parameter scale and the shape parameter. By changing the value of shape parameter Weibull distribution can be converted into many other distribution as Rayleigh distribution, exponential distribution, normal distribution and lognormal distribution.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

Generally any research start with collection of particulars from the existing literature to get the equipped with the latest development in the area of research. This help us in building a theoretically background needed to propose a new research hypothesis. This hypothesis is then tested on the suitable platform, and the results are evaluated to prove the proposed work as a distribution in the concerned area. The main purpose of this chapter is to give an overview of research methodology used in this research in order to the research question and fulfill the research objectives.

3.2 Proposed research methodology

Literature reveals that a very few effort have been made to develop a probabilistic model for capturing the fatigue life of the product. Usually, degradation model are formulated to predict the reliability and future life of the product. Very few researchers have tried to consider the load sequencing and load interaction effect for deterministic model. And not done the same for the nonlinear damage accumulation. This has motivated this study to develop a degradation model which can be very useful in prediction of the reliability.

Moreover, fatigue is recognized as one of the main reason for failure of the mechanical components. Therefore in this study, attempt is made to develop a methodology for the probabilistic modeling of nonlinear fatigue damage accumulation for single stress level and multi-stress level loading condition. The proposed probabilistic damage accumulation model is then tested for predicting the reliability of a mechanical component subjected to fatigue loading.

An outline of the research methodology is presented in the figure 3.1. The research methodology consists of 3 stages. The goal of the very first stage is to build a theoretical framework through the literature review and propose a degradation model. In stage two, the proposed degradation model is used and tested for the different purpose. In the third stage conclusion and future work is summarized.

3.2.1 Phase 1: literature review and development of probabilistic damage accumulation model

Collect the necessary literature first. Then by carefully by study find the research gap. As the research gap in hand we see the probability of doing that work. Then start probabilistic damage accumulation modeling for the reliability prediction of any component. The methodology used for modeling is S-N curve approach and one to one PDF transformation techniques.

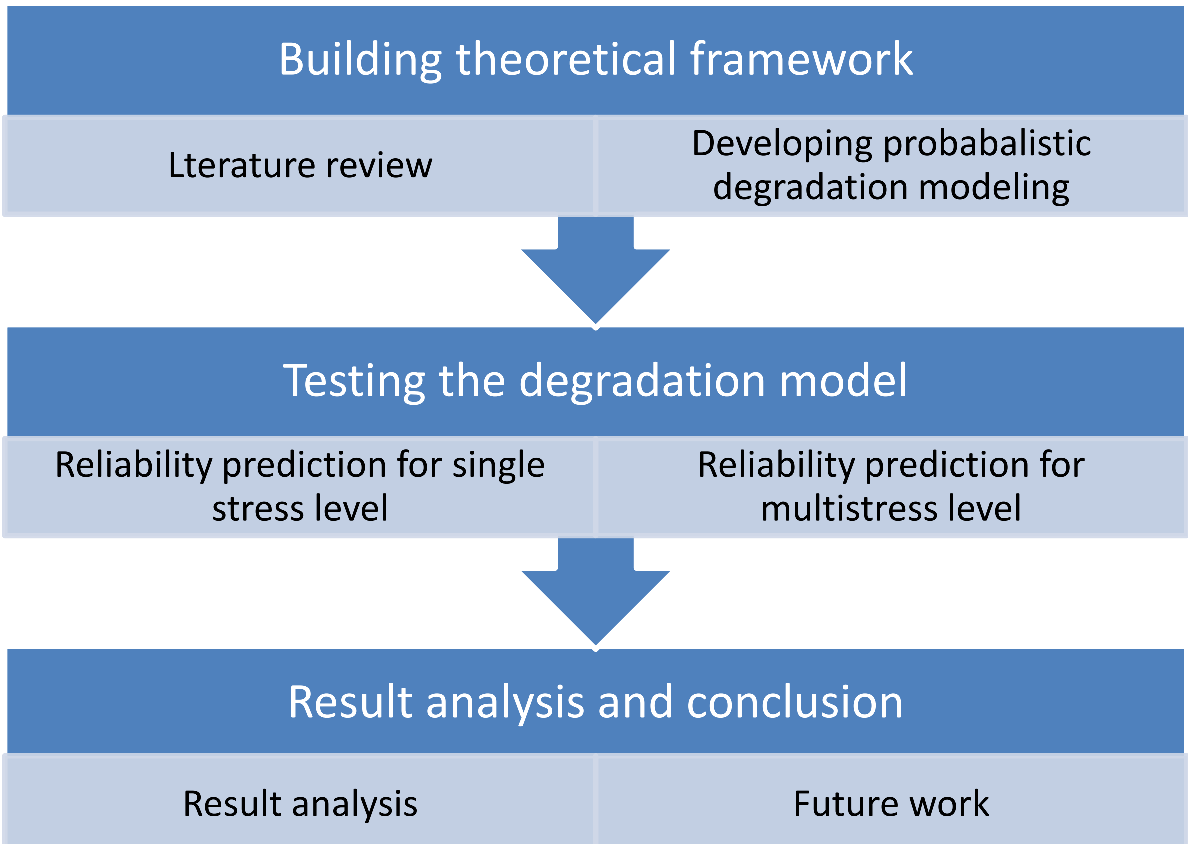

Figure 3.1: Outline of research

3.2.2 Phase 2: Reliability prediction

Earlier models of fatigue damage accumulation reported in literature focus on mainly deterministic nature of the process, whereas in practice, damage accumulation is of stochastic nature. Very few researchers work on probabilistic process. Due to this stochasticity results from the randomness of the fatigue resistance of material as well as that of the loading process. Therefore, the capability of degradation model is checked for capturing the degradation phenomenon probabilistically and predicts the reliability at any point of time is considered. Chapter 4 illustrates the procedural details of this stage.

3.3 Conclusion

The proposed research work consists of three steps and can be divided into three different stages:

1. Development of a probabilistic degradation model to predict the degradation behavior of fatigue strength and reliability of mechanical components subjected to fatigue.
2. Prediction of reliability at single stress level and multi level loading condition.
3. Give summary and future work for the proposed work.

CHAPTER 4

PROBABLILISTIC DAMAGE ACCUMULATION MODELING

This chapter presents a methodology for probabilistic modeling of fatigue damage accumulation for the single or multi-stress level loading. The damage accumulation is modeled as a non-stationary process and used for the reliability prediction under the single or multi-stress level loading, utilizing the dynamic statistical model of cumulative fatigue damage. The reliability prediction under both type of loading is demonstrated with the help of examples.

4.1 Introduction

Fatigue is one of the most persistent problems in the engineering design, as most of the mechanical components are subjected to the fatigue during their service life. This has led to a need for developing the new approaches to predict the reliability and the useful life of the mechanical components, which are subjected to the fatigue damage. This has been the primary focus of the designer since many years and the field still presents many different types of challenges (Liu and Mahadevan, 2007). Moreover, earlier models of the fatigue damage accumulation reported in the literature focus on the deterministic nature of the process, while in the real world situation damage accumulation is of stochastic nature. This stochasticity results from the randomness in fatigue resistance of the material as well as that of the loading process (Shen et al., 2000). As a result of this, even under the constant amplitude fatigue test, at any given stress level, fatigue shows the stochastic behavior with a specific type of distribution. Literature shows that fatigue life data follow either normal or lognormal distribution under the constant amplitude or random loading (Wirching and Chen, 1987; Albrecht, 1983; Wu et al., 1997). Weibull distribution has also been reported to fit fatigue life data (Zaccone, 2001; Munse et al., 1983). Researchers have proposed different modeling approaches to the probabilistic damage accumulation paradigm. (Shen et al., 2000) proposed a probabilistic distribution model of stochastic fatigue damage, wherein they have considered that the randomness of the loading process as well as the randomness of the fatigue resistance of the material by introducing a random variable of the single cycle fatigue damage. Liu and Mahadeven (2007) proposed a general methodology for the stochastic fatigue life under variable amplitude loading by combing a nonlinear fatigue damage accumulation rule and the stochastic S-N curve representation techniques. Nagode and Fajdiga (1998) have modeled a PDF of failure cycles at any stress level as a normal distribution based on the DeMoivre-Laplace principle to reliably predict endurance limit of a randomly loaded structural components. Liao et al., (1995) he proposed a new cumulative fatigue damage dynamic interference model assuming that cumulative fatigue damage follow normal or lognormal distribution. Wu and Huang (1993) modeled fatigue damage and fatigue life of the structural components subjected to the variable loading as a Gaussian random process. Ben-Amoz (1990) proposed a cumulative damage theory based on the concepts of the bounds on the residual fatigue life in two-stage cycling. Castillo et al.,(2008) proposed a general model for predicting the fatigue behavior for any stress level and the range by generalizing the Weibull model. Sethuraman and Young (1986) proposed a cumulative damage threshold model. This model considered a multi component product which undergoes deterioration/damage at the regular interval of time and failure occurs as soon as the maximum damage to some components crosses a certain threshold value. Time to failure data is used to estimates the model parameters. A review of cumulative fatigue damage and life prediction theories can be found in Fatemi and Yang (1998).

As mention earlier, to aspect are significantly important from the point of you modeling of the probabilistic fatigue damage. First, and accurate physical damage accumulation models needs to be in the place to predict the expected or the nominal fatigue damage. Second an appropriate uncertainty modeling techniques is required to account for the stochasticity (Liu and Mahadevan, 2007). A review of literature has indicated that handling of the stocahasticity in modeling uncertainty involves complex mathematics. This fact is the primary motivation behind the development of the simpler approach for handling the stocashticity in the fatigue damage accumulation modeling in the proposed research work. This chapter proposes a simpler approach to deduce the distribution of a fatigue damage accumulation from the fatigue damage life distribution using a one to one transformation methodology and to the certain extent attempts to minimize the mathematical complexity. It also proposes a simple and unique way to model the damage accumulation process treating it as a non-stationary probabilistic process to capture the damage accumulation and its variability at any given point of time. The proposed methodology can be effectively used to predict the reliability of the mechanical components subjected to the fatigue loading due to the single and mutli-stress levels. Rathod et al,(2011) developed a method for probabilistic modeling of fatigue damage accumulation for single stress and multi-level stress loading by using the Palmgren-Miner' rule and a probabilistic S-N curve. The major limitation of this method is that it models damage accumulation process as a linear phenomenon where as in real damage accumulation in engineering structures could be a nonlinear phenomenon, which is particularly prone to uncertainty.

Recently Zhu et al.,(2012) proposed a model for nonlinear damage accumulation. The major limitation of this model is that it does not consider the effect of the load interaction effect.

Recently Gao et al., (2014) proposed a methodology for load interaction and load sequencing effect for damage calculation for the nonlinear phenomenon.

In the present thesis we are presenting a nonlinear damage accumulation model considering both load sequencing and load interaction effect while fatigue follows a Weibull distribution. It is a common distribution in which by changing the value of the shape parameter different other distribution can be achieved.

4.2 Modeling probabilistic fatigue damage accumulation

Damage accumulation is a complex and irreversible phenomenon, wherein the damage of the product under consideration gradually accumulates and over a period of the time leads to its failure. So, damage accumulation can be treated as a measure of degradation in fatigue resistance of the materials. Moreover, damage accumulation is probabilistic in nature and it can be depicted graphically as shown in figure 4.1.

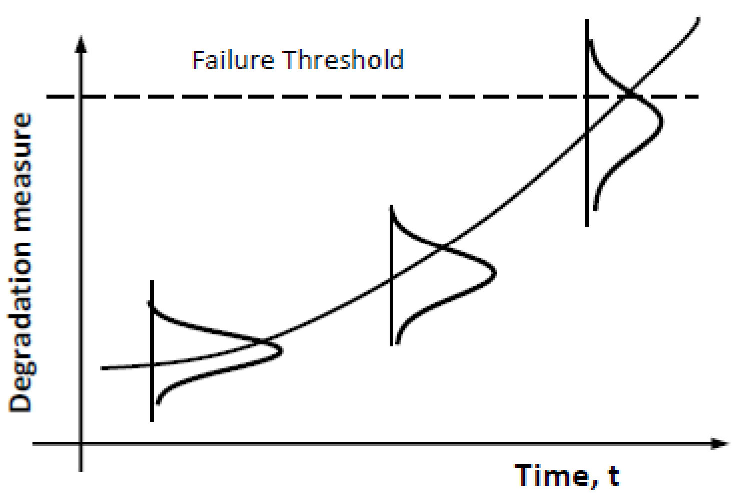

Figure 4.1: Degradation path example

The figure 4.1 shows monotonically increasing degradation path where the degradation measure is increasing probabilistically with the time.

Wang and Coit (2007) they have explained that at any specific time, there exists a distribution of the degradation measurements considering a population of similarly degrading components. They also explain that the variability in any given degradation measure increase with the usage time. Since the damage accumulation is a measure of degradation, the reasoning given by the Coit et al., (2005) and Wang and Coit (2007) can be applied in assuming that the damage accumulation follows a certain probability distribution and that the distribution parameters of any damage accumulation measures will change with the usage time. (Zaccone, 2001; Wu et al., 1983) shows that damage accumulation also follow the Weibull distribution. Therefore, the damage accumulation can be modeled as a non-stationary probabilistic process based on Weibull distributed fatigue life data. This is achieved by establishing non-linear functional relationship between the damage accumulation and the fatigue life or usage as advocated by Benjamin and Cornell (1970) about the functionally related random variable and their distribution.

The non-stationary probabilistic process of damage accumulation (based on the Weibull distributed fatigue failure data) can be given as:

$$D(t) \approx N\{\mu_D(t),\ {\sigma_D}^2(t)\} \tag{4.1}$$

Where $D(t)$ is a damage accumulation measures that varies probabilistically with time t, and $\mu_D(t)$ and ${\sigma_D}^2(t)$ are its mean and variance. The proposed probabilistic modeling of damage accumulation is elaborated in the subsequent sections.

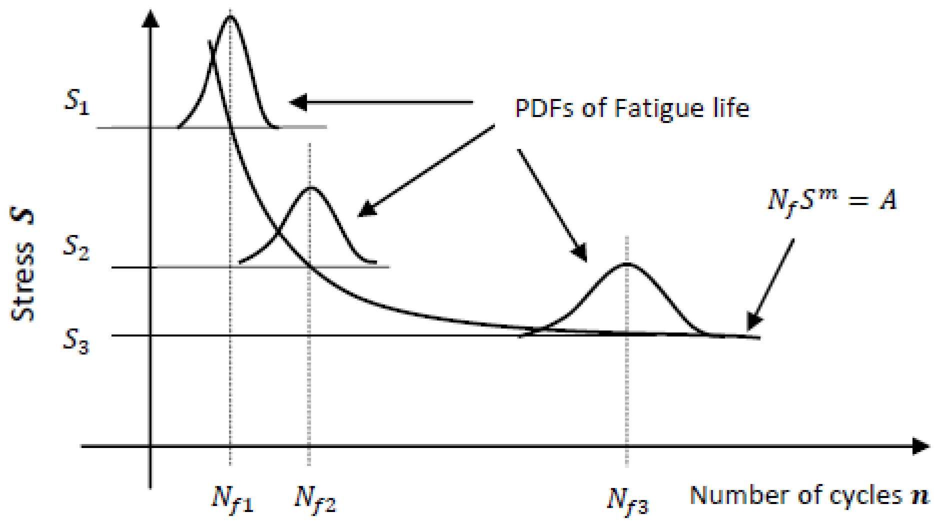

Figure 4.2: Probabilistic $S - N$ curve

4.2.1 Modeling for mean value of cumulative fatigue damage

The two most widely used models for the fatigue loading are S-N curve and Palmgren-Miner's damage accumulation models (Liu and Mahadevan, 2007; Hwang and Han, 1986). The S-N curve model is used to express the relationship between fatigue life (N_f) and stress level (S) and is expressed by the well known S-N curve equation as given below

$$N_f S^m = C \tag{4.2}$$

Where C is the fatigue strength constant and m represents slope of the S-N curve. Figure shows a probabilistic interpretation of the S-N curve, wherein PDFs (on normal scale) of fatigue lives are depicted at the different stress level S_1, S_2 andS_3.

The linear damage accumulation model, which is also known as Palmgren-Miner's rule, defines damage as the ration of the number of cycle of operation to the number of cycle to failure at any given stress level (Hwang and Han, 1986). Assuming no initial damage, the damage accumulation at single stress is given as:

$$D = \frac{n}{N_f}$$

D_c is the critical threshold value which vary appreciably among components/specimens in practice. Thus a probabilistic rather than a deterministic threshold value is more appropriate such as the critical cumulative damage at the fatigue failure point.

Consider a mechanical component subject to constant amplitude loading. The cumulative fatigue damage at loading cycle n, $D(n)$ initially equal to 0, and is assumed to increase monotonically, which is a basic physical condition. If the environmental and frequency effect on $D(n)$ are not considered. The rate of damage accumulation should depends on D_c and the loading stress magnitude S. Although a linear relationship between damage accumulation and number of loading cycles is absolutely reasonable in the some cases, there are many situations where a nonlinear description is likely to be more appropriate to the nature of fatigue damage. Based on the above definition the general form of the cumulative fatigue damage curve in figure 4.3 according to Zhu et al (2013) can be written as

$$D(n) = \mathrm{f}(S, D_C)\, n^a \tag{4. 3}$$

Where $\mathrm{f}(S, D_C)$ describes the rate of damage accumulation associated with the cyclic loading, a is a yet to be determined "damage accumulation exponent", which depends on the amplitude of alternating stress. The function $\mathrm{f}(S, D_C)$ is determined based on the boundary conditions and failure criterion. It is assumed that failure will occur when the cumulative damage $D(n)$ equals to the critical threshold value D_c , and the number of lading cycle n equals to the constant amplitude fatigue life N_f . Substituting these condition into equation 1 gives

$$\mathrm{f}(S, D_C) = \frac{D_C}{{N_f}^a} \tag{4.4}$$

Substituting equation 4.4 into 4.3 we can get

$$D(n) = D_C \left(\frac{n}{N_f}\right)^a \tag{4.5}$$

Combining equation 4.5 and 4.4 can be rewritten as

$$D(n) = D_C \left(\frac{S^m}{C}\right)^a n^a \tag{4.6}$$

It should be noted that equation 4.6 satisfies the two basic physical conditions as explained by Ye and Wang (2001). Figure 4.3 illustrate possible damage accumulation curve as a function of fatigue loading cycle as described by (5). Moreover note that damage accumulation curve begins initially damage 0 and passes through the location under the failure condition, i.e. $D(N_f) = D_C$. Considering that failure occurs when total damage accumulation reaches unity, $D_C = 1$, it is noted that equations (4.6) reduce to Marco and Starkey's (1954) model. Corten Dolan's theory (1956) and Plmgren-Miner's rule(1954) when a=1, so by considering both S-N curve model and physical perspective, (4.6) can be extended for the multi-stress level as

$$D = \sum_{i=1}^{j} D_i = \sum_{i=1}^{n} D_C . \left(\frac{{S_i}^m}{C}\right)^{a_i} n^{a_i} \tag{4.7}$$

Using equation (4.6) and (4.7), the mean value of cumulative fatigue damage at any given point of loading cycle can be calculated for the constant amplitude and variable amplitude loading condition, respectively. However, fatigue cycle loading is a probabilistic process in nature. It is extremely important to treat fatigue damage accumulation as a random variable and to calculate the distribution of the damage accumulation.

4.3 Distribution of cumulative fatigue damage

Fatigue damage increases with the applied loading cycles in constant/variable loading. Up to now, lots of models have been developed to describe the average or typical the fatigue damage accumulation behavior. The individual fatigue damage accumulation paths may diverge significantly from the mean, thereby, the distributions of cumulative fatigue damage depicted in Fig. 1 need to be modeled. By treating fatigue failure life as a random variable which follows a certain distribution, the distribution of damage accumulation can be established using the one-to-one PDF transformation technique as described below:

In order to establish the PDF of the damage accumulation measure (*D*) and to estimate the distribution parameters, first the fatigue failure life is treated as the random variable which follows the certain distribution. Thereafter, the distribution of *D* is derived using the one-to-one PDF transformation methodology proposed by Benjamin and Cornell (1970). As per Benjamin and Cornell (1970), the unknown PDF of a random variable can be derived using this transformation technique, if that variable is directly or functionally related to another random variable whose PDF is already known. Since the cumulative damage accumulation is a function of usage life (or fatigue failure life), the PDF transformation methodology provides an effective means to establish distribution of damage accumulation measure. A probabilistic interpretation of general damage accumulation curve is shown in Fig 4.3, which describes that how to obtain the PDF of damage accumulation based on the known PDF of fatigue life at any of the stress level. In Figure 4.3, curve $c_1$is the trend curve of mean cumulative damage as given by (5) at a given stress level S, which depicts the nonlinear relation between the cumulative damage and loading cycles. Note that initial variability of loading cycles is zero and it increases with the increase of the loading cycles. Considering that cumulative damage at a given stress level S and no initial damage, (4.7) can be simplified as

$$D(n) = kn^a \tag{4.8}$$

Where $k = D_C . \left(\frac{S^m}{C}\right)^a$

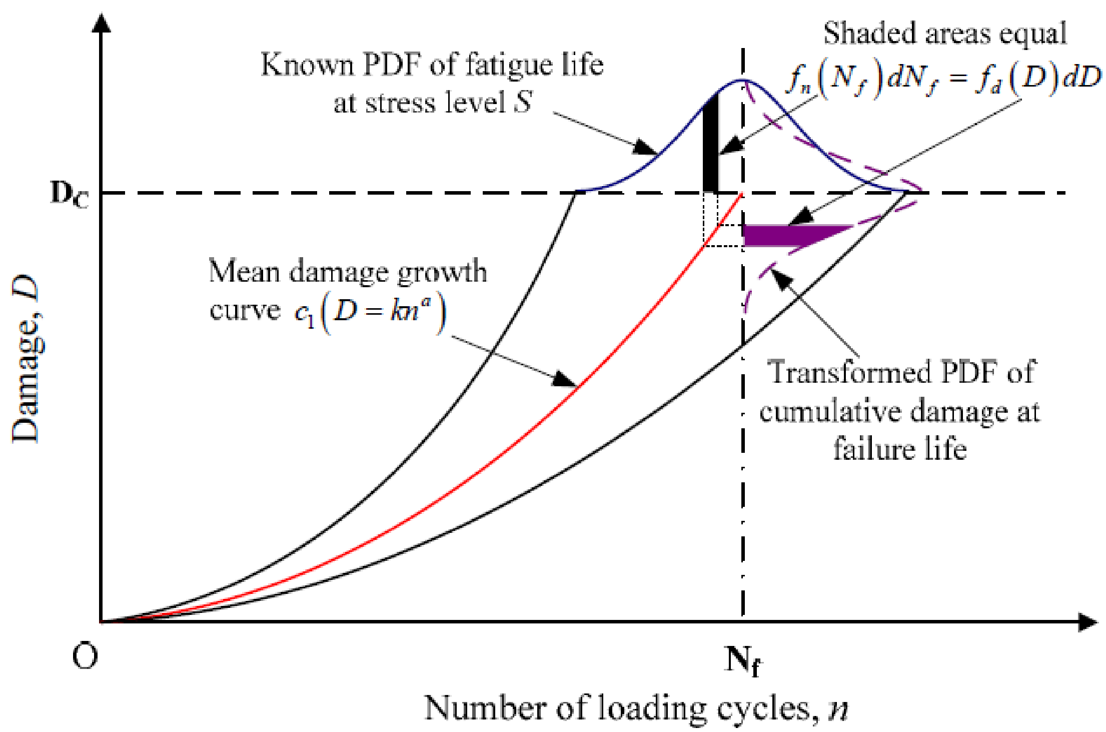

Figure 4.3: One to one transformation of PDF (Adopted from Zhu et al.,2012)

From the above discussion, it is clear that in order to derive the distribution of D using Benjamin and Cornell (1970) PDF transformation technique, there are two basic requirements that need to be fulfilled:

(i) A clearly defined relation between damage accumulation and usage cycles and

(ii) The knowledge of the distribution or PDF of the usage cycle.

As mentioned earlier the variability of fatigue life N_f can be described by a Weibull distribution

$$f_n(N_f) = \frac{\beta}{\eta}\left(\frac{N_f}{\eta}\right)^{\beta-1} exp - \left(\frac{N_f}{\eta}\right)^{\beta} \tag{4.9}$$

Where β= shape parameter (independent of the applied stress)

η= scale parameter (dependent of applied stress)

The functional relationship between damage accumulation measure (D) and fatigue life (N_f) can be generically expressed as given below:

$$D = g(N_f) \tag{4.10}$$

The inverse relation of equation (4.10) can be expressed as:

$$N_f = g^{-1}(D) \tag{4.11}$$

The cumulative distribution function (CDF) of the dependent variable D can be obtained from the CDF of N_f as:

$$F_d(D) = F_n(g^{-1}(D)) \tag{4.12}$$

Subsequently to obtain the PDF of the damage accumulation measures(D), we simply need to take the derivatives of its CDF as given below:

$$f_d(D) = \frac{d}{dD} f_d(D)$$

$$f_d(D) = \frac{d}{dD} F_n(g^{-1}(D))$$

$$f_d(D) = \frac{d}{dD}\left[\int_{-\infty}^{g^{-1}(D)} f_n\left(N_f\right) dN_f\right]$$

$$f_d(D) = \frac{dg^{-1}(D)}{dD} f_n(g^{-1}(D)) \tag{4.13}$$

Using equation (4.11) equation (4.13) can be written in a more suggestive form as follows:

$$f_d(D) = \frac{dN_f}{dD} f_n\left(N_f\right) \tag{4.14}$$

$$f_d(D). dD = f_n\left(N_f\right). dN_f \tag{4.15}$$

Further, differentiating equation expressing functional relation between $D \; and \; N_f$ is given by

$$D = D_c. \left(\frac{S^m}{C}\right)^a N_f{}^a$$

$$\frac{dN_f}{dD} = \frac{1}{a}\left(\frac{D}{k}\right)^{\frac{1}{a}-1}$$

Putting value of above differentiation in equation (4.15)

$$f_d(D) = \frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{aD} \frac{\beta}{\eta} \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1} exp - \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta} \tag{4.16}$$

From the above equation it is clear that damage accumulation is also follow Weibull distribution. Now we have to estimate the mean and variance of the above PDF of damage accumulation

The mean of Weibull is given by

$$\mu_D = \int_0^{\infty} D f_d(D) \; dD \tag{4.17}$$

$$\mu_D = \int_0^{\infty} D \frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{aD} \frac{\beta}{\eta} \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta} \right)^{\beta-1} exp - \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta} \right)^{\beta} dD$$

$$\mu_D = \int_0^{\infty} \frac{\beta}{\eta a} \left(\frac{D}{k}\right)^{\frac{1}{a}} \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta} \right)^{\beta-1} exp - \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta} \right)^{\beta} dD$$

Putting

$$\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta} = t$$

$$\frac{1}{k.a.\eta} \left(\frac{D}{k}\right)^{\frac{1}{a}-1} dD = dt$$

$$dD = \frac{dt\ (k.a.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}}$$

$$\mu_D = \int_0^{\infty} \frac{\beta}{\eta a} \frac{(k.a.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}}\ \eta t\ (t)^{\beta-1} exp - (t)^{\beta}\ dt$$

$$\mu_D = \frac{(k.\beta.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \int_0^\infty t\ (t)^{\beta-1} exp - (t)^\beta\ dt$$

$$\mu_D = \frac{(k.\beta.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \int_0^\infty (t)^\beta exp - (t)^\beta\ dt$$

Putting

$$t^\beta = z$$

$$\beta t^{\beta-1} dt = dz$$

$$dt = \frac{dz}{\beta t^{\beta-1}}$$

$$\mu_D = \frac{(k.\beta.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \int_0^\infty \frac{1}{\beta t^{\beta-1}} z\ exp - z\ dz$$

$$\mu_D = \frac{(k.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}} \int_0^\infty z\ exp - z\ dz$$

$$\mu_D = \frac{(k.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}} \tag{4.18}$$

Thus mean of the PDF of damage accumulation line is given by the above equation. Now we want to find the variance of the damage accumulation function. Using the form for variance estimation

$$\text{Variance}=E[X^2] - E[X]^2 \tag{4.19}$$

Where $E[X]^2$ square of the mean

Now estimate the value of $E[X^2]$, it is given by

$$E[X^2] = \int_0^\infty D^2 \frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{aD} \frac{\beta}{\eta} \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1} exp - \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta} dD \tag{4.20}$$

$$E[X^2] = \int_0^\infty D \frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{a} \frac{\beta}{\eta} \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1} exp - \left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta} dD$$

Putting

$$\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta} = t$$

$$\frac{1}{k.a.\eta} \left(\frac{D}{k}\right)^{\frac{1}{a}-1} dD = dt$$

$$dD = \frac{dt\ (k.a.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}}$$

$$D = k.(t.\eta)^a$$

$$E[X^2] = \int_0^\infty \frac{k.(t.\eta)^a}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \frac{(\eta.a.k.\beta)}{\eta}\ \frac{\eta t}{a} t^{\beta-1} exp - t^\beta\ dt$$

$$E[X^2] = \frac{k^2 \eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \int_0^\infty t^a\ t.t^{\beta-1} exp - t^\beta\ dt$$

$$E[X^2] = \frac{k^2 \eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \int_0^\infty t^a\ \ t^\beta exp - t^\beta\ dt$$

Putting

$$t^\beta = z$$

$$t = z^\beta$$

$$\beta t^{\beta-1} dt = dz$$

$$dt = \frac{dz}{\beta t^{\beta-1}}$$

$$E[X^2] = \frac{k^2\eta^{a+1}\,\beta}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} . \frac{1}{\beta t^{\beta-1}} \int_0^\infty z^{\frac{a}{\beta}} exp - zdt$$

$$E[X^2] = \frac{k^2\eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} . \frac{1}{t^{\beta-1}} \int_0^\infty z^{\frac{a}{\beta}} exp - zdt$$

$$E[X^2] = \frac{k^2\eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} . \frac{1}{t^{\beta-1}} \Gamma\left(\frac{a}{\beta} + 2\right)$$

$$E[X^2] = \frac{k^2\eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} . \frac{1}{\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\,\beta-1} \Gamma\left(\frac{a}{\beta} + 2\right) \qquad (4.21)$$

Putting equation (4.18) and (4.21) into equation (4.19)

Then the variance of the damage accumulation nonlinear line is given by

Variance

$$\sigma^2 = \frac{k^2\eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \cdot \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}} \Gamma\left(\frac{a}{\beta}+2\right) - \left(\frac{(k.\eta)}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}}\right)^2$$

$$\sigma^2 = \frac{k^2\eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \cdot \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}} \left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right) - \left(\frac{\eta^2}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}}\right)\right]$$

Standard deviation of the damage accumulation line is give by

$$\sigma_D = \sqrt{\left(\frac{k^2\eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \cdot \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}} \left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right) - \left(\frac{\eta^2}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}}\right)\right]\right)} \qquad (4.22)$$

Thus D damage accumulation is a function of above mean and variance. Where the standard deviation of the damage accumulation is given by equation (4.22)

Using the mathematical form

$$\frac{\sigma_D}{\sigma_{N_f}} = \frac{x}{y}$$

$$\sigma_D = \frac{x}{y}\sigma_{N_f} \tag{4.23}$$

So σ_D is $\left(\frac{x}{y}\right)$ time of σ_{N_f}

4.4 Modeling trend line of variance

As shown in the equation (4.8), the cumulative damage increase nonlinearly with the loading cycles at any given stress level. Many researchers have demonstrated that the variability or the standard deviation of cumulative damage increases monotonically with the increase in loading cycles, while the variability of fatigue lives increases with the decreasing stress levels, Wang and Coit (2007); Coit and Vogt (2005); Pascual and Meeker (1990) Based on the above discussion, the variability in cumulative damage can be derived as a function of that in the fatigue lives.

Assuming that the variability in loading cycles is equal to zero at the initial point, it increases continuously to a certain value at the fatigue failure life. Using the geometric reasoning technique proposed in Rathod et al.,(2011) , the change rate of the variability can be interpreted as shown in figure 4.4

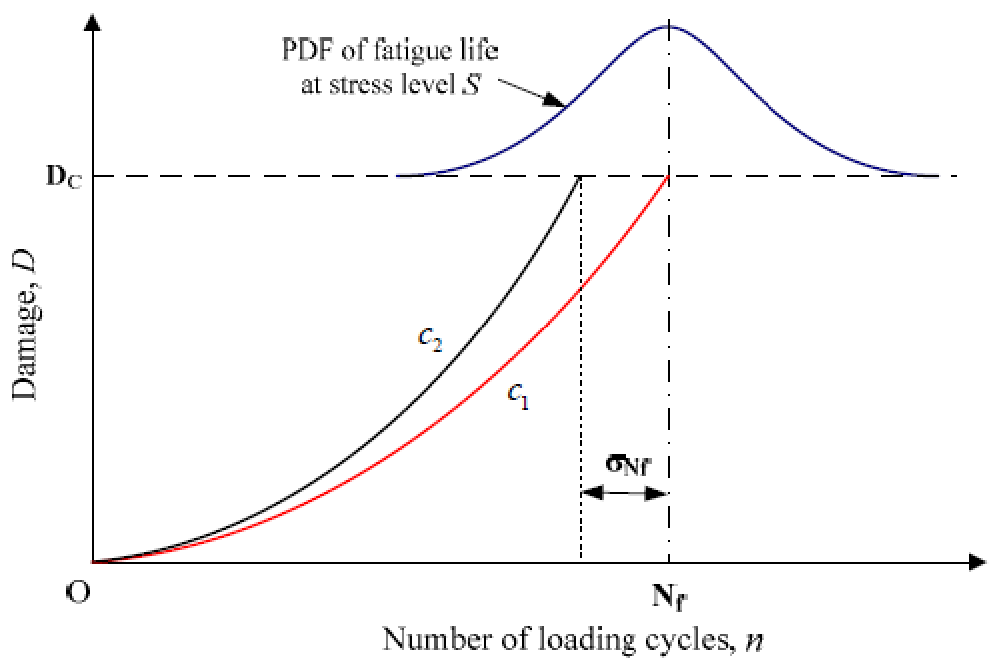

Figure 4.4: Graphical interpretation of change rate of variability in loading cycles

In Figure 4.4, $c_1$represents the mean cumulative damage trend curve and $c_2$is the 1- σ curve of fatigue life distribution. From the geometric construction shown in Figure 4.4, the rate of change of standard deviation (r_σ) of loading cycle can be derived as

$$r_\sigma = \frac{\sigma_{N_f}}{N_f} \tag{4.24}$$

Further, the standard deviation of loading cycle n can be obtained as

$$\sigma_n = \left(\frac{\sigma_{N_f}}{N_f}\right) n$$

Then

$$\sigma_{D=} \left(\frac{\sigma_{N_f}}{N_f}\right) n \frac{x}{y} \tag{4.25}$$

$$\sigma_{D=} \left(\frac{n}{N_f}\right) x$$

$$\sigma_{D=} \sqrt{\left(\frac{k^2\eta^{a+1}}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \cdot \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}}\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right) - \left(\frac{\eta^2}{\left(\frac{D}{k}\right)^{\frac{1}{a}-1}} \frac{1}{\left(\frac{\left(\frac{D}{k}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}}\right)\right]\right)} \left(\frac{n}{N_f}\right)$$

$$\sigma_{D}=\sqrt{\left(\frac{k^{2}\eta^{a+1}}{\left(N_{f}{}^{a}\right)^{\frac{1}{a}-1}}\cdot\frac{1}{\left(\frac{\left(N_{f}{}^{a}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}}\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right)-\left(\frac{\eta^{2}}{\left(N_{f}{}^{a}\right)^{\frac{1}{a}-1}}\frac{1}{\left(\frac{\left(N_{f}{}^{a}\right)^{\frac{1}{a}}}{\eta}\right)^{\beta-1}}\right)\right]\right)\left(\frac{n}{N_{f}}\right)}$$

$$\sigma_{D}=\sqrt{\left(\frac{k^{2}\eta^{a+1}}{\left(N_{f}\right)^{1-a}.\left(\frac{N_{f}}{\eta}\right)^{\beta-1}}\cdot\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right)-\left(\frac{\eta^{2}}{\left(N_{f}\right)^{1-a}}\frac{1}{\left(\frac{N_{f}}{\eta}\right)^{\beta-1}}\right)\right]\right)\left(\frac{n}{N_{f}}\right)}$$

$$\sigma_{D}=\sqrt{\left(\frac{k^{2}\eta^{\beta-1}}{\left(N_{f}\right)^{\beta-a}.}\cdot\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right)-\left(\frac{\eta^{2}}{\left(N_{f}\right)^{\beta-a}}\eta^{\beta-1}\right)\right]\right)\left(\frac{n}{N_{f}}\right)}$$

$$\sigma_{D}=\sqrt{\left(\left(N_{f}\right)^{-(\beta-a)}.\,k^{2}\eta^{\beta-1}\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right)-\left(\eta^{\beta+1}.\left(N_{f}\right)^{-(\beta-a)}\right)\right]\right)\left(\frac{n}{N_{f}}\right)} \qquad (4.26)$$

Putting the value of k

$$\mathrm{k}=D_{c}.\left(\frac{S^{m}}{C}\right)^{a}$$

$$\sigma_{D}=\sqrt{\left(\left(N_{f}\right)^{-(\beta-a)}.\,D_{c}.\left(\frac{S^{m}}{C}\right)^{a^{2}}\eta^{\beta-1}\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right)-\left(\eta^{\beta+1}.\left(N_{f}\right)^{-(\beta-a)}\right)\right]\right)\left(\frac{n}{N_{f}}\right)}$$

(4.27)

Now for the multi stress level

$$\sigma_D = \sqrt{\sum_{i=1}^{j}\left(\sqrt{\left(D_c\left(\frac{S_i{}^m}{C}\right)^{a_i}\right)^2 \eta_i{}^{\beta_i-1} N_{f_i}{}^{-(\beta_i-a)}\left[\eta_i{}^{a_i+1}\,\Gamma\left(\frac{a_i}{\beta_i}+2\right)-\eta_i{}^{\beta_i+1} N_f{}^{-(\beta_i-a_i)}\right]\left(\frac{n}{N_f}\right)}\right)^2} \tag{4.28}$$

Where i=1,2,….j Is the no of stress level under multi-level loading.

4.5 Framework of fatigue reliability analysis

The well-known stress-strength interference model considers the product reliability from the probabilistic point of view. This concept has been used by many researchers for developing the models to predict product reliability in the past (Liao et al. 1995; Kapur and Lamberson, 1977; Rao, 1992; Place et al., 1999) have classified the existing cumulative fatigue damage models for reliability prediction into two groups based on the fundamental assumptions and hypothesis as:

(i) Static statistical models and

(ii) Dynamic statistical models.

Unlike static models, dynamic models treat both the expected value and variance of random variable as the time dependent and their values continuously change with the time. However, these dynamic statistical models are developed on the existing classical stress-strength interference model considering the random variable as dynamic random variable (Liao et al., 1995). In the present work, the damage accumulation is treated as dynamic random variable whose distribution parameters (mean and variance) are dependent on usage life (time) as given above.

The fatigue failure of materials is reflected specifically in the evolution and distribution of the damage, which compromises reliability and safety. Based on the current state of structure, assurance of the reliability and continued safety requires a quantitative assessment of the structure in its projected future state. For this assessment, the proposed probabilistic modeling method in above section can be used to estimate the distribution of damage accumulation over its projected period of operation. Moreover, the framework for fatigue reliability analysis is depicted in figure 4.5

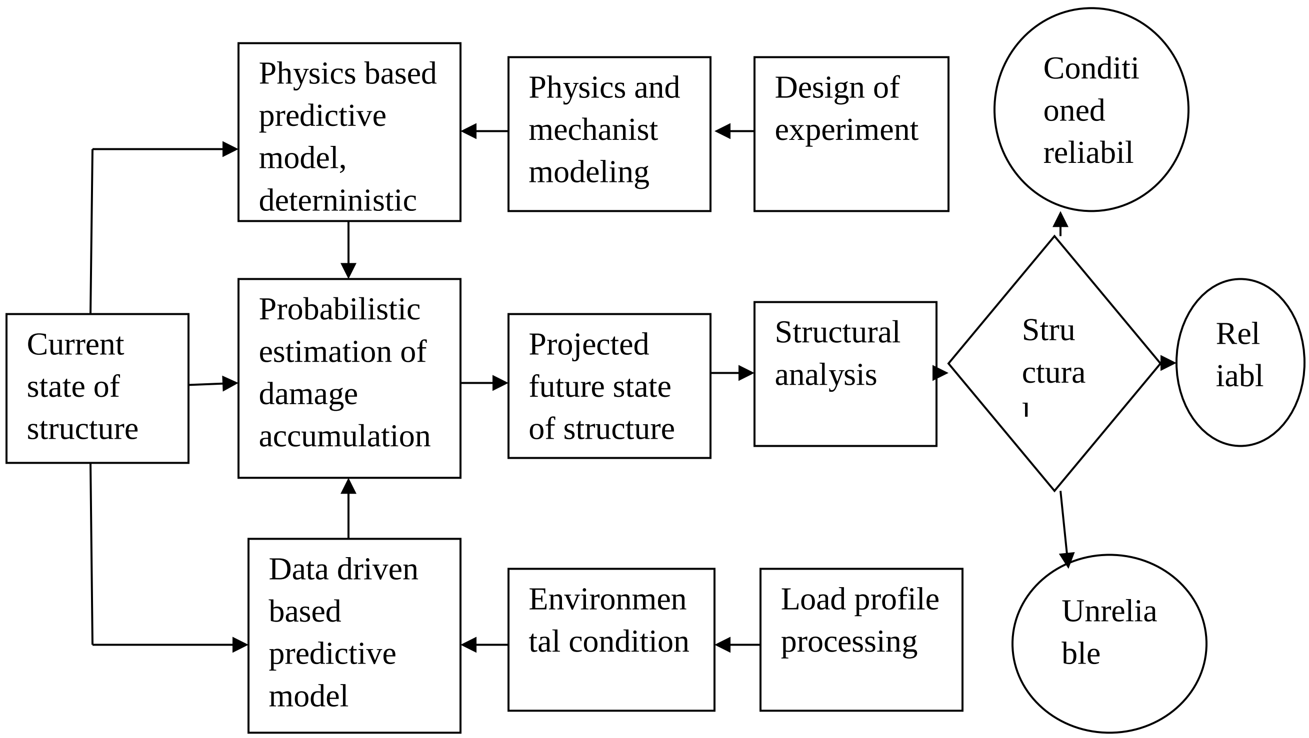

Figure 4.5: A flow diagram for fatigue reliability analysis of engineering structures (adopted from Zhu et al.,2012)

This thesis proposes a dynamic reliability prediction model considering the probabilistic damage accumulation developed in the previous section of this paper. The following assumptions have been made while formulating a dynamic reliability prediction model.

(1) Fatigue failure occurs when the damage accumulation (D) reaches the threshold damage (D_c), where $E(D_c) = 1$.

(2) The threshold damage or critical damage has the same distribution as the damage accumulation measure.

(3) When usage life is equal to the fatigue failure life ($n = N_f$), the variability of threshold damage accumulation ${\sigma_{D_c}}^2$ is equal to the variability of damage accumulation measure ${\sigma_D}^2$The variability of the damage accumulation measure continuously increases with the usage life but when usage cycle reaches to the fatigue failure level, the corresponding variability is assumed to be the same as the variability of threshold damage accumulation. However, it is statistically independent of (D).

At any given D_c , the critical threshold damage of the structure under consideration, the failure occurs when the random cumulative damage D is larger than D_c. The limit state function $G(n)$ associated with this problem is

$$G(n) = D_c - D(n)$$

Following the Weibull assumption of the fatigue damage accumulation, given the model for $D(n)$ one is able to drive the reliability of a component in terms of the general damage accumulation curve is

$$R = prob(G(n) > 0) = 1 - \phi\left(-\frac{\mu_{D_c} - \mu_D}{\sqrt{\left({\sigma_{D_c}}^2 + {\sigma_D}^2\right)}}\right) \tag{4.29}$$

A diagrammatic representation of the above concept is shown in figure 4.6 It is important to note that when the usage cycle is equal to failure life ($n = N_f$), the variability of the threshold damage accumulation will be equal to the variability of damage accumulation $({\sigma_{D_c}}^2 = {\sigma_D}^2)$.

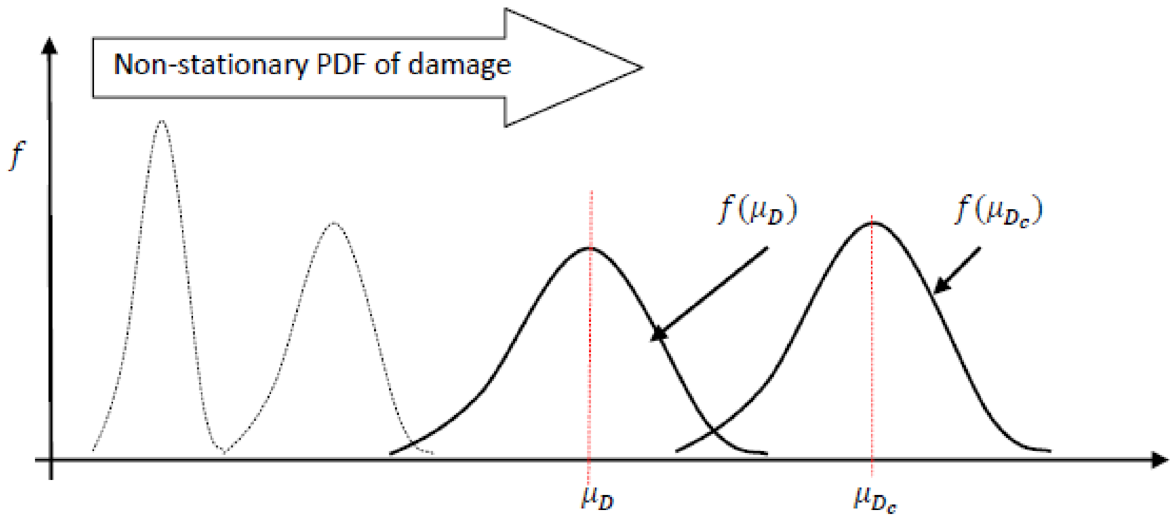

Figure 4.6: Dynamic Stress-strength Interference model for damage accumulation (adopted from Rathod et al.,2011)

Substituting equation (4.7) and (4.28) in equation (4.29) , the reliability can be expressed in the more suggestive form as

$$R = 1 - \phi\left(- \frac{\mu_{D_c} - \sum_{i=1}^{n} D_c \cdot \left(\frac{{S_i}^m}{C}\right)^{a_i} n^{a_i}}{\sqrt{\left({\sigma_{D_c}}^2 + \sum_{i=1}^{j}\left(\sqrt{\left(D_c\left(\frac{{S_i}^m}{C}\right)^{a_i}\right)^2 {\eta_i}^{\beta_i - 1} {N_{f_i}}^{-(\beta_i - a)}\left[{\eta_i}^{a_i+1}\,\Gamma\left(\frac{a_i}{\beta_i}+2\right) - {\eta_i}^{\beta_i+1} {N_f}^{-(\beta_i - a_i)}\right]}\left(\frac{n}{N_f}\right)\right)^2\right)}}\right) \tag{4.30}$$

The above model provides a dynamic reliability prediction considering dynamic behavior or continuous degradation phenomenon of the product with the usage cycle. In essence, the proposed dynamic reliability prediction model captures the product life cycle and assesses the product reliability for a given time period or usage cycle. The proposed model can be used for predicting reliability of a product subjected to both single stress and multi-stress level scenarios. The applicability of the proposed model is demonstrated with the help of a experimental data of 45 steel used in the railway vehicle example.

CHAPTER 5

NUMERICAL ANALYSIS

5.1 Introduction

To demonstrate the applicability of the proposed reliability approach, fatigue test data is required to model $S - N$ curve is adopted from Zhu et al.,(2012) which were obtained after conducting fatigue test on 45 steel used in the railway wheel. Table 5.1 shows the fatigue test data at different amplitude test level and corresponding standard deviation considered.

The 45 steel-1 data adopted from (Zheng et al., 2005) and 45 steel-2 data adopted from (Yan et al., 2000)

Table 5.1: Fatigue life data

Material	Stress amplitude	Mean (N_f)	Standard deviation (N_f)
45 steel-1	525	207	1.378
	500	245	1.404
	475	268	1.336
	450	337	1.419
	400	699	1.2969
45 steel-2	750	90	1.1618
	650	149	1.5102
	630	155	1.1274
	590	189	1.1051
	520	285	1.2712

Using the above data in table 5.1the model parameters for 45 steel were obtained by fitting the $S - N$ curve model in (4.5)

$$m = 2.43604; \quad C = 9.851\times10\text{^}8 \qquad 10$$

5.1.1 Define the $\boldsymbol{\beta}$ shape parameter

Many probability distributions are not a single distribution, but are in fact of the family of the distribution. This is due to the distribution having one or more shape parameter.

Shape parameters allow a distribution to take on the variety of shapes, depending on the value of shape parameter. These distributions are practically useful in the modelling application since they are flexible enough to model a variety of data sets.

The value of shape parameter is varies as 0.5, 1, 2, 3, 5 Figure 5.1 shows as the value of beta change how the shape of the curve change.

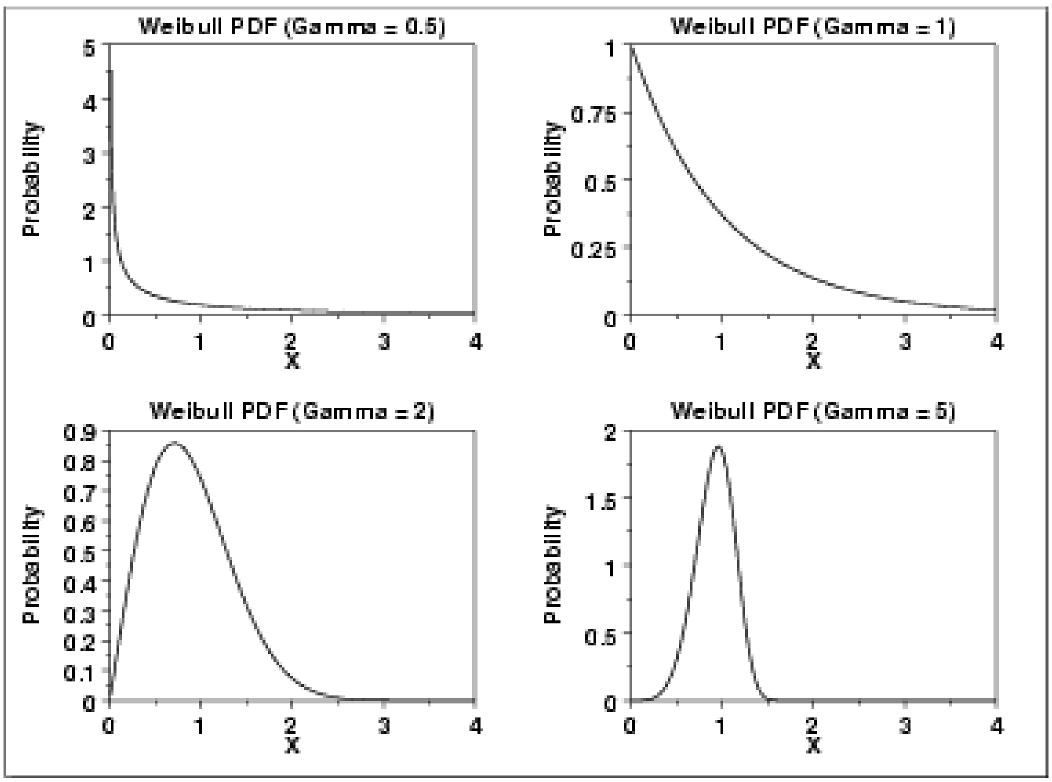

Figure 5.1: PDF of Weibull distribution at different shape parameter

The shapes above include an exponential distribution, a right-skewed distribution, and a relatively symmetric distribution.

The Weibull distribution has a relatively simple distributional form. However, the shape parameter allows the Weibull to assume a wide variety of the shapes. This combination of simplicity and flexibility in the shape of the Weibull distribution has made it an effective distributional model in the reliability applications. This ability to model a wide variety of distributional shapes using a relatively the simple distributional form is possible with many other distributional families as well. With the change in the value of shape parameter the curve or distribution is changes as give below:

1. $\beta = 1$ Represents the exponential distribution
2. $\beta = 2$ Represents the Rayleigh distribution
3. $\beta = 3$ Represents the lognormal distribution
4. $\beta = 5$ Represents the normal distribution

5.1.2 Define the scale parameter

The scale parameter, η, defines where the bulk of the distribution lies. A change in the scale parameter, η, has same effect on the distribution as a change of the abscissa scale. Increasing the value of scale parameter η while holding β constant has the effect of stretching out the PDF Since the area under the PDF curve is a constant value of one, the "peak" of the PDF curve will also decrease with the increase of η.

The 100 p^{th} percentile is given by

$$t_p = \eta[-\ln(1-p)]^{1/\beta} \tag{5.1}$$

η is equal to the 63.2th percentille of the no of total cycle etc.

$$\eta = t_{0.632}$$

η has the same unit as *T*, such as hours, miles, cycles, actuations, etc.

5.1.3 Define exponent parameter a

(Gao et al., 2014) proposed a nonlinear damage accumulation model for fatigue life prediction while considering the load interaction effects. He proposed two methods for calculating the exponent parameter

1) When there is load sequencing effect is considered then a is given by

$$a_{i-1,i} = \left(\frac{N_{f(i-1)}}{N_{fi}}\right)^{0.4} \tag{5.2}$$

Where the subscript 1,2,3….,i-1,I are the sequence no of loading stress, $n_1, n_2, \ldots\ldots, n_{i-1}, n_i$ are the cycle number under different loading stress, and $N_{f1} = N_{f2} = N_{f3} \ldots \ldots . = N_{fi-1} = N_{fi}$ represents the fatigue life under $\sigma_1, \sigma_2, \sigma_3, \ldots . \sigma_{i-1}, \sigma_i$ respectively. 0.4 is the material constant.

2) When there is load interaction effect is considered then a is given by

$$a_{i-1,i} = \left(\frac{N_{f(i-1)}}{N_{fi}}\right)^{0.4.min\left\{\sigma_{i-1}/\sigma_{i'}\frac{\sigma_i}{\sigma_{i-1}}\right\}} \tag{5.3}$$

For high-loading conditions, $0 < \frac{N_{f1}}{N_{f2}} < 1;$ then 0 <∝< 1

5.2 Validation of study

Validation of the study can be done in two parts:

5.2.1 Exponent parameter depends on the load sequencing effect

$$a_{i-1,i} = \left(\frac{N_{f(i-1)}}{N_{fi}}\right)^{0.4}$$

Then the following table is obtained while considering load sequencing effects

Table 5.2: For steel 45-1

Material	Stress	N_f	σ_f	Exponent a
Steel 45-1	525	207	1.378	1
	500	245	1.404	0.9348
	475	268	1.336	0.9647
	450	337	1.419	0.9124
	400	699	1.2969	0.7468

$$m = 2.43604; \quad C = 9.851\times 10\text{^}8$$

$$\mu_{D=} \ \sum_{i=1}^{n} D_c.\left(\frac{S_i{}^m}{C}\right)^{a_i} n^{a_i}$$

$$\mu_{D=} \ \sum_{i=1}^{n} 1.\left(\frac{S_i{}^{2.43604}}{9.851\times 10\text{^}8}\right)^{a_i} n^{a_i}$$

5.2.1.1 Reliability prediction for single stress level for steel 45-1

First, the applicability of the proposed model is demon stared by estimating the reliability for the single stress level. A single stress level of 525 Mpa is considered. For that purpose one has to estimate the variability of the threshold damage (σ_{D_c}) at the fatigue failure life and the variability of the damage accumulation at any given usage cycle. The variability of the threshold damage at the fatigue failure life is calculated by considering third assumption ($\sigma_{D_c} = \sigma_D$) at the failure life(N_f) and using the equation (4.28)

$$\sigma_{D=}\sqrt{\left(\left(N_f\right)^{-(\beta-a)}.D_c.\left(\frac{S^m}{C}\right)^{a^2}\eta^{\beta-1}\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right)-\left(\eta^{\beta+1}.\left(N_f\right)^{-(\beta-a)}\right)\right]\right)}\left(\frac{n}{N_f}\right)$$

Where $C = 9.851\times 10\text{^}8$, $S = 525\ MPa$;

Fatigue failure life $n = N_f = 207\ cycles$;

And $\sigma_{N_f} = 1.378 \quad a = 1$

(a) Firstly taking the value of $\boldsymbol{\beta = 1}$ (exponential distribution)

$$\sigma_{D_{c}=}0.0489$$

Similarly we can calculate the variability in damage accumulation at any given time period or the usage cycle (n). Once this elements of variability is estimated, equation (4.28) can be used to estimate the reliability of the steel 45-1 subjected to the single stress level for any given time period as below:

$$R = 1 - \phi\left(- \frac{\mu_{D_C} - \sum_{i=1}^{1} 1.\left(\frac{{S_i}^{2.43604}}{9.851\times10\text{^}8}\right)^{ai} n^{ai}}{\sqrt{\left(0.0489^2 + \sum_{i=1}^{j}\left(\sqrt{\left(D_c\left(\frac{{S_i}^m}{C}\right)^{a_i}\right)^2 {\eta_i}^{\beta_i - 1} {N_{f_i}}^{-(\beta_i - a)}\left[{\eta_i}^{a_i+1}\,\Gamma\left(\frac{a_i}{\beta_i}+2\right) - {\eta_i}^{\beta_i+1} {N_f}^{-(\beta_i - a_i)}\right]}\left(\frac{n}{N_f}\right)\right)^2\right)}}\right)$$

Considering the above reliability function the graph can be plot for any given stress level. A separate graph is plot for all stress level in one combine figure.

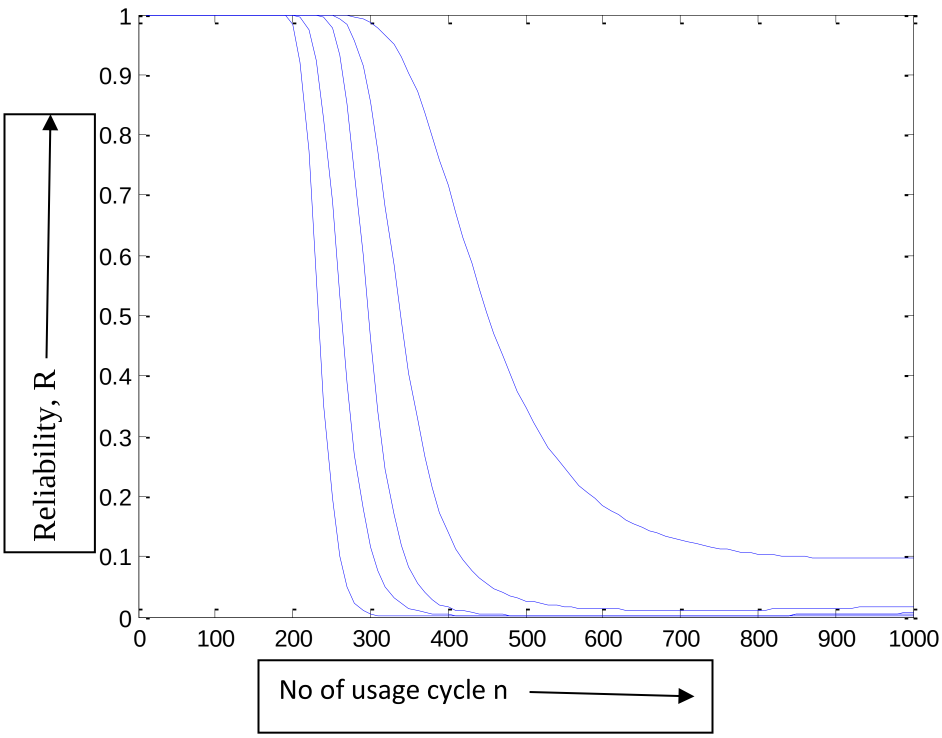

Figure 5.2: No of usage cycle versus reliability plot for single stress level when $\beta = 1$

In the above figure stress value vary from high to low from left to right. By using the above reliability plot we can estimate the reliability at any point of any stress level. These reliability plots clearly reveals that the trend of the reliability loss with the increase in the usage cycle. The careful analysis of these reliability plots indicate that reliability remains constant for some period at initial but later on it start decrease with the usage cycle. This phenomenon explains the existing understanding of crack initiation and the crack propagation period. The higher and stable reliability phase, although it varies with the stress levels, represents the crack initiating period and the reliability loss phase is the indicative of the crack propagating period. From the above diagram it is clear that the crack initiation period is smaller for the higher stress level while it is more for the lower stress level. The total life of the product also varies with the stress level.

(b) Taking the value of $\boldsymbol{\beta = 1.5}$

Similarly process for change the value of beta can be done as discussed above for the beta value one. The reliability plot for single stress level is given below:

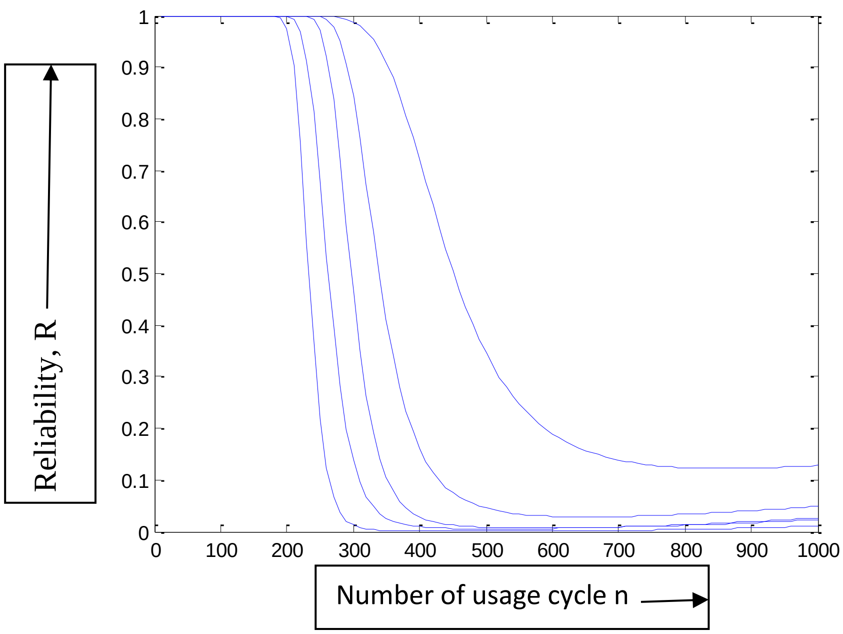

Figure 5.3: No of usage cycle versus reliability plot for single stress level when $\beta = 1.5$

From the above figure it is clearly indicated that as the value of beta increase curve will be shifted towards the left side means that failure or decline in the reliability is fast compare to the less value of beta. Other all significance will be same as for the beta is equal to one value.

(c) Taking the value of $\beta = 2$ (Rayleigh distribution)

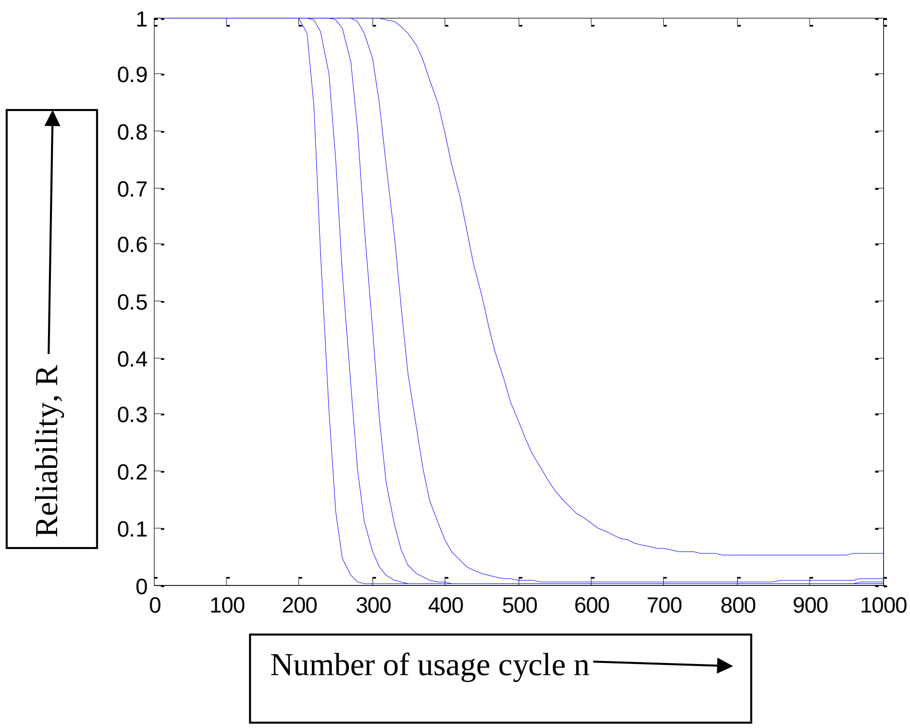

Figure 5.4: No of usage cycle versus reliability plot for single stress level when $\beta = 2$

From the above figure it is clearly indicated that as the value of beta is increase curve will become more flatten and shifted towards the left side. Mean decrease in reliability is faster compare to less value of the beta.

(d) Taking value of $\beta = 3$ (log-normal distribution)

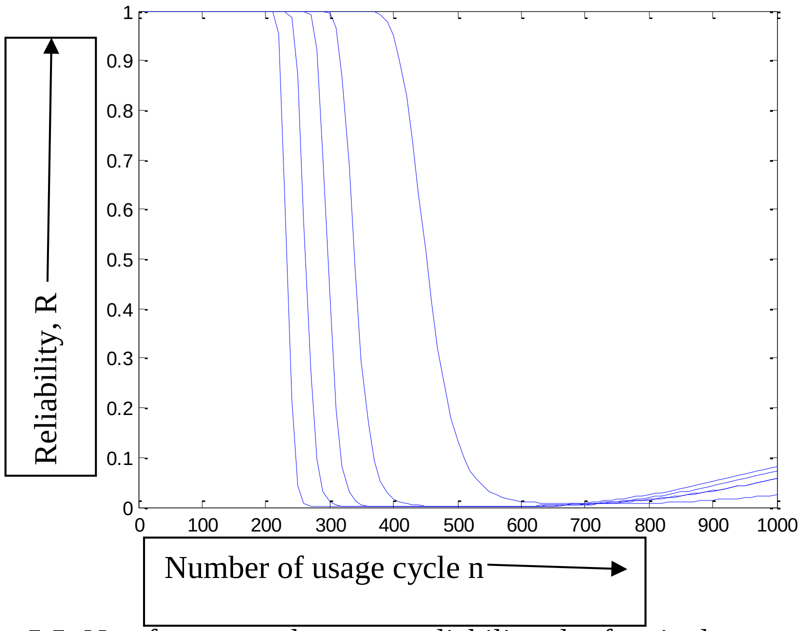

Figure 5.5: No of usage cycle versus reliability plot for single stress level when $\beta = 3$

When the value of beta is 3 Weibull distribution behaves like lognormal distribution and according to the above example the curve may shifted more towards left and become more flatten also. And the drop in the reliability is more here compare to less value of beta.

(e) Taking value of $\boldsymbol{\beta} = 5$ (Normal distribution)

Form the figure it is indicated that as the value of beta is five reliability plot become almost flatten for these data and also shifted more towards the left side. As from the many literature it reveals that normal distribution fits the data more suitable or in a better way but as the curve is more flatten it also indicated that these data not much suitable for the normal distribution. Since normal distribution needs more no of sample with the higher value to fit the data more suitably, But also up-to some extent it can be apply to the above data.

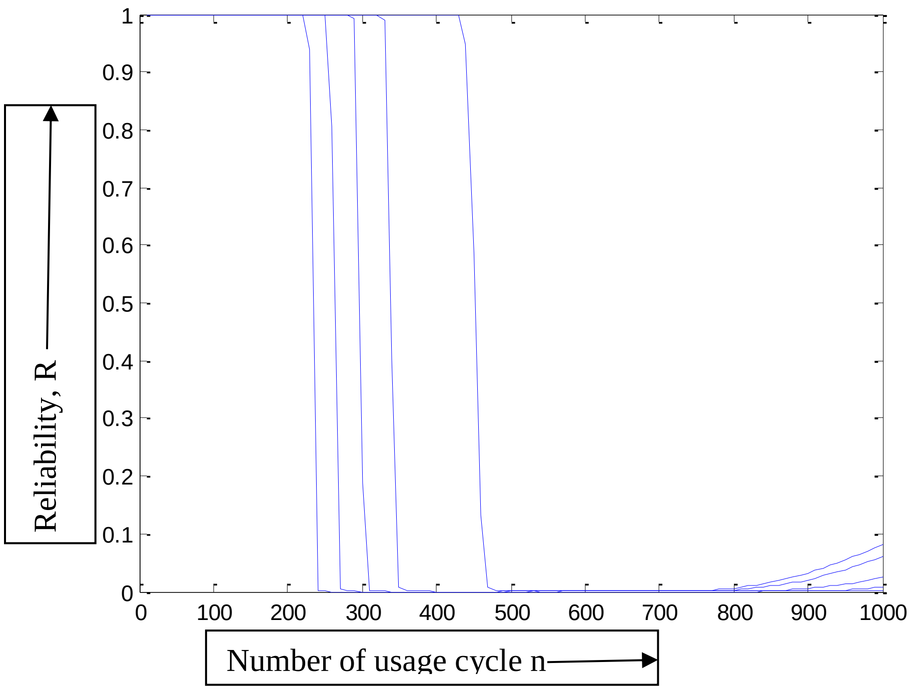

Figure 5.6: No of usage cycle versus reliability plot for single stress level when $\beta = 5$

From the above all figure for the different beta value it reveals that as the value of beta increase the reliability plots shifted towards the right side and become more flatten so it indicates that with higher value of beta drop in the reliability faster or more comparatively. It also due to that the no of cycle required for the failure is less, and higher value of failure cycle is required for higher value of beta to get efficient curve or slowly drop in the reliability.

5.2.1.2 Reliability prediction for multi-stress level for steel 45-1

To demonstrate multilevel loading condition, consider the same data as in the above table for steel 45-1 and taking five successive stress levels. To estimate reliability under multi-stress level loading, the fatigue life of steel 45-1 need to be predicted under multi-stress loading condition.

(a) Taking value of $\boldsymbol{\beta = 2}$

Figure 5.7 shows the combine reliability plot for high-low loading condition in right side and low-high loading condition in the left side. It reveals that when high-low loading condition occurs reliability is more with the same no of usage cycle compare to low-high loading condition since in low-high loading condition the value of exponent parameter is more than one. And effect reliability plot more. By using this figures we can easily calculate the reliability of high-low or low-high at any given usage cycle.

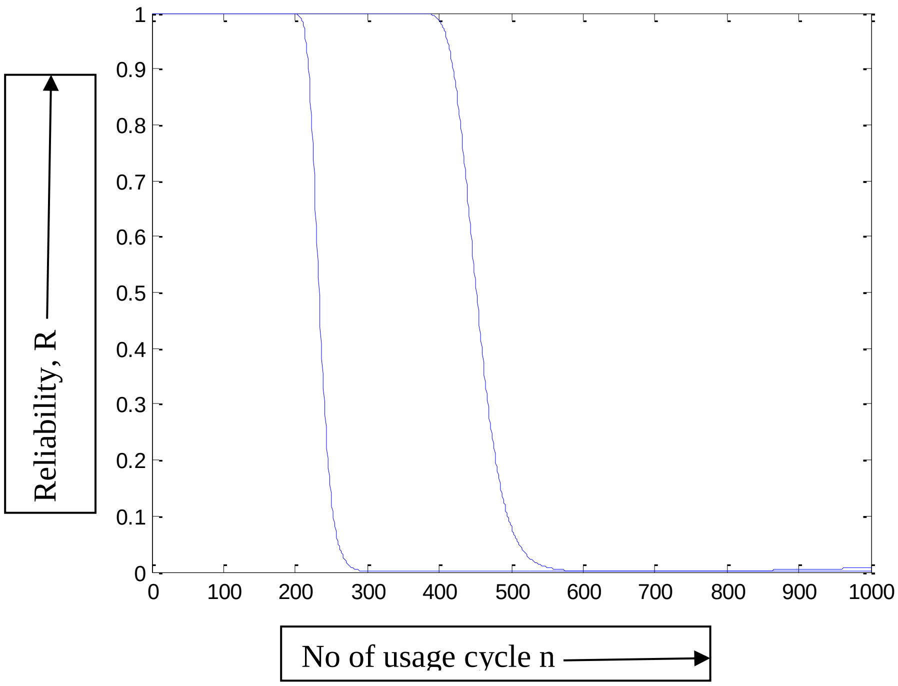

Figure 5.7: No of usage cycle versus reliability plot for multi-stress level when $\beta = 2$

(b) Taking value of $\boldsymbol{\beta = 1.5}$

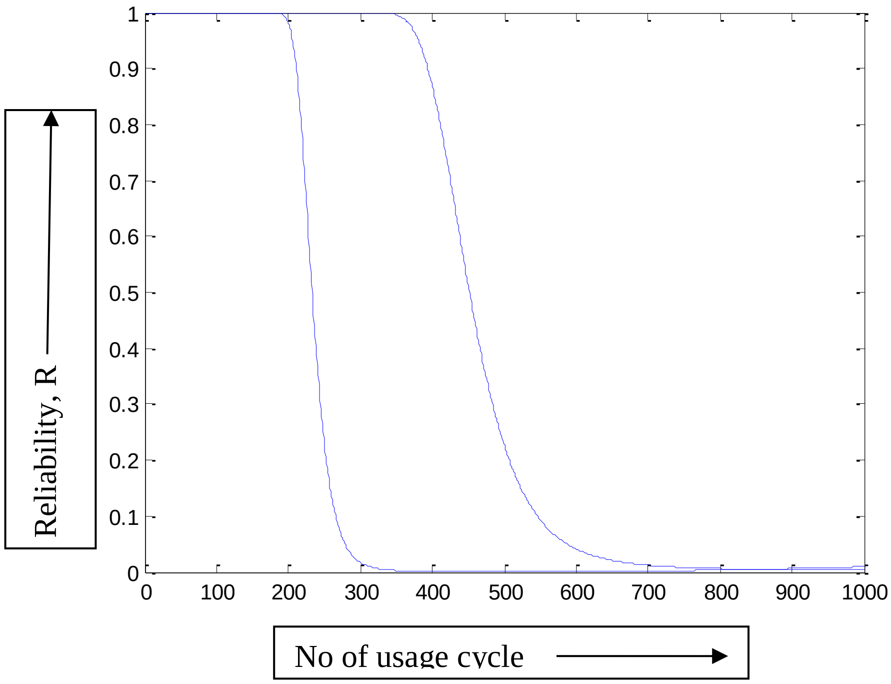

Figure 5.8: No of usage cycle versus reliability plot for multi-stress level when $\beta = 1.5$

(c) Taking the value of $\beta = 3$

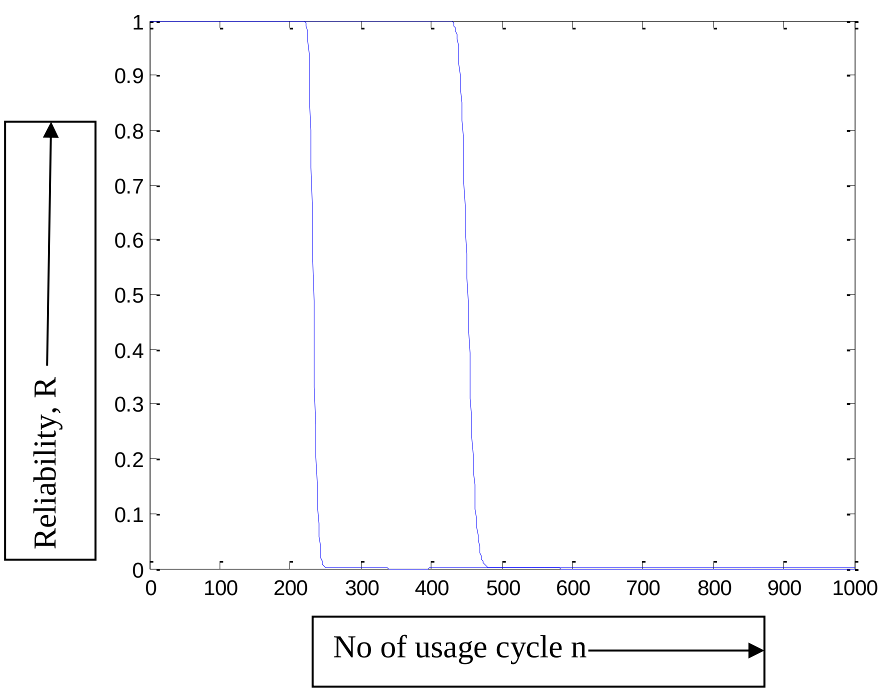

Figure 5.9: No of usage cycle versus reliability plot for multi-stress level when $\beta = 3$

(d) Taking value of $\boldsymbol{\beta} = 5$

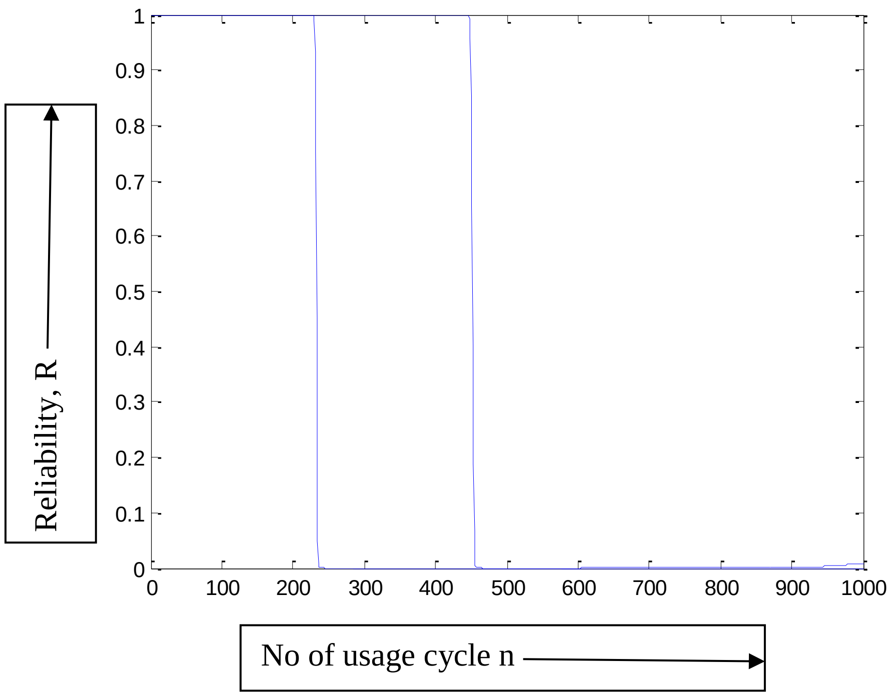

Figure 5.10: No of usage cycle versus reliability plot for multi-stress level when $\beta = 5$

From the above all figures it reveals that as the value of shape parameter increase there is curve will become flatten for both type of loading condition high-low and low-high. It is due to low no of cycle used here since at the higher value of shape parameter it will become normal distribution and it requires higher no of cycle for estimating the best reliability plot. And in all the above figure there is combine reverse loading condition is left side and combine (high-low) loading condition plot is right side.

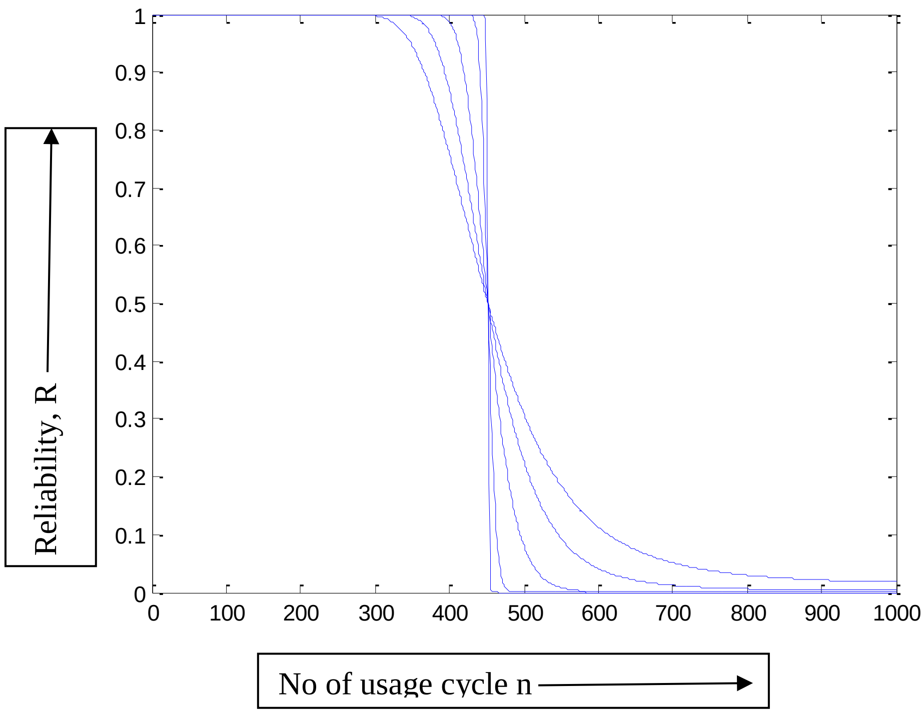

Figure 5.11: No of usage cycle versus reliability plot for single stress level when for all β

Figure 5.11 indicates the combine reliability plots for all value of beta. In the figure 5.11 the value of shape parameter increases from left hand side to the right hand side. These plots are very useful for comparison of reliability at different value of shape parameter at the same value of usage cycle.

Table 5.3: Fatigue data value for steel 45-2

Material	Stress	N_f	σ_f	Exponent a
	750	90	1.1618	1
	650	149	1.5102	0.8173
Steel 45-1	630	155	1.274	0.9843
	590	189	1.1051	0.9237
	520	285	1.2712	0.8484

$$m = 2.43604; \quad C = 9.851 \times 10^{\wedge}8$$

$$\mu_{D=}\ \sum_{i=1}^{n} D_c.\left(\frac{{S_i}^m}{C}\right)^{a_i} n^{a_i}$$

$$\mu_{D=}\ \sum_{i=1}^{n} 1.\left(\frac{{S_i}^{2.43604}}{9.851\times10\hat{}8}\right)^{a_i} n^{a_i}$$

5.2.1.3 Reliability prediction for single stress level for steel 45-2

First, the applicability of the proposed model is demon stared by estimating the reliability for the single stress level. A single stress level of 750 Mpa is considered. For that purpose one has to estimate the variability of the threshold damage (σ_{D_c}) at the fatigue failure life and the variability of the damage accumulation at any given usage cycle. The variability of the threshold damage at the fatigue failure life is calculated by considering third assumption ($\sigma_{D_c} = \sigma_D$) at the failure life(N_f) and using the equation (4.28)

$$\sigma_{D=}\sqrt{\left(\left(N_f\right)^{-(\beta-a)}.D_c.\left(\frac{S^m}{C}\right)^{a^2}\eta^{\beta-1}\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right)-\left(\eta^{\beta+1}.\left(N_f\right)^{-(\beta-a)}\right)\right]\right)\left(\frac{n}{N_f}\right)}$$

Where $C = 9.851\times10\hat{}8$, $S = 750\ MPa$;

Fatigue failure life $n = N_f = 90\ cycles$;

And $\sigma_{N_f} = 1.1618$ $a = 1$

(a) Firstly taking the value of $\boldsymbol{\beta = 1}$ (exponential distribution)

$$\sigma_{D_c=}0.0319$$

Similarly we can calculate the variability in damage accumulation at any given time period or the usage cycle (n). Once this elements of variability is estimated, equation (4.30) can be used to estimate the reliability of the steel 45-1 subjected to the single stress level for any given time period as below:

$$R = 1 - \phi\left(- \frac{\mu_{D_c} - \sum_{i=1}^{1} 1.\left(\frac{S_i^{2.43604}}{9.851\times10^{\wedge}8}\right)^{a} n^{a}}{\sqrt{\left(0.0319^2 + \sum_{i=1}^{j}\left(\sqrt{\left(D_c\left(\frac{S_i^{m}}{C}\right)^{a_i}\right)^2 \eta_i^{\beta_i - 1} N_{f_i}^{-(\beta_i - a)}\left[\eta_i^{a_i+1}\,\Gamma\left(\frac{a_i}{\beta_i}+2\right) - \eta_i^{\beta_i+1} N_f^{-(\beta_i - a_i)}\right]}\left(\frac{n}{N_f}\right)\right)^2\right)}}\right)$$

Considering the above reliability function the graph can be plot for any given stress level. A separate graph is plot for all stress level in one combine figure.

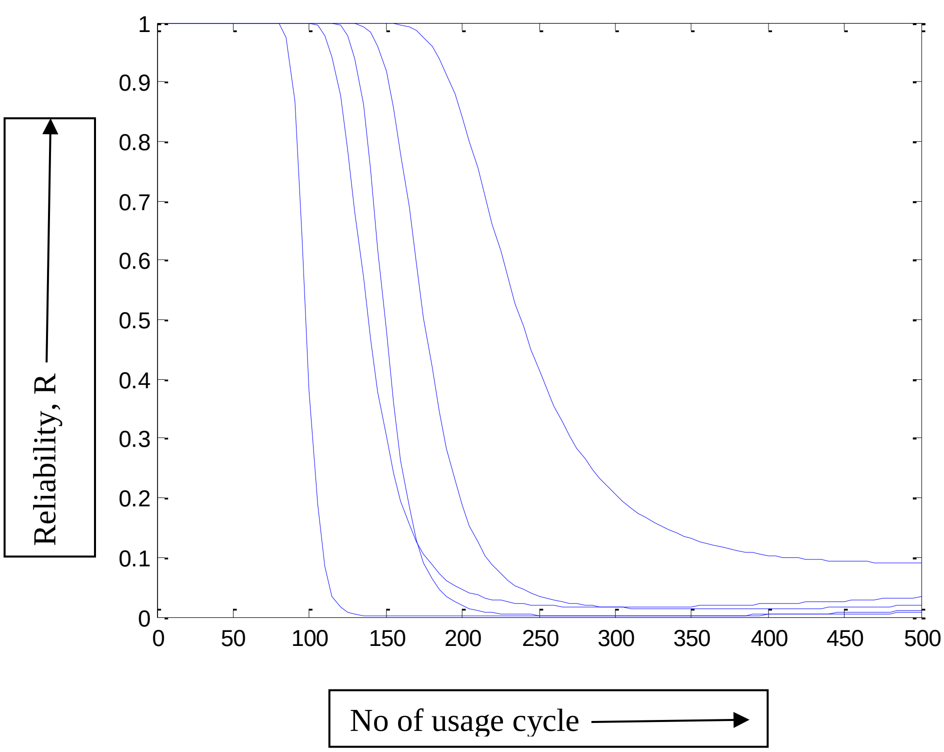

Figure 5.12: No of usage cycle versus reliability plot for single stress level when $\beta = 1$

(b) $\boldsymbol{\beta = 1.5}$

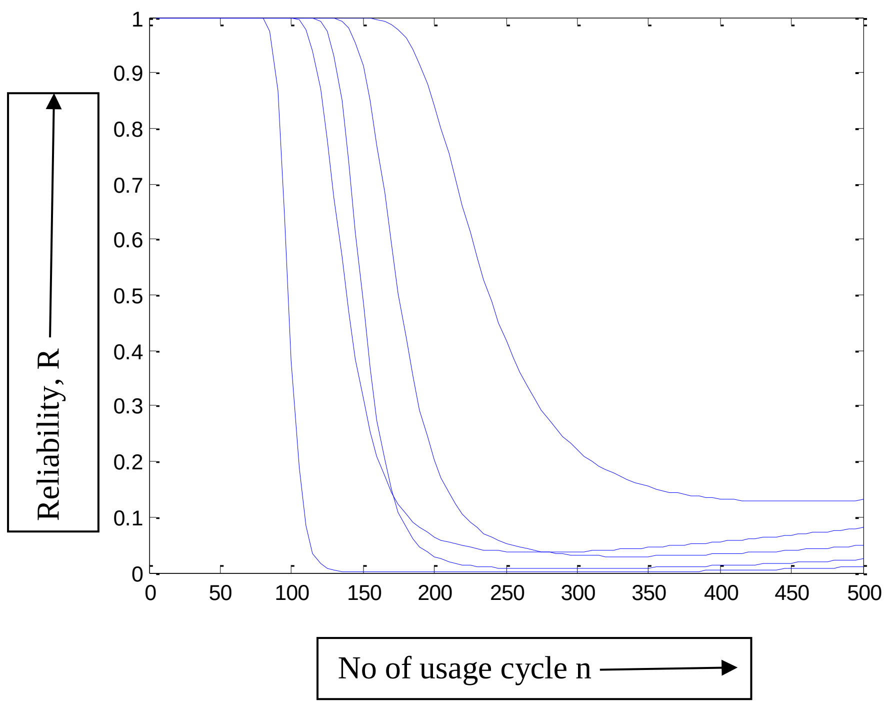

Figure 5.13: No of usage cycle versus reliability plot for single stress level when $\beta = 1.5$

(c) $\boldsymbol{\beta} = \mathbf{2}$

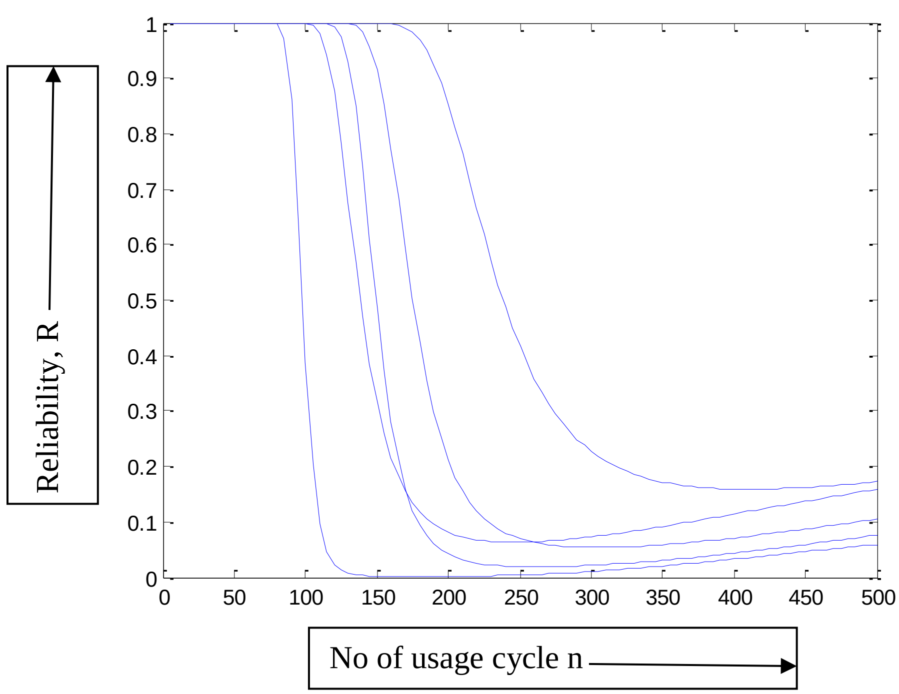

Figure 5.14: No of usage cycle versus reliability plot for single stress level when $\beta = 2$

(d) $\boldsymbol{\beta = 3}$

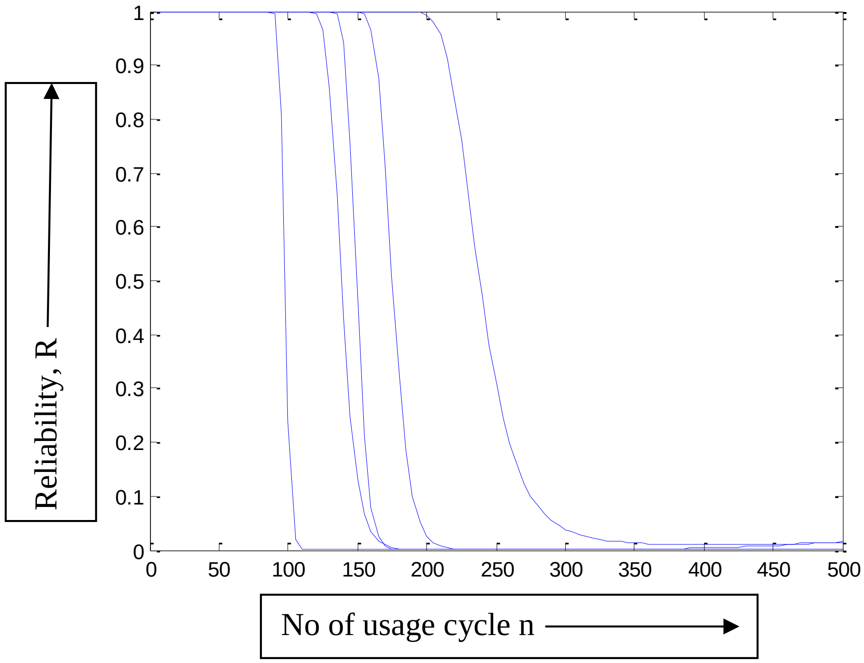

Figure 5.15: No of usage cycle versus reliability plot for single stress level when $\beta = 3$

(e) $\boldsymbol{\beta = 5}$

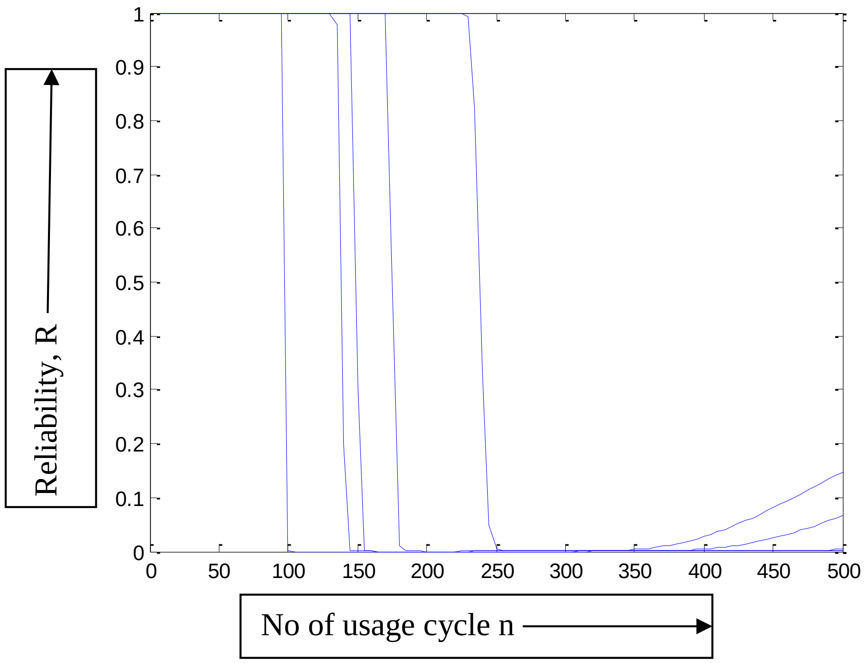

Figure 5.16: No of usage cycle versus reliability plot for single stress level when $\beta = 5$

In steel 45-2 variation in loading is random compare to steel 45-1 in which variation in loading is fixed. Steel 45-2 plots shows that the drop in the reliability plot is less with the usage cycle compare to the steel 45-1. Also the flatness of curve is less in these plots comparatively. Other all the conclusion remains same as steel 45-1 as curve will become shifted towards left as the value of shape parameter increases.

5.2.1.4 Reliability plot for multi stress level

To demonstrate multilevel loading condition, consider the same data as in the above table for steel 45-2 and taking five successive stress levels. To estimate reliability under multi-stress level loading, the fatigue life of steel 45-2 need to be predicted under multi-stress loading condition.

(a) $\boldsymbol{\beta = 1.5}$

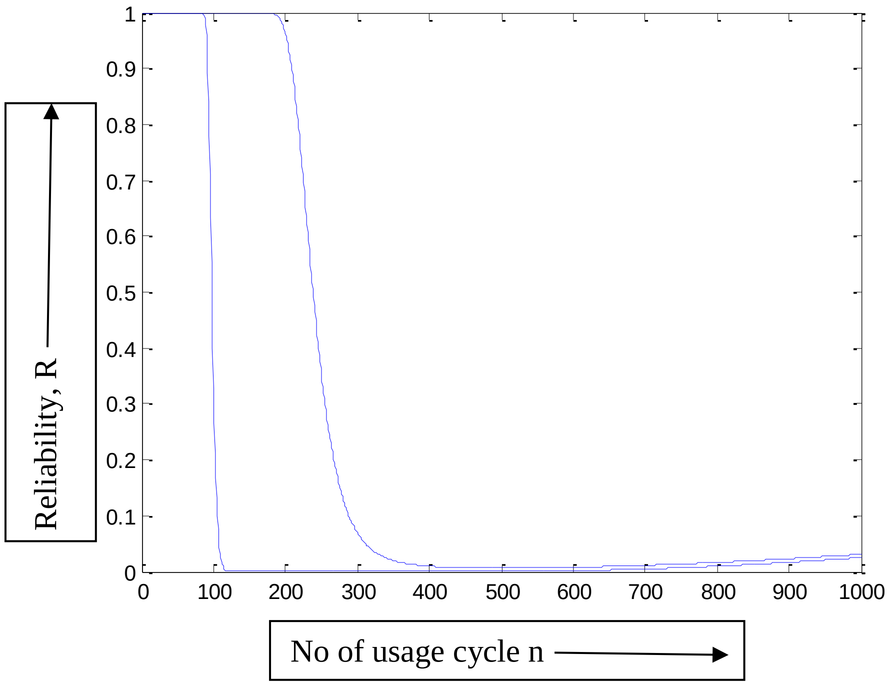

Figure 5.17: No of usage cycle versus reliability plot for multi-stress level when $\beta = 1.5$

(b) $\boldsymbol{\beta = 2}$

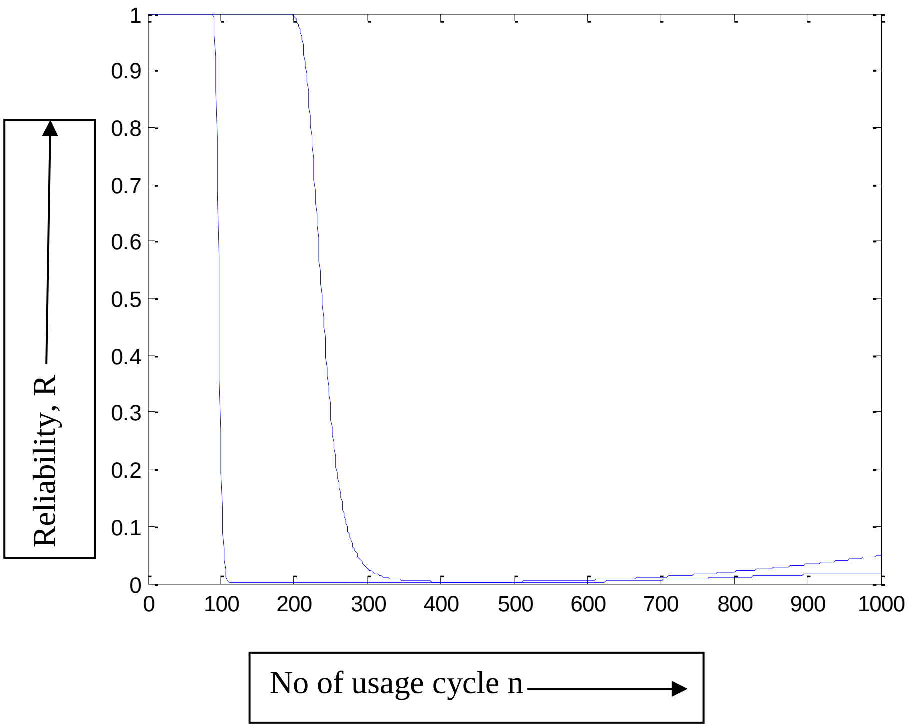

Figure 5.18: No of usage cycle versus reliability plot for multi-stress level when $\beta = 2$

(C) $\boldsymbol{\beta = 3}$

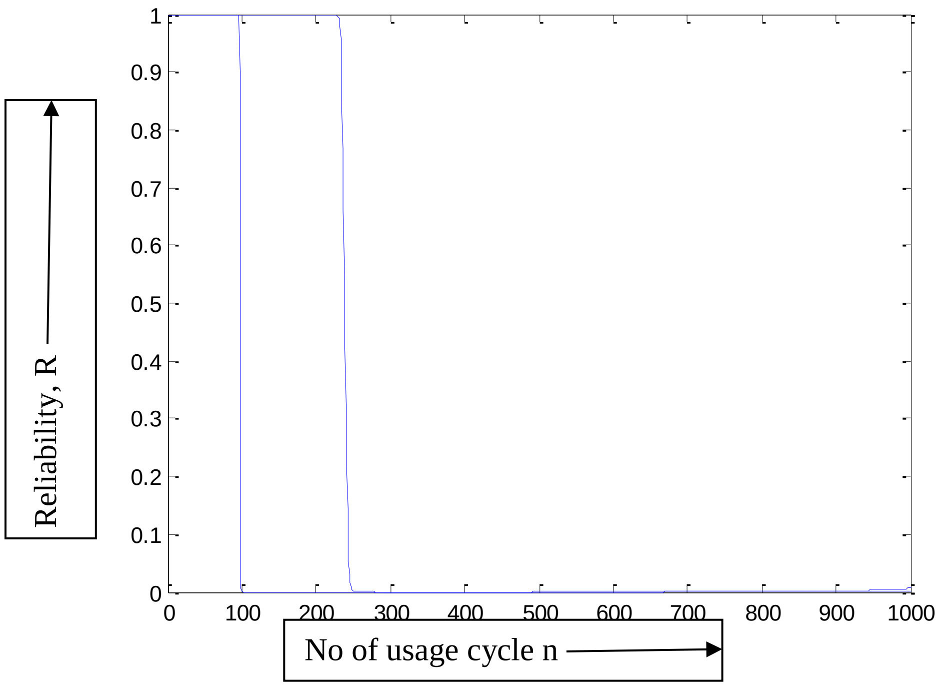

Figure 5.19: No of usage cycle versus reliability plot for multi-stress level when $\beta = 3$

(d)$\boldsymbol{\beta} = \mathbf{5}$

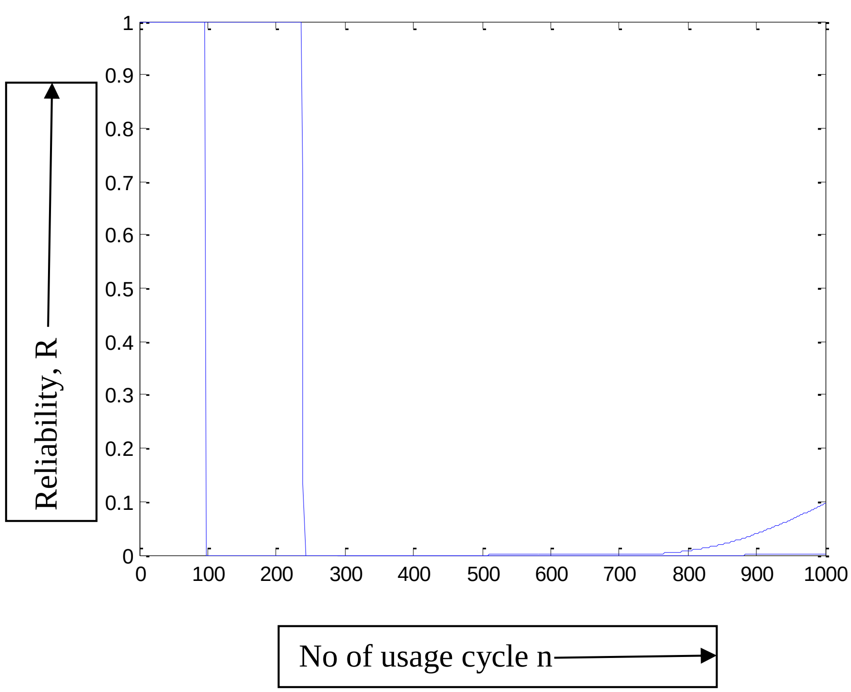

Figure 5.20: No of usage cycle versus reliability plot for multi-stress level when $\beta = 5$

For steel 45-2 all combine figures shows the plot of multi-stress level loading. The left hand plot shows low-high loading condition. While the right hand plot shows high-low loading condition. Here the curve more flatten compare to steel 45-1 since here maximum value of stress is more and also change in the stress value is different each time. Since at the higher value of stress level the no of cycle for failure is less so these curve will become more flatten for the shape parameter (5) since then it will become normal reliability plot. It also some-how flatten for the lognormal distribution because this distribution requires more no of fatigue failure cycle.

(e)Combine forward plot

This graph shows for all value of shape parameter for steel 45-2. The value of shape parameter increase from left to right. These plots are very useful tool for predicting the reliability of any distribution at the same no of usage cycle.

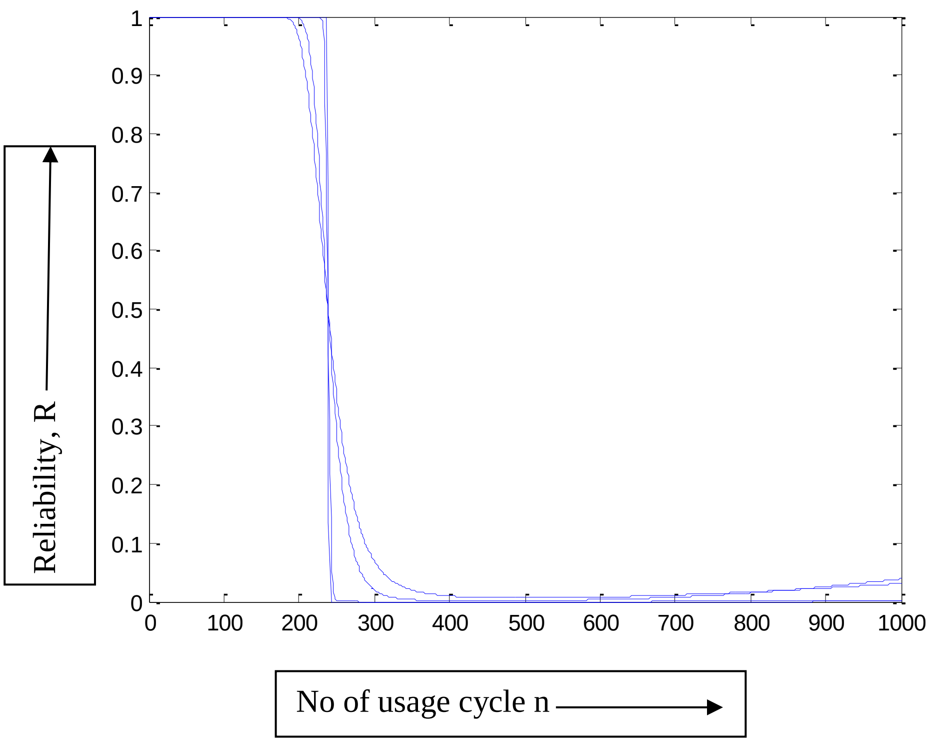

Figure 5.21: No of usage cycle versus reliability plot for single stress level when for all β

5.2.2 Exponent parameter depends on load interaction effect

$$a_{i-1,i} = \left(\frac{N_{f(i-1)}}{N_{fi}}\right)^{0.4.min\left\{\sigma_{i-1}/\sigma_i,\frac{\sigma_i}{\sigma_{i-1}}\right\}}$$

Then the following table is obtained while considering load sequencing effects

Table 4.4 Fatigue life data for load interaction for steel 45-1

Material	Stress	N_f	σ_f	Exponent a
Steel 45-1	525	207	1.378	1
	500	245	1.404	0.9378
	475	268	1.336	0.9664
	450	337	1.419	0.9168
	400	699	1.2969	0.7715

$$m = 2.43604; \quad C = 9.851\times10\text{^}8$$

$$\mu_{D=} \ \sum_{i=1}^{n} D_c.\left(\frac{S_i{}^m}{C}\right)^{a_i} n^{a_i}$$

$$\mu_{D=} \ \sum_{i=1}^{n} 1.\left(\frac{S_i{}^{2.43604}}{9.851\times10\text{^}8}\right)^{a_i} n^{a_i}$$

5.2.2.1 Reliability prediction for single stress level (Load interaction)

First, the applicability of the proposed model is demon stared by estimating the reliability for the single stress level. A single stress level of 525 Mpa is considered. For that purpose one has to estimate the variability of the threshold damage (σ_{D_c}) at the fatigue failure life and the variability of the damage accumulation at any given usage cycle. The variability of the threshold damage at the fatigue failure life is calculated by considering third assumption ($\sigma_{D_c} = \sigma_D$) at the failure life(N_f) and using the equation (4.28)

$$\sigma_{D=}\sqrt{\left(\left(N_f\right)^{-(\beta-a)}.D_c.\left(\frac{S^m}{C}\right)^{a^2}\eta^{\beta-1}\left[\eta^{a+1}\Gamma\left(\frac{a}{\beta}+2\right)-\left(\eta^{\beta+1}.\left(N_f\right)^{-(\beta-a)}\right)\right]\right)\left(\frac{n}{N_f}\right)}$$

Where $C = 9.851\times10\text{^}8$, $S = 525\ MPa$;

Fatigue failure life $n = N_f = 207\ cycles$;

And $\sigma_{N_f} = 1.378$ $a = 1$

(a) Firstly taking the value of $\boldsymbol{\beta = 1}$ (exponential distribution)

$$\sigma_{D_c=}0.0512$$

Similarly we can calculate the variability in damage accumulation at any given time period or the usage cycle (n). Once this elements of variability is estimated, equation can be used to estimate the reliability of the steel 45-1 subjected to the single stress level for any given time period as below:

$$R = 1 - \phi\left(-\frac{\mu_{D_c} - \sum_{i=1}^{1} 1.\left(\frac{{S_i}^{2.43604}}{9.851\times10\text{^}8}\right)^a n^a}{\sqrt{\left(0.0512^2 + \sum_{i=1}^{j}\left(\sqrt{\left(D_c\left(\frac{{S_i}^m}{C}\right)^{a_i}\right)^2 {\eta_i}^{\beta_i-1}{N_{f_i}}^{-(\beta_i-a)}\left[{\eta_i}^{a_i+1}\Gamma\left(\frac{a_i}{\beta_i}+2\right)-{\eta_i}^{\beta_i+1}{N_f}^{-(\beta_i-a_i)}\right]\left(\frac{n}{N_f}\right)}\right)^2\right)}}\right)$$

Considering the above reliability function the graph can be plot for any given stress level. a separate graph is plot for all stress level in one combine figure.

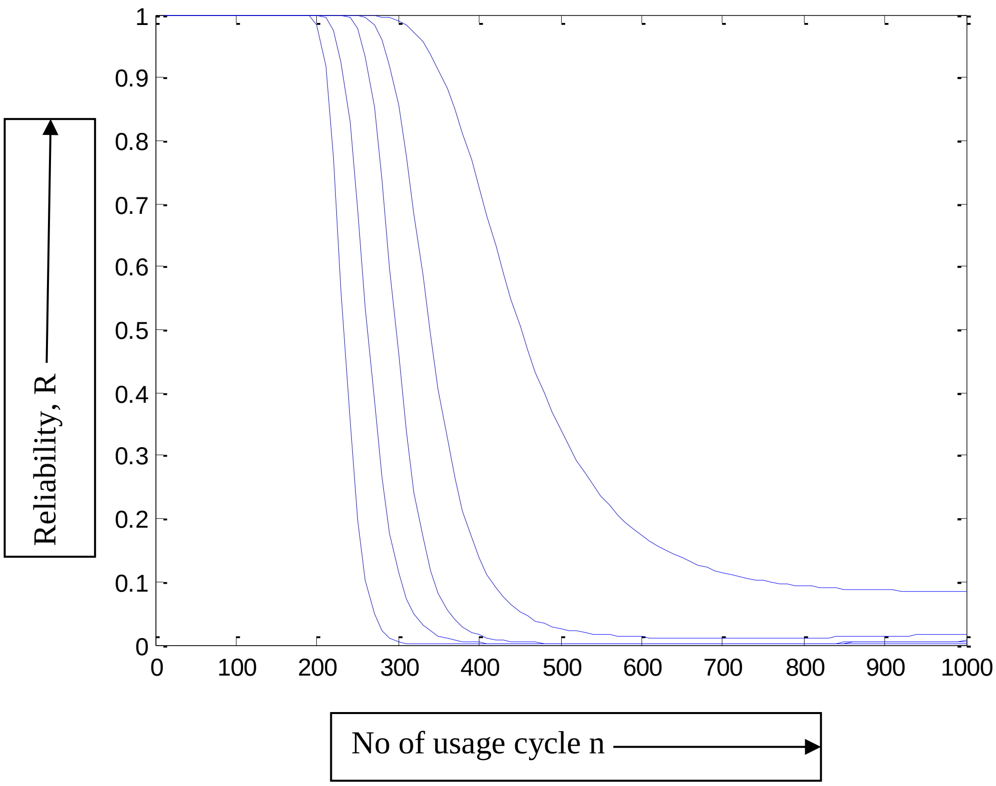

Figure 5.22: No of usage cycle versus reliability plot for single stress level (load interaction) $\beta = 1$

(b) $\beta = 2$

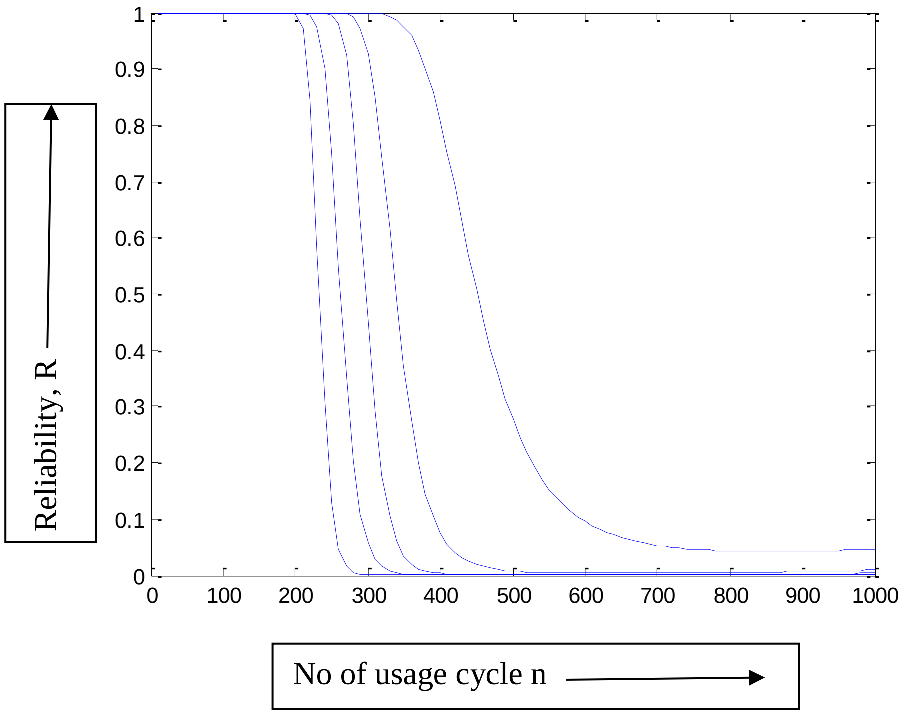

Figure 5.23: No of usage cycle versus reliability plot for single stress level when $\beta = 2$

(c) $\boldsymbol{\beta} = \mathbf{3}$

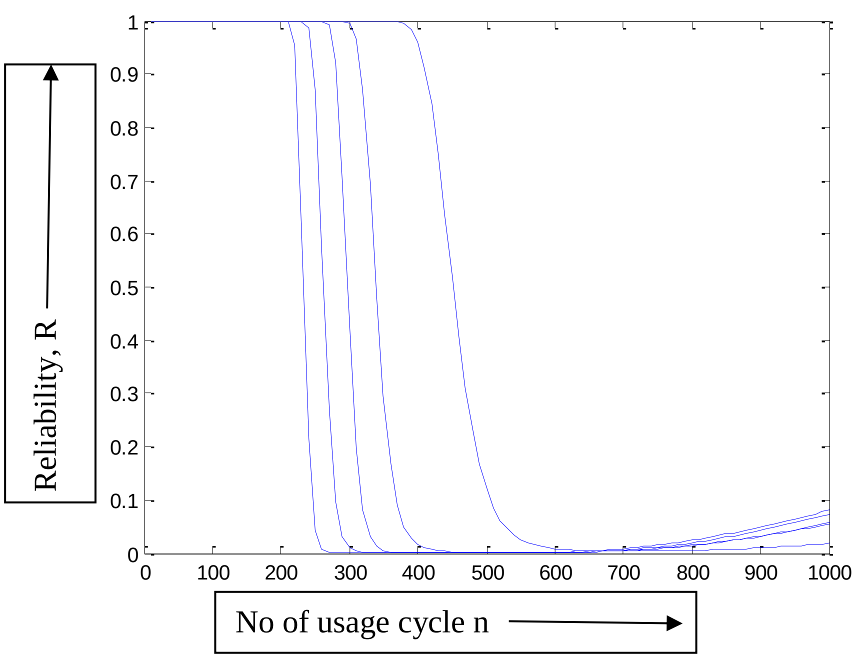

Figure 5.24: No of usage cycle versus reliability plot for single stress level when $\beta = 3$

(d) $\beta = 5$

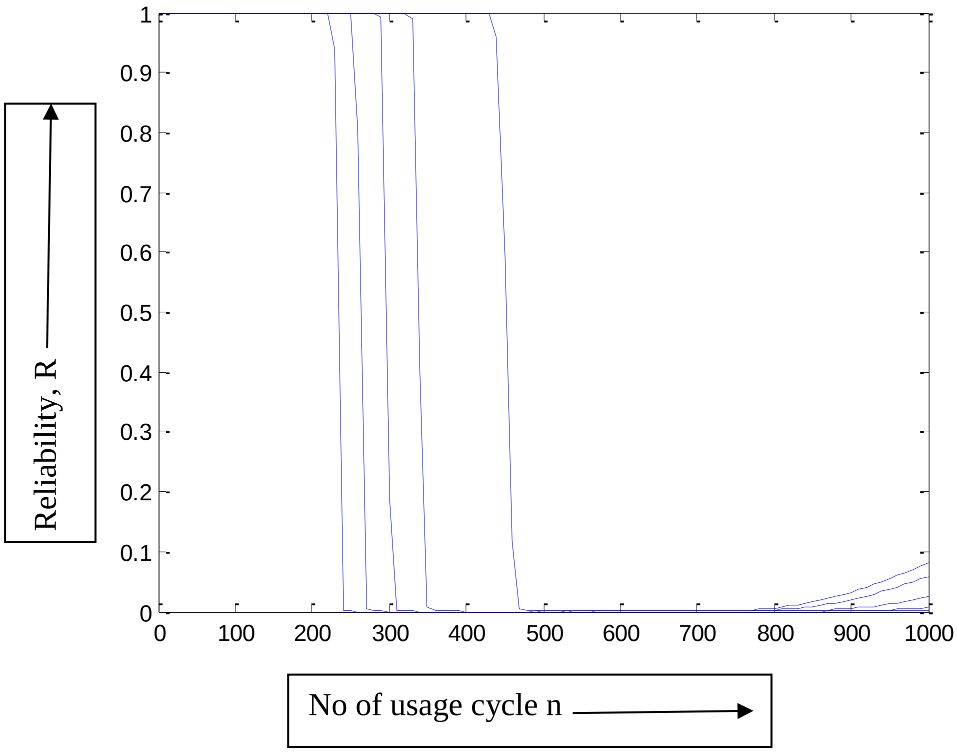

Figure 5.25: No of usage cycle versus reliability plot for single stress level when $\beta = 5$

Load interaction effect is an important phenomenon when any component is subjected to fatigue. Since any load effect in some manner second load so it becomes an important factor for consideration while we are doing plot for the reliability prediction. For all the values of shape parameter by considering load interaction effect, these plots give more accurate reliability for any usage cycle. These plots are more accurate compare to load sequencing effect. And become very useful in real life scenario.

5.2.2.2 Reliability prediction for multi-stress level (load interaction)

To demonstrate multilevel loading condition, consider the same data as in the above table for steel 45-1 and taking five successive stress levels. To estimate reliability under multi-stress level loading, the fatigue life of steel 45-1 need to be predicted under multi-stress loading condition.

(a) $\boldsymbol{\beta = 1}$

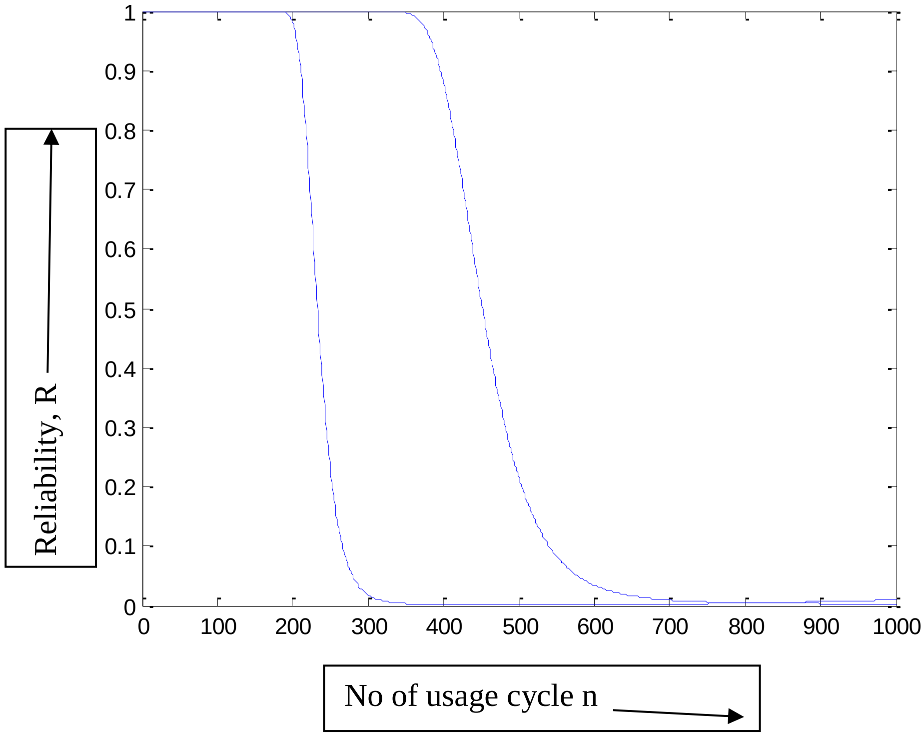

Figure 5.26: No of usage cycle versus reliability plot for multi-stress level when $\beta = 1$

(b) $\boldsymbol{\beta = 2}$

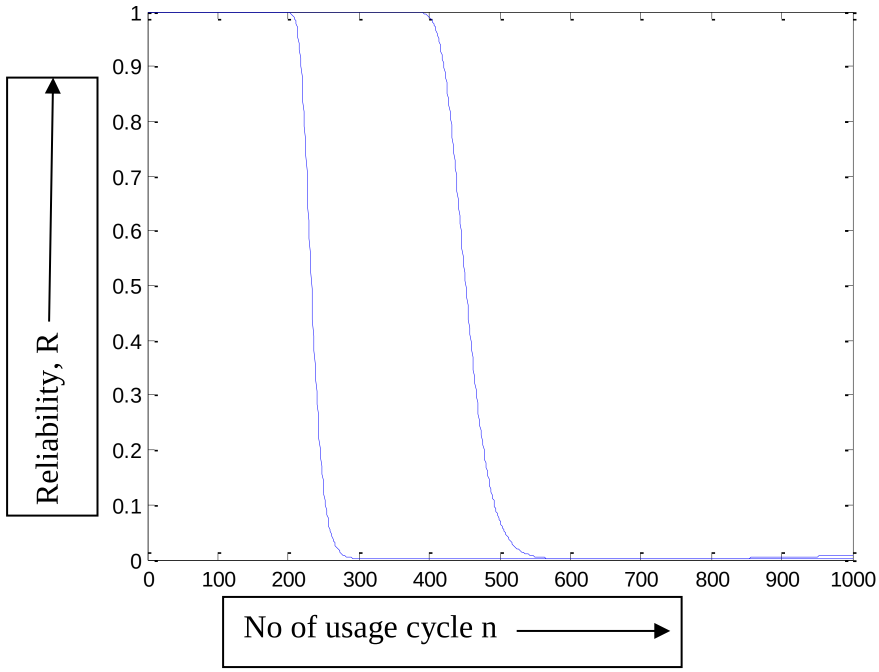

Figure 5.27: No of usage cycle versus reliability plot for multi-stress level when $\beta = 2$

(c) $\beta = 3$

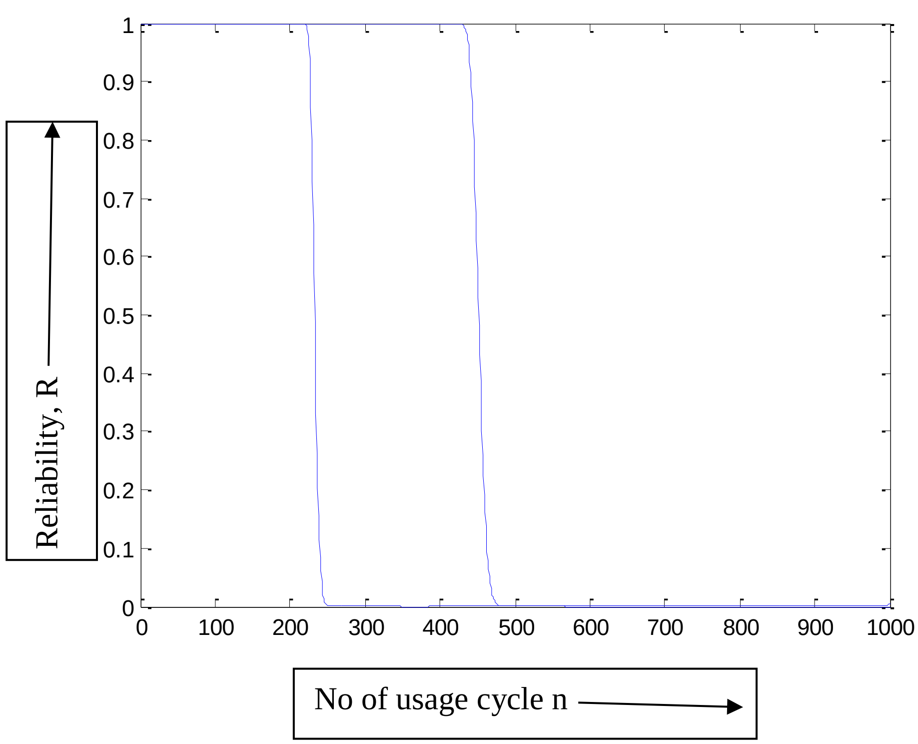

Figure 5.28: No of usage cycle versus reliability plot for multi-stress level when $\beta = 3$

(d) Combine high-low loading for all beta

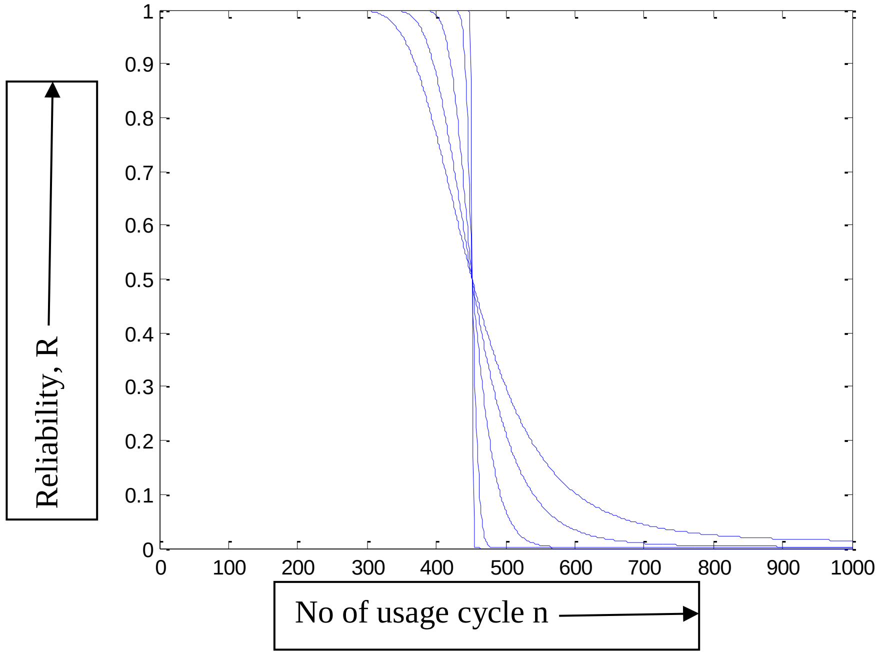

Figure 5.29: No of usage cycle versus reliability plot for single stress level combine

The reliability plot for high-low and low-high loading is shown in all figures. There is high-low loading is right side while low-high or reverse loading is left side. These curves gives more accurate evaluation of reliability at any usage cycle compare to load sequencing plot, since load interaction effect is considered here.

Table 5.5 fatigue life data for steel 45-2 (load interaction)

Material	Stress	N_f	σ_f	Exponent a
Steel 45-2	750	90	1.1618	1
	650	149	1.5102	0.8371
	630	155	1.274	0.9848
	590	189	1.1051	0.9284
	520	285	1.2712	0.8651

$$m = 2.43604; \quad C = 9.851\times10^{\wedge}8$$

$$\mu_{D=} \ \sum_{i=1}^{n} D_c . \left(\frac{{S_i}^m}{C}\right)^{a_i} n^{a_i}$$

$$\mu_{D=} \ \sum_{i=1}^{n} 1. \left(\frac{{S_i}^{2.43604}}{9.851\times10^{\wedge}8}\right)^{a_i} n^{a_i}$$

5.2.2.3 Reliability prediction for single stress level

Similarly procedure can be applied as for steel 45-1 for load sequencing effect only change the value of exponent parameter a so reliability plot are given below for single stress level for different value of β

(a) β =1

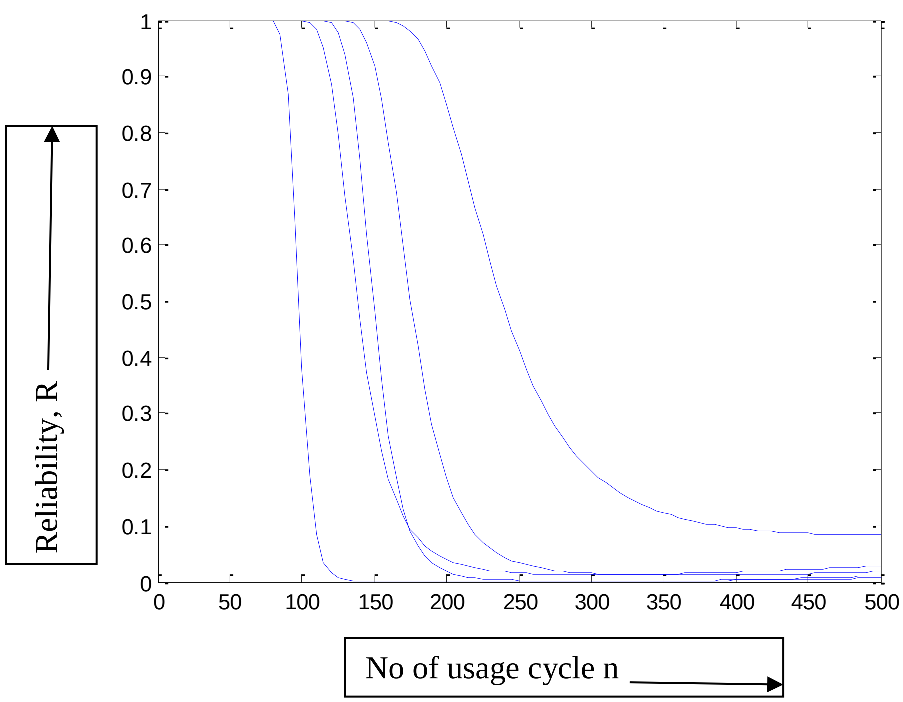

Figure 5.30: No of usage cycle versus reliability plot for single stress level when $\beta = 1$

(b) $\beta = 1.5$

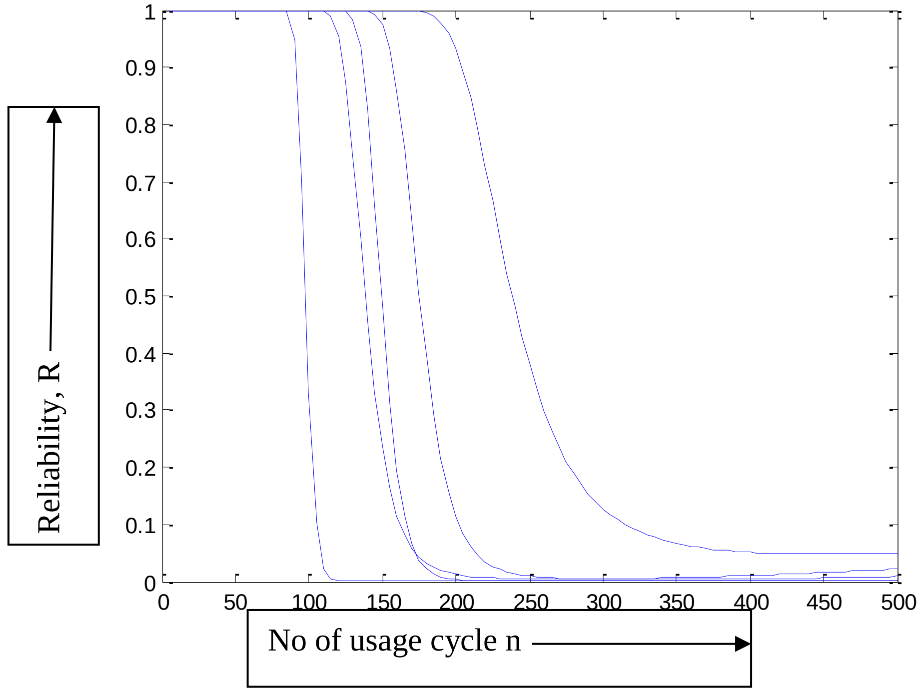

Figure 5.31: No of usage cycle versus reliability plot for single stress level when $\beta = 1.5$

(c) $\boldsymbol{\beta = 2}$

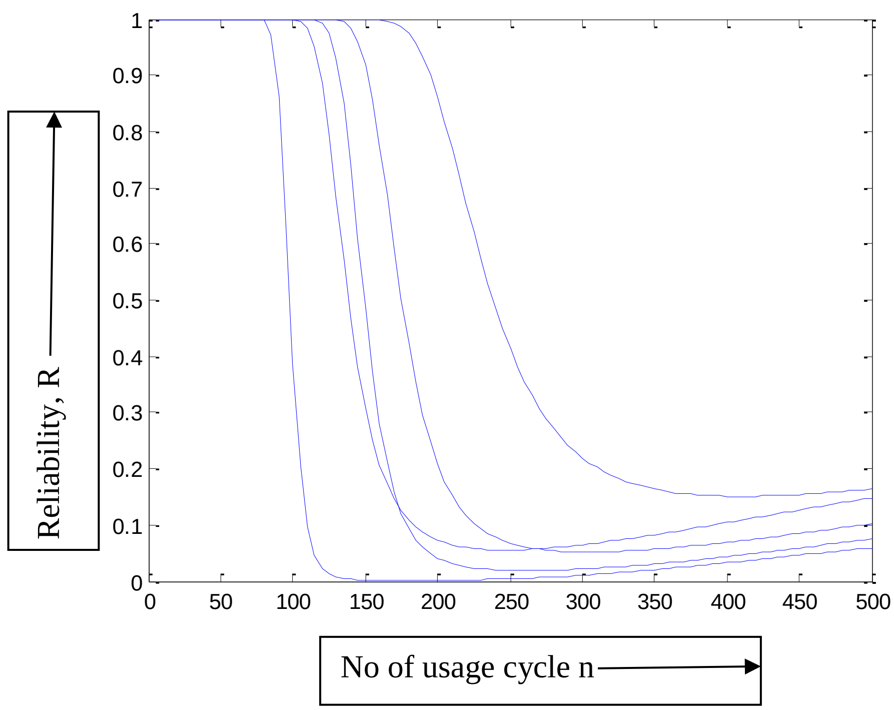

Figure 5.32: No of usage cycle versus reliability plot for single stress level when $\beta = 2$

(d) $\boldsymbol{\beta = 3}$

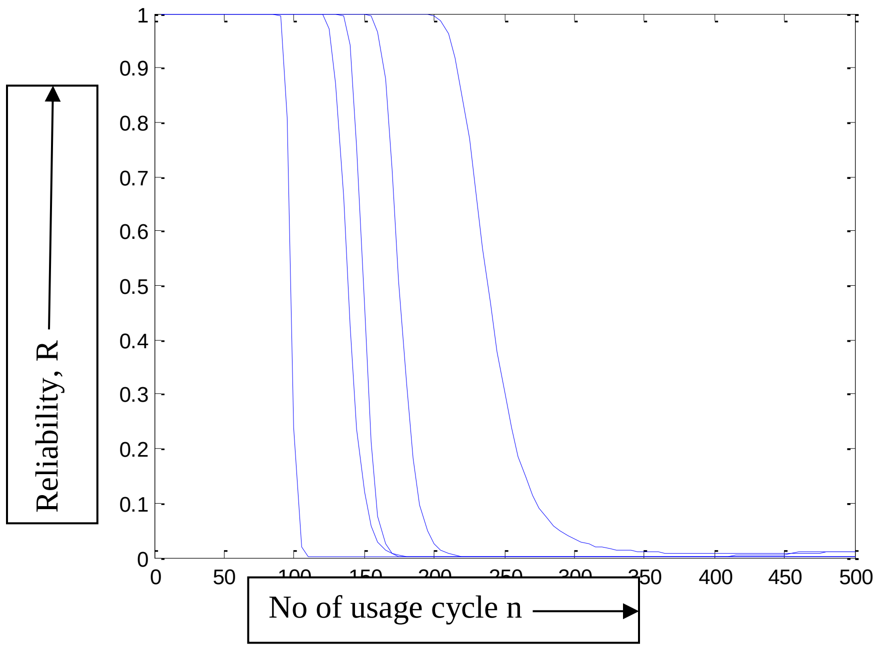

Figure 5.33: No of usage cycle versus reliability plot for single stress level when $\beta = 3$

(e) $\boldsymbol{\beta = 5}$

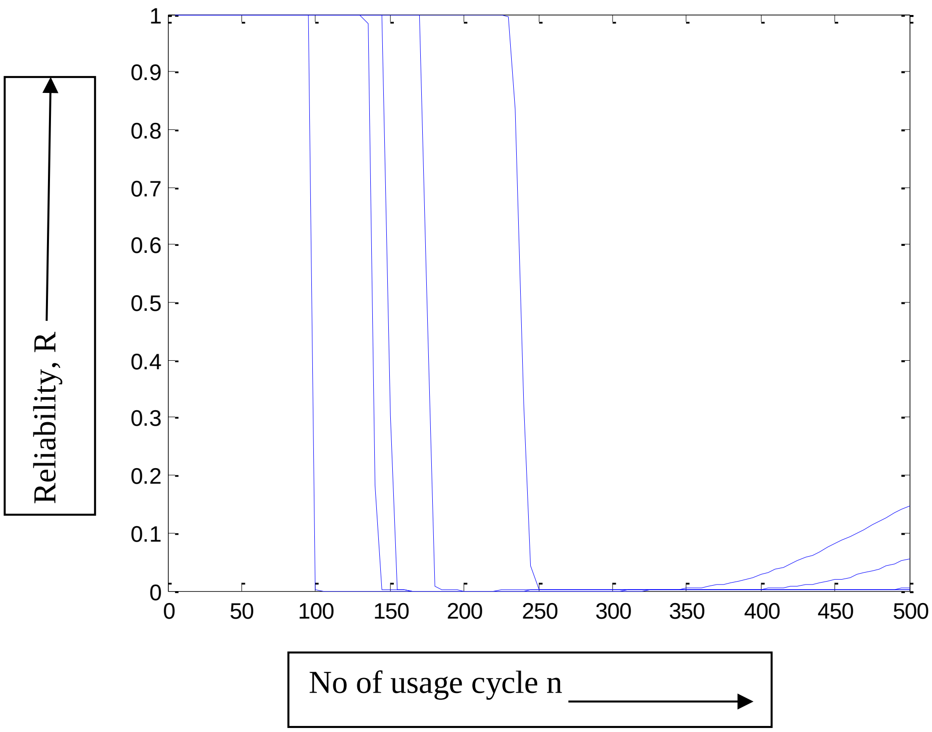

Figure 5.34: No of usage cycle versus reliability plot for single stress level when $\beta = 5$

These plots are very useful for predicting the reliability at any value of usage cycle and at any value of the shape parameter for steel 45-2. The curve will become more flatten as the value of shape parameter increases since the no of cycle is less and more no of fatigue failure cycle is required for smoother curve. Thus these plots are very useful for reliability prediction while considering the load interaction effect.

5.2.2.4 Reliability prediction for multi-stress level

To demonstrate multilevel loading condition, consider the same data as in the above table for steel 45-1 and taking five successive stress levels. To estimate reliability under multi-stress level loading, the fatigue life of steel 45-1 need to be predicted under multi-stress loading condition.

(a) $\boldsymbol{\beta = 2}$

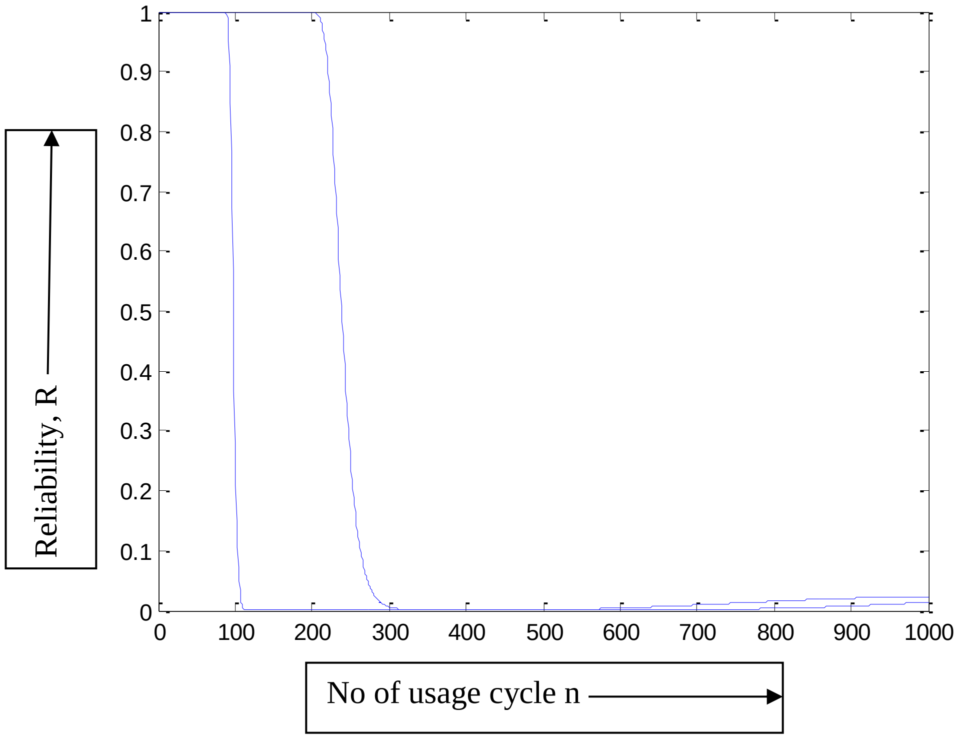

Figure 5.35: No of usage cycle versus reliability plot for multi-stress level when $\beta = 2$

(b) $\beta = 3$

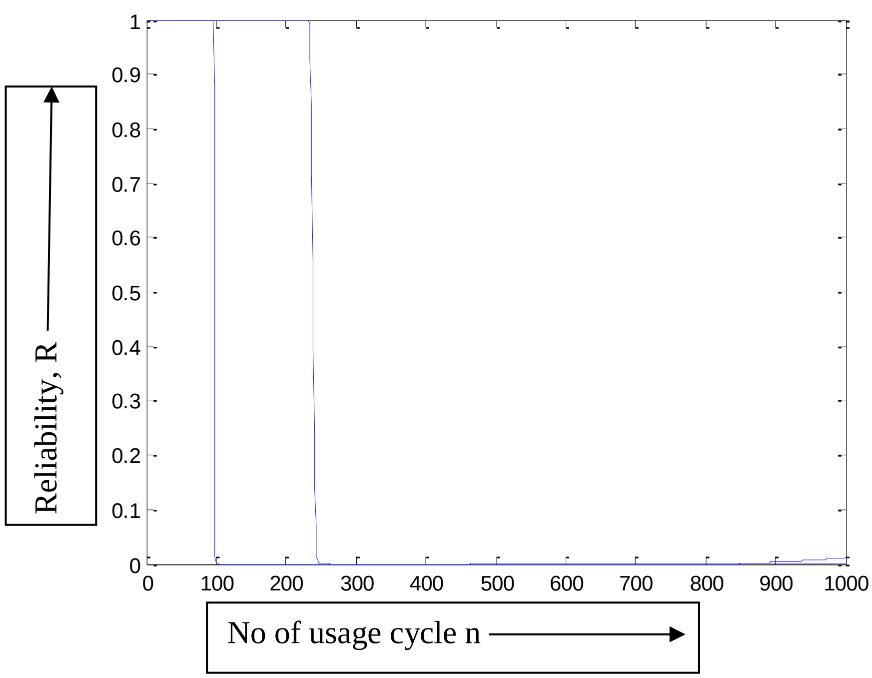

Figure 5.36: No of usage cycle versus reliability plot for multi-stress level when $\beta = 3$

(c) Combine for all beta value

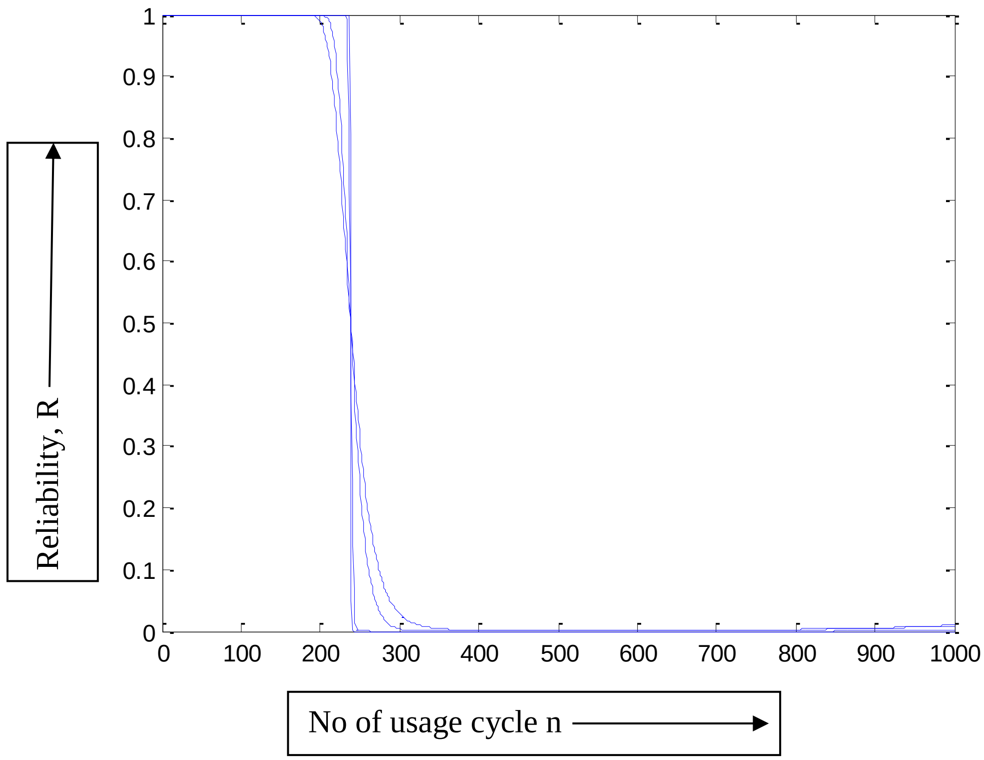

Figure 5.37: No of usage cycle versus reliability plot for single stress level for all β

5.3 Conclusion

From the all above figures it reveals that curve will become more flatten as the value of shape parameter increase. And also the effect of load interaction is taking into accounts. So these plots are more useful to find the reliability at any know of usage cycle with more accuracy.

The proposed approach in this chapter provides an easy methodology for modeling probabilistic of the damage accumulation measure and hence capturing the real life behavior of the product. Since here damage accumulation is considered as a nonlinear phenomenon. It also proposed a simple way to model the damage accumulation process treating it as a non-stationary process to capture the damage accumulation at any given point of the time period. Here damage accumulation considered as a nonlinear phenomenon in a general degradation path, which can be extended to model the probabilistic damage accumulation under other degradation behaviors.

The proposed methodology is very useful tool for predicting the reliability of any component by converting this model into different other type distribution by changing the value of shape parameter. For the high value of stress scale parameter is less while at lower value of stress scale parameter is more. It also reveals that scale parameter depends highly on the shape parameter more the value of shape parameter less the value of scale parameter to capture the real life behavior. It also considered by deeply consideration of all the plots that as the value of shape parameter increases the curve will become more flatten or shifted towards the left side. It means that drop in the reliability of any component is faster. And if the value of shape parameter is less, curve is smooth and the drop in the reliability of product takes place very slowly. Both the load sequencing effect and also load interaction effect is taken into consideration here. Since in the case of multi-level loading condition these factor play an important role for the prediction of reliability. Since there are change take place in the value of fatigue from time to time so it become necessary to consider both the effect.

By doing deeply consideration it indicates that load interaction effect plots are more suitable for the reliability prediction of any component. Thus it has been concluded that proposed model is a common model for all type of distribution for nonlinear damage accumulation and also it considered both type of effect load sequencing as well as load interaction effects.

CHAPTER 6

SUMMARY AND FUTURE WORK

6.1 Summary

In the present scenario of highly competitive design community mostly believes on the proactive quality and the reliability improvement approaches and tools. So, use of these approaches and tools is the only way to survive in the competitive market and satisfy customer demands. Consequently various approaches and tools have been emerged in the past few decades to help the design community. The research work presented in the present thesis has been carefully carried out after conducting the detailed study of existing probabilistic design techniques and the approaches use to tackle prominent design issue at an early design stage. Usually, the degradation model is formulated to predict the reliability and future life of product. Literature also reveals that, very few researchers have tried to use the degradation information generated t an early design stage to ensure the desired quality and reliability for the intended life of the product.

Literature review also reveals that, majority of the mechanical component are subjected to the fatigue loading and consequently fails due to the fatigue. This fatigue failure is the outcome of the irreversible damage accumulation phenomenon. A numbers of researchers work on the deterministic nature of the damage accumulation only few researchers' proposed methodology for the probabilistic nature of damage accumulation. And some effect as load interaction is not considered well on multi-level loading condition. This motivated to propose a simple probabilistic damage accumulation model which is common for all type of distribution for real life behavior and also considered the load interaction effect into consideration. Here we use one to one pdf transformation methodology and also minimize the mathematical calculation. It also proposed a simple and unique way to model damage accumulation process by considering it as a non-stationary process to capture the damage accumulation and also its variability at any given point of the time. The proposed methodology is then effectively used to predict the reliability of steel 45-1 and steel 45-2 for railway vehicle.

The proposed methodology in this thesis is very useful for looking the effect of load sequencing and load interaction in any component. And also it becomes a very useful tool for different type of distribution by changing the value of only shape parameter. The proposed methodology can be easily used in many type of loading condition where fatigue failure occurs. Hence the proposed methodology is very useful for estimating the life of any mechanical component subjected to fatigue while damage accumulation phenomenon is considered to be nonlinear.

6.2 Future work

Since the proposed methodology have certain advantage due to considering load sequencing and load interaction effect into consideration. But it has some limitations which should direct us towards future work.

1. In the proposed methodology both the effect load sequencing and interaction has been taken separately so develop a model which take both the effect together.
2. In the proposed methodology loading is in sequence to validate the model for the random loading condition.
3. In the proposed methodology only certain value of shape parameter is taken into consideration so develop software which take so many value of shape parameter and become an effective tool for reliability prediction.
4. This applicability of the proposed model to the life assessment of components or structural systems also need to further study.

.

REFERENCES

A.Heng, S.Zhang, A.Tan and J.Mathew(2009),“ Rotating machinery prognostics: State of the art, challenges and opportunities,”mechanical system and signal processing,vol.23,pp.724-739

A.Wiesel, Y.Eldar and A.Yeredor(2008),“ Linear Regression With Gaussian Model Uncertainty:Algorithms and Bounds,” IEEE Transactions on signal processing,vol.56,no.6,pp.2194-2205

Ai-ling.S, Wen-hua.Y, Zhou.D and Yu-mei.L, "Assessment of Residue Fatigue Life of Crankshaft Based on Theory of Fatigue Damage," Optoelectronics and Image Processing (ICOIP), 2010 International Conference on , vol.1, no., pp.298-301, 11-12 Nov. 2010

Albrecht, P., (1983), "S-N fatigue reliability analysis of highway bridges, probabilistic fracture mechanics and fatigue methods: Applications for structural design and maintenance", *American Society for Testing and materials* , Vol. 789, pp. 184-204

Augustine, M., (2011), *Investigating system interaction failure behaviour of a complex system at early design stage*, Ph.D dissertation, MNIT, Jaipur, India.

Ben-Amoz, M., (1990), "A cumulative Damage Theory for Fatigue Life Prediction", *Engineering Fracture Mechanics* , Vol. 37, No. 2, pp. 341-347.

Benjamin, J. R., and Cornell, C. A., (1970), *Probability, Statistics, and Decision for Civil Engineers*, McGraw-Hill, Inc., New York.

Bhamare. S.S,Yadav O.P,Rathore .A.(2007)’Evolution of reliability engineering discipline over the last six decades: a comprehensive review’International journal of reliability and safety ,Vol.1,No.4

Boehm, F., and Lewis, E.E., (1992), “A stress–strength interference approach to reliability analysis of ceramics: Part II – delayed fracture,” *Probabilistic Engineering Mechanics*, Vol. 7, pp. 9–14.

C.Peng and S.Tseng(2009),“ Mis-Specification Analysis of Linear Degradation model,” IEEE Transactions on reliability,vol.58,no.3,pp.444-455

Castillo, E., Fernandez-Canteli, A., and Ruiz-Ripoll, M. L., (2008), "A general model for fatigue damage due to any stress history", *International Journal of Fatigue* ,Vol. 30, No. 1, pp. 150-164.

Coit, D. W., Evans, J. I., Vogt, N. T., and Thompson, J. R., (2005), "A method for correlating field life degradation with reliability prediction for electronic modules", *Quality and Reliability Engineering International* , Vol. 21, pp.715-726.

Coit, D. W., Evans, J. I., Vogt, N. T., and Thompson, J. R., (2005), "A method for correlating field life degradation with reliability prediction for electronic modules", *Quality and Reliability Engineering International* , Vol. 21, pp.715-726.

D. Y. Ye and Z. L. Wang, “A new approach to low-cycle fatigue damage based on exhaustion of static toughness and dissipation of cyclic plastic strain energy during fatigue,’ International Journal of fatigue, vol. 23, no. 8, pp. 679-687, 2001.

Dasgupta, A., (1993), “Failure mechanism models for cyclic fatigue,” *IEEE Transactions on Reliability*, Vol. 42, No. 4, pp. 548–555.

Fatemi, A., and Yang, L., (1998), "Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials", *International Journal of Fatigue,* Vol. 20, No. 1, pp. 9-34.

Gao, H., Huang, H. Z., Zhu, S. P., Li, Y. F., & Yuan, R. (2014). A Modified Nonlinear Damage Accumulation Model for Fatigue Life Prediction Considering Load Interaction Effects. *The Scientific World Journal*, *2014*.

Gao, H., Zuo, F., Huang, H. Z., Li, Y. F., & Zhu, S. P. (2013, July). A modified non-linear damage accumulation model considering load interaction effects under two-level loading. In *Quality, Reliability, Risk, Maintenance, and Safety Engineering (QR2MSE), 2013 International Conference on* (pp. 1774-1776). IEEE.

Giancane.S,Nobile.R,Panella.F.W and Dattoma.V(2010),“ Fatigue life prediction of notched components based on a new nonlinear Continuum Damage Mechanics model,”Procedia engineering,vol.2,no.1,pp.1317-1325

Gorjian.N,Ma.L,Mittinty.M,Yarlagadda.P,and Sun.Y(2009)'A review on degradation models in reliability engineering'proceeding of the 4th world congress on engineering asset management,Athens,Greece, 28-30 sept

Gupta.S and Rychlik.I(2007)," Rain-flow fatigue damage due to nonlinear combination of vector Gaussian loads," Probabilistic Engineering Mechanics,vol.22,pp.231-249

H. T. Corten and T. J. Dolan, "Cumulative fatigue damage, In Proceedings of the International Conference on Fatigue of Metals," Institution of Mechanical Engineers, ASME, pp. 235-246, 1956

H.Yip,H.Fan and Y.Chiang(2014)," Predicting the maintenance cost of construction equipment: Comparison between general regression neural network and Box–Jenkins time series models,"automation in construction,vol.38,pp.30-38

Hernandez-Mangas.J.M, Pelaz.L,Marques.L.A and Bailón. L, "Damage buildup model with dose rate and temperature dependence," Electron Devices, 2005 Spanish Conference on , vol., no., pp.435-437, 2-4 Feb. 2005

Hooke, F.R., (1987), "Aircraft structural reliability and risk analysis," in J.W. Provan (Ed). *Probabilistic Fracture Mechanics and Reliability*, Martinus Nijhoff Publishers.

Huang, W., and Askin, R.G., (2004), "A generalised SSI reliability model considering stochastic loading and strength aging degradation," *IEEE Transactions on Reliability*, Vol. 53, pp. 77–82.

Huang,H.Z;Gong,J;Zuo,M.J;Zhu,S.P and Liao.Q(2012)," Fatigue Life Estimation of an Aircaft Engine Under Different Load Spectrums," Int. J. Turbo Jet-Engines,vol.29,pp.259-267

HZ.Huang and Z.W.An(2009)," A Discrete Stress-Strength Interference Model With Stress Dependent Strength," IEEE Transactions on reliability,vol.58,no.1,pp.118-122

J. H. Yan, X. L. Zheng, and K. Zhao, "Prediction of fatigue life and its probability distribution of notched friction welded joints under variable amplitude loading," International Journal of fatigue, vol. 22, no. 6, pp. 481-494, 2000

J.Fan,K.Yung and M.Pecht(2012),“ Lifetime Estimation of High-Power White LED Using Degradation-Data-Driven Method,” IEEE Transactions on device and material reliability,vol.12,no.2,pp.470-477

Kapur, K. C., and Lamberson, L. R., (1977), *Reliability in Engineering Design*, John Wiley & Sons, New York.

L.Jiang,Q.Feng and D.Coit(2012),“ Reliability and Maintenance Modeling for Dependent Competing Failure Processes With Shifting Failure Thresholds,” IEEE Transactions on reliability,vol.61,no.4,pp.932-948

Leve, H. L., (1960), “Cumulative Damage Theories," *Metal Fatigue: Theory and Design*, A. F. Madayag, Ed., pp. 170-203.

Lewis, E.E., and Chen, H.C., (1994), “Load-capacity interference and the bathtub curve,” *IEEE Transactions on Reliability*, Vol. 43, No. 3, pp. 470–475.

Liao, M., Xu, X., Yang, Q., (1995), “Cumulative fatigue damage dynamic interference statistical model,” *International Journal of Fatigue*, Vol. 17, No. 8, pp. 559-566

Liao.Q, Yang.Y, Yang.J, Huang.H.Z and He.L, "A general energy-based model for fatigue life prediction of high-temperature structural materials," Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), 2012 International Conference on , vol., no., pp.236,239, 15-18 June 2012

Liu, N., Gong, Z., Tang, Y., Huang, H. Z., & Zhu, S. (2012, June). A study on fatigue damage accumulation based on continuum damage mechanics. In*Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), 2012 International Conference on* (pp. 867-870). IEEE

Liu, Y. and , Mahadevan, S., (2007), “Stochastic fatigue damage modeling under variable amplitude loading”, *International Journal of Fatigue* ,Vol. 29, pp. 1149-1161.

Lv, Z., Gao, H., Zuo, F., Huang, H. Z., & Zhu, S. P. (2013, July). A nonlinear fatigue damage accumulation model accounting for load interaction effects. In*Quality, Reliability, Risk, Maintenance, and Safety Engineering (QR2MSE), 2013 International Conference on* (pp. 885-887). IEEE

M. Arulampalam,S.Maskell,N.Gordon and T.Clapp(2002),“ A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking,”IEEE Transcations on signal processing,vol.50,no.2,pp.174-188

Marin, J., (1962), *Mechanical Behavior of Engineering Materials*, Prentice-Hall.

Melchers, R. E., (1999), Structural Reliability Analysis and Prediction, 2nd edn. Wiley New York.

Miner, M. A., (1945),“Cumulative damage in fatigue”, Journal of Applied Mechanics, Vol. 12, pp. A159-A164.

Munse, W. H., Wilbur, T. W., Tellalian, M. L., and K, W. (1983), "Fatigue Characterization of fabricated ship details for design", *Ship Structure Committee*, SSC-318

Nagode, M., and Fajdiga, M., (1998), "On a new method for prediction of the scatter of loading spectra", *International Journal of Fatigue* ,Vol. 20, pp. 271-277

P.Xue,XG.Li(2012),“ A Storage Life Modeling Based on Fuzzy Stress-Strength Interface Reliability Model,” IEEE Transactions on reliability,pp.1010-1014

Pascual, F. G., and Meeker, W. Q., (1999), “Estimating Fatigue Curves with the Random Fatigue-Limit Model”, *Technometrics,* Vol. 41, NO. 4, pp. 277-290.

Pecht, M., and Dasgupta, A., (1996), “Physics-of-failure: an approach to reliable product Development,” IRW Final Report, IEEE.

Pelaz. L, Marques.LA, Aboy.M, Lopez.P and Santos.I, "Physics based models for process optimization." In *Electron Devices, 2007 Spanish Conference on*, pp. 5-8. IEEE, 2007

Place, C. S., Strutt, J. E., Allsopp, K., Irving, P. E., and Trille, C. (1999), "Reliability Prediction of Helicopter Transmission System Using Stress-Strength Interference with Underlying Damage Accumulation", *Quality and Reliability Engineering International* , Vol. 15, pp. 69-78.

R. Jiang(2010),“ Optimization of alarm threshold and sequential inspection scheme,” Reliability Engineering and System Safety,vol. 95,pp.208-215

Raghunathan, U.S.; Chakraborty, P.S.; Wier, B.; Cressler, J.D.; Yasuda, H.; Menz, P., "TCAD modeling of accumulated damage during time-dependent mixed-mode stress," *Bipolar/BiCMOS Circuits and Technology Meeting (BCTM), 2013 IEEE* , vol., no., pp.179,182, Sept. 30 2013-Oct. 3 2013

Rao, S. S., (1992), *Reliability Based Design*, McGraw-Hill, Inc., New York.

S. Marco and W. Starkey, "A concept of fatigue damage," Trans. ASME, vol. 76, no. 4, pp. 627-632, 1954.

S.Eryilmaz(2008)," Consecutive k-Out-of-n : G System in Stress-Strength Setup," Communications in Statistics – Simulation and Computation,vol.37,pp.579-589

S.KN and M.Chan(2009)," Modeling Link Lifetime Data with Parametric Regression Models in MANETs,"IEEE Communications,vol.13,no.12,pp.983-985

Sethuraman, J., and Young T. R., (1986), "Cumulative damage threshold crossing models", *Reliability and Quality Control*, Basu AP (ed.), Elsevier: Amsterdam , pp. 309-319.

Sethuraman, J., and Young T. R., (1986), "Cumulative damage threshold crossing models", *Reliability and Quality Control*, Basu AP (ed.), Elsevier: Amsterdam , pp. 309-319.

SH.Z,H.N.Liu,D.W.Zhang and ZH.Wang(2009)," Reliability Calculation Model of Gears Considering Strength Degradation," IEEE Transactions on reliability ,pp.1200-1203

Shao,X; Cao,L; Liu,J; Liu,G; Yu,G and Tian,G, "A new kind of reliability analysis method in coupled case of high-cycle fatigue and low-cycle fatigue," Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), 2012 International Conference on , vol., no., pp.966,970, 15-18 June 2012

Shen, H., Lin, J., and Mu, E., (2000), "Probabilistic model on stochastic fatigue damage", *International Journal of Fatigue* , Vol. 22, No. 7, pp. 569-572.

Shen, H., Lin, J., and Mu, E., (2000), "Probabilistic model on stochastic fatigue damage", *International Journal of Fatigue* , Vol. 22, No. 7, pp. 569-572.

Singh, A., Mourelatos, Z. P., and Li, J., (2010), "Design for lifecycle Costing using Time-Dependent Reliability", *ASME Journal of Mechanical Design,*Vol. 132, No. 9, pp. 091008-1-11.

Snook, I., Marshall, J.M., and Newman, R.M., (2003), "Physics of failure as an integrated part of design for reliability," *Proceedings of Annual Reliability and Maintainability Symposium*, pp. 46–54.

Stoyanov.S ,Bailey.C, Mackay. W,Jibb.D and Gregson.C, "Lifetime assessment of electronic components for high reliability aerospace applications," Electronics Packaging Technology Conference, 2004. EPTC 2004. Proceedings of 6th , vol., no., pp.324,329, 8-10 Dec. 2004

Sun,Y; Hu,L, "Low cycle fatigue life prediction of steam turbine rotor based on a new CDM model," *Control Conference (CCC), 2014 33rd Chinese* , vol., no., pp.6814,6819, 28-30 July 2014

Tryon. R.G,Dey. A,Krishnan.G and Zhao. Y,"Microstructural-based physics of failure models to predict fatigue reliability," Reliability and Maintainability Symposium, 2006. RAMS '06. Annual , vol., no., pp.520-525, 23-26 Jan. 2006

Tucker, J. P., Chan, D. K., Subbarayan, G., & Handwerker, C. A. (2012, May). Maximum entropy fracture model and its use for predicting cyclic hysteresis in solder alloys. In *Thermal and Thermomechanical Phenomena in Electronic Systems (ITherm), 2012 13th IEEE Intersociety Conference on* (pp. 207-215). IEEE..

V. Rathod, O.P. Yadav, A. Rathore, and et al, "Probabilistic modeling of fatigue damage accumulation for reliability prediction," International Journal of Quality, Statistics, and Reliability, doi:10.1155/2011/718901, 2011

Wang, P, and Coit, D., W,(2007), "Reliability and degradation modeling with random or uncertain failure threshold", *Reliability and Maintainability Symposium*, pp. 392-397

Wei, W., Fei, Q., Cha, G., Wenhui, Z., & Guofeng, X. (2012, August). Thermal fatigue life optimization of QFN package based on Taguchi Method. In*Electronic Packaging Technology and High Density Packaging (ICEPT-HDP), 2012 13th International Conference on* (pp. 1128-1132). IEEE

Wen, Y.K., and Chen, H.C., (1989), “Systems reliability under time varying loads: I,” *Journal of Engineering Mechanics*, Vol. 115, pp. 808–823.

White, G. J., and Ayyub, B. M. (1997), "A reliability conditioned approach for the fatigue design of marine structures", *Marine Structural reliability Symposium* , Oct 5-6, pp. 99-113.

Wirsching, P. H., and Chen, Y. N., (1987), “Considerations of probability-based fatigue design for marine structures” *Marine Structural reliability Symposium*, pp. 31-42.

Wu, W. F., and Huang, T. H., (1993), "Prediction of fatigue damage and fatigue life under random loading", *International Journal of Pressure Vessels and Piping* , Vol. 53, No. 2, pp. 273-298.

Wu, W. F., Liou, H.Y., and Tse, H. C., (1997), “Estimation of fatigue damage and fatigue life of components under random loading”, *International Journal of Pressure Vessels and piping,* Vol. 72, No. 3, pp. 243-249.

X. L. Zheng and J.F. Wei, “On the prediction of P-S-N curves of 45 steel notched elements and probability distribution of fatigue life under variable amplitude loading from tensile properties,” International Journal of fatigue, vol. 27, no. 6, pp. 601-609, 2005

X.Si and D.Zhou(2014),“ A Generalized Result for Degradation Model-Based Reliability Estimation,”IEEE Transactions on automation science and engineering,vol.11,no.2,pp.632-637

X.Yuan and M.Pandey(2009),“ A nonlinear mixed-effects model for degradation data obtained from in-service inspections,”reliability engineering and system safety,vol.94,pp.509-519

Xue, J., and Yang, K., (1997), “Upper and lower bounds of stress-strength interference reliability with random strength-degradation,” IEEE Transactions on Reliability, Vol. 46, No. 1, pp. 142–145

Y.Zhang and H.Liao(2014),“ Analysis of Destructive Degradation Tests for a Product With Random Degradation Initiation Time,” IEEE Transactions on reliability,vol.

Yuan.R, Li. H,Huang. H. Z, Liu.Y and Zhu.S (2012, June). A new fatigue damage accumulation model based on nonlinear continuum damage mechanics. In Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), 2012 International Conference on (pp. 875-878). IEEE.

Z.Wang,H.Huang,Y.Li,Y.Pang and N.Xiao(2012),“ An approach to system reliability analysis with fuzzy random variables,”mechanism and machine theory,vol.52,pp-35-46

Zaccone, M. A. (2001), "Failure Analysis of Helical springs under Compressor Start/Stop Conditions", *ASM International* , Vol. 1 No. 3, pp. 51-62.

Zengah,S;Aid,A and Benguediab,M(2013),“Comparative Study of Fatigue Damage Models Using Different Number of Classes Combined with the Rainflow Method,” *Engineering, Technology & Applied Science Research* ,vol.3,no.3,pp.446-451

Zhang,Z; Cui,P; Lv,Z; Liu,N; Zhu,S.N; and Huang,H.Z, "Notice of Retraction Creep life prediction model of aircraft turbine disc alloy based on continuum damage mechanics," *Quality, Reliability, Risk, Maintenance, and Safety Engineering (QR2MSE), 2013 International Conference on* , vol., no., pp.1018,1021, 15-18 July 2013

Zhu, S. P., Huang, H. Z., Liao, Q., Yuan, R., & He, L. P. (2012, June). Probabilistic modeling of damage accumulation for fatigue reliability analysis. In*Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), 2012 International Conference on* (pp. 918-923). IEEE

Zhu, S. P., Huang, H. Z., Liao, Q., Yuan, R., & He, L. P. (2012, June). Probabilistic modeling of damage accumulation for fatigue reliability analysis. In*Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), 2012 International Conference on* (pp. 918-923). IEEE

Zhu, S. P., Huang, H. Z., Liu, Y., Yuan, R., & He, L. (2012). An efficient life prediction methodology for low cycle fatigue–creep based on ductility exhaustion theory. *International Journal of Damage Mechanics*, 1056789512456030

Zhu,S.P;Huang,H.Z;He,L.P;Liu,Y and Wang,Z(2012),“A generalized energy based fatigue -creep damage parameter for life prediction of turbine disk blade,”engineering fracture mechanics,vol.90,pp.89-100

Zhu,S.P;Huang,H.Z;Liu,Y;He,L.P and Liao,Q(2012),“ A Practical Method for Determining the Corten–Dolan Exponent and Its Application to Fatigue Life Prediction,” *Int. J. Turbo Jet-Engines*,vol.29,pp.79-87

Zhu,S.P;Huang,H.Z;Ontiveros,V;He,L.P and Modarres,M(2012),“ Probabilistic Low Cycle Fatigue Life Prediction Using an Energy-Based Damage Parameter and Accounting for Model Uncertainty,”International journal of damage mechanics,vol.21,no.8,pp.1128-1152

Zhu.S.P and Huang.H.Z(2009),“ A generalized frequency separation–strain energy damage function model for low cycle fatigue–creep life prediction,” Fatigue & Fracture of Engineering Materials & Structures,vol.33,no.4,pp.227-237

Zhu.S.P,Huang.H.Z,Li.H,Sun.R and Zuo.M.J(2011),“ A New Ductility Exhaustion Model for High Temperature Low Cycle Fatigue Life Prediction of Turbine Disk Alloys,”International journal Turbo jet engine,vol.28,pp.119-131

Zhu.S.P,Huang.H.Z,Wang.Z.L(2011),“ Fatigue Life Estimation Considering Damaging and Strengthening of Low amplitude Loads under Different Load Sequences Using Fuzzy Sets Approach,” International Journal of damage mechanics,vol.20,pp.876-899

Zibdeh, H.S., and Heller, R.A., (1989), “Rocket motor service life calculations based on first passage methods,” *Journal of Spacecraft and Rockets*, Vol. 26, pp. 279–284.