`

INVENTORY OPTIMIZATION IN DISTRIBUTION NETWORK UNDER UNCERTAIN ENVIRONMENT

Ph.D. Thesis

PEEYUSH VATS

I.D: 2013RME9005

DEPARTMENT OF MECHANICAL ENGINEERING

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

JANUARY 2019

INVENTORY OPTIMIZATION IN DISTRIBUTION NETWORK UNDER UNCERTAIN ENVIRONMENT

Submitted by

Peeyush Vats

(2013RME9005)

(Department of Mechanical Engineering)

Under the supervision of

Dr. GUNJAN SONI

Assistant Professor

Department of Mechanical Engineering

M.N.I.T. Jaipur

Dr. AJAY PAL SINGH RATHORE

Professor

Department of Mechanical Engineering

M.N.I.T. Jaipur

Submitted in the fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

to the

DEPARTMENT OF MECHANICAL ENGINEERING
MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

2019

`

`

Respectfully dedicated to my parents, my wife, my teachers

&

Both of my sons

`

DECLARATION

I, **Peeyush Vats**, declare that this thesis titled, **“Inventory Optimization in Distribution Network under Uncertain Environment”** and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this university.

- Where any part of this thesis has previously been submitted for a degree or any other qualification at this university or any other institution, this has been clearly stated.

- Where I have consulted the published work of others, this is always clearly attributed.

- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.

- I have acknowledged all main sources of help.

- Where the thesis is based on work done by myself, jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Date: 15/01/2019 | Peeyush Vats

Place: Jaipur | (2013RME9005)

`

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY
JAIPUR-302017 (RAJASTHAN), INDIA

CERTIFICATE

This is to certify that the thesis entitled **"Inventory Optimization in Distribution Network under Uncertain Environment"** being submitted by **Mr. Peeyush Vats (ID No. 2013RME9005)** to the **Malaviya National Institute of Technology Jaipur** for the award of Doctor of philosophy in **Mechanical Engineering** is a bonafide record of original research work carried out by him. He has worked under our guidance and supervision and has the fulfilled the requirement for the submission of this thesis, which has reached the requisite standard.

The result contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.

Date:

Dr. GUNJAN SONI

Assistant Professor

Department of Mechanical Engineering

M.N.I.T. Jaipur, INDIA

Dr. AJAY PAL SINGH RATHORE

Professor

Department of Mechanical Engineering

M.N.I.T. Jaipur, INDIA

ACKNOWLEDGEMENTS

It is very difficult for me to know where to begin because there are so many people that I want to thank. First of all I would like to thank the Almighty God for all blessings in my life and for giving me the opportunity to follow my dreams.

Next, my warm regards, gratitude and thanks go to both of my supervisors, **Dr. Gunjan Soni** and **Prof. A. P. S. Rathore** for providing me this excellent opportunity to undertake this Ph.D. and for introspective guidance with constructive suggestions, deliberative discussions, and continuous encouragement to the last moment and all the last minute editing.

I want to show my sincere gratitude to **Prof. G. S. Dangayach** (DPGC, Coordinator, and Prof. in Mechanical Engineering Department), **Prof**. **Dillip Sharma** (Head, Mechanical Engineering Department), and **Late Prof. Awadesh Bhardwaj** (Mechanical Engineering Department).

I feel indebted to my DREC Members **Prof. Rakesh Jain** (Mechanical Engineering Department) and **Prof. M.L. Mittal** (Mechanical Engineering Department) for their continuous help, constructive inputs, and research insights given during Ph.D. progress report presentations.

I want to express my sincere thanks to **Dr. B. C. Sharma** (Director, IPEC Ghaziabad) and **Dr. Satyendra Singh** (BTKIT, Dwarahat) for their continuous motivation to me.

I wish to express my overwhelming thanks to my friends, **Dr. Surya Prakash Yadav**, **Dr. Jayant Kishor Purohit, Dr. Om ji Shukla, Dr. Man Mohan Siddh, Dr. Vinod Yadav, Dr. Avanish Chauhan, Mr. Gaurav Badhotiya, Mr. Manish Kumar, Mr. Manoj Kumar Pal, Mr. Anniruddh Wargie etc.,** and for their constant moral support and generous help.

I also extend my sincere thanks to other fellow Ph.D. scholars of MNIT whose names I could not mention above.

`

My parents deserve special mention for their inseparable support and prayers. I want to express my heart felt reverence to my **parents**, wife **Rukmani**, sons **Anshmaan** and **Shashank** for their constant support. Their presence is the most precious blessing in my life and it is not possible for me to express my love and gratitude in few words.

I feel privileged to take the opportunity to express my sincere gratitude to AIMMS team for giving me full academic version of their software. I am also grateful to all those people, whom I have not mentioned, but the helped me directly or indirectly.

Date: (PEEYUSH VATS)

`

ABSTRACT

Management of inventories and related issues in supply chain distribution network is an integral part of supply chain management. In the current market situations which are highly dynamic and uncertain, maintaining the proper inventory level by reducing operational cost and to achieve best level of responsiveness is the top priority of any manufacturing firm. The higher inventory level increase the responsiveness of supply chain but decrease its operational efficiency because of holding the inventory. In this situation, it becomes essential to manage proper inventory levels and responsiveness in supply chain. A small change in demand and supply of any product may lead to price fluctuations at various stages of a supply chain which can adversely affect the operational efficiency of the network. Most of the manufacturing companies along with their distribution networks face dynamic challenges of uncertain demand and supply that require not only good inventory planning, but also robust supply chain distribution networks with tight coordination mechanisms.

Risk pooling is an effective approach for maintaining proper inventory level under uncertain demand and it suggests that demand variability can be reduced if a supply chain manager aggregates the demand of different facilities. This approach is also very useful for reducing the safety stock and the average inventory level which reduces the overall cost of system. In this approach, demands of various facilities (retailers) are fulfilled by a single distributor (warehouse) which is called centralization. Due to this centralization, there is an unwanted increment in the lead time and it produces adverse effects on the supply chain responsiveness. Therefore there is a natural tradeoff between cost and responsiveness. The basic aim of a supply chain manger is to maintain proper balance between cost and responsiveness. To explore the concept of risk pooling approach, a supply chain distribution network is considered among different distributors and retailers. Due to some geographical and business constraints, a retailer can fulfill its demand by a single distributor, which means that the complete shipment will be received by a retailer from a fixed distributor. By applying this condition different combinations of networks among various distributors and retailers are generated hence providing the best level of aggregation between various distributors and retailers.

`

For obtaining optimal inventory level, a mathematical model is developed by using risk pooling approach incorporating different combinations of networks among various distributors and retailers. Initially this model is limited for single product and single period scenario. Later on this model is extended for multi-product and multi-periods conditions. A numerical example is illustrated for solving the model. The computational results show that the proposed model is capable to handle the uncertain demand effectively in order to provide reduced operational cost and appropriate level of responsiveness. Some other performance parameters are also evaluated with the help of these computational results. The proper weights are provided to these performance parameters with the help of some decision makers from industry and academia. Finally the best alternative network is selected from the entire alternative networks along with best inventory policy. This alternative network provides reduced operational cost and appropriate level of responsiveness.

There are four highlights of this research. First one is to obtain proper inventory level in terms of reorder point and ordering quantity. Second is to maintain proper balance between cost and responsiveness by providing different weights to various performance parameters. The third key component of this research is to obtain best level of aggregation among various facilities. Finally, two different case studies for two different types of products are provided to demonstrate practical applicability of the proposed models. First case study is taken from a sugar mill (for general type of product), second case study taken from milk plant (for perishable product). The purpose of two different case studies from different business firm are to represent the proposed models in a most generalized way and to prove that the proposed model is valid for different types of products under uncertain conditions of demand and supply.

`

CONTENTS

		Page No.
Declaration		i
Certificate		ii
Acknowledgement		iii-iv
Abstract		v-vi
Contents		vii-x
List of Figures		xi-xii
List of Tables		xiii-xvii
Abbreviations		xviii
CHAPTER 1: INTRODUCTION		1-8
1.1	Introduction	1
1.2	Overview of research problem	2
1.3	Research scope, Motivation and Objectives	4
1.4	Research Methodology	6
	1.4.1 Literature Review	6
	1.4.2 Finding Research Gaps	6
	1.4.3 Design and Development of Modeling	6
	1.4.4 Extension of Modeling	6
	1.4.5 Numerical Test	6
	1.4.6 Case Study	6
1.5	Overview of Thesis	7
CHAPTER 2: LITERATURE REVIEW		9-51
2.1	Introduction	9
2.2	Definition of Inventory and Inventory Management	9

`

2.3	Inventory Control Problems under Demand and Supply Uncertainty	11
	2.3.1 Inventory Management in Supply Chains	13
	2.3.2 Literature Review: From 1996 to 2018	15
	2.3.2.1 Mathematical Modeling Technique	15
	2.3.2.2 Mathematical Modeling and Other Techniques	17
	2.3.2.3 METRIC Modeling Techniques	20
	2.3.2.4 Markov Decision Process Technique	20
	2.3.2.5 Simulation	21
	2.3.2.6 Stackelberg Game	22
	2.3.2.7 Literature Review Technique	22
	2.3.2.8 Other Techniques	22
2.4	Inventory Control Problems handling by Risk Pooling Approach	44
	2.4.1 Characteristics of risk pooling	45
	2.4.1.1 Increasing returns	45
	2.4.1.2 Diminishing marginal returns with increasing application as well as increasing benefits	46
	2.4.1.3 Increasing Demand Variability	46
	2.4.1.4 Decreasing Demand Correlations	47
	2.4.1.5 Decreasing Concentration of Uncertainty	47
	2.4.2 Inventory Pooling	47
2.5	Findings of Literature Review	51
CHAPTER 3: SINGLE PRODUCT SINGLE PERIOD RISK POOLING INVENTORY CONTROL MODEL IN DISTRIBUTION NETWORK		52-95
3.1	Introduction	52
3.2	Single product Single period Risk Pooling Inventory Control Model in Distribution Network	54
3.3	Numerical Illustration	60
3.4	Solution Procedure	63
3.5	Optimization Results	66
3.6	TOPSIS Approach for Selecting Best Inventory Policy	78
3.7	Discussion	83

`

	3.7.1 Effect of Demand Uncertainty on Performance Parameters	84
	3.7.2 Effect of Supply Uncertainty on Performance Parameters	87
	3.7.3 Combined Effect of Demand and Supply Uncertainty on Performance Parameters	90
3.8	Summary of the Chapter	95
CHAPTER 4: MULTI-PRODUCT MULTI-PERIOD RISK POOLING INVENTORY CONTROL MODEL IN DISTRIBUTION NETWORK		96-149
4.1	Introduction	96
4.2	Multi-product Multi-period Risk Pooling Inventory Control Model in Distribution Network	96
4.3	Numerical Illustration and Solution Procedure	103
4.4	Optimization Results	105
4.5	Selection of preeminent inventory policy through Fuzzy-TOPSIS approach	109
4.6	Fuzzy Set Theory	111
	4.6.1 Fundamental Definitions:	112
	4.6.1.1 Definitions 1: Fuzzy Set	112
	4.6.1.2 Definitions 2: Fuzzy Numbers	112
	4.6.1.3 Algebraic Operations with Fuzzy Numbers	113
	4.6.1.4 Fuzzy TOPSIS	114
4.7	Application of Fuzzy TOPSIS for finding best option of distribution network for the best inventory policy	115
4.8	Discussion	133
	4.8.1 Effect of Demand Uncertainty on Performance parameters	146
	4.8.2 Effect of Supply Uncertainty on Performance parameters	147
	4.8.3 Combined Effect of Demand and Supply Uncertainty on Performance parameters	147
4.9	Summary of the Chapter	149
CHAPTER 5: CASE STUDIES		150-183
5.1	Introduction	150
5.2	Case Study-1	151-165

`

5.3	Case Study-2	165-182
5.4	Summary of chapter	182
CHAPTER 6: CONCLUSION AND FUTURE RESEARCH DIRECTIONS		184-188
6.1	Introduction	184
	6.1.1 Managerial Implications	184
6.2	Concluding remarks and discussion	185
6.3	Limitations of Present Study	187
6.4	Future Research Directions	187
6.5	Summary of chapter	188
REFERENCES		189-217
Appendix-I:	AIMMS Codes and screen shots for single product, single period risk pooling inventory control model	218-224
Appendix-II:	AIMMS Codes and screen shots for multi- product, multi- period risk pooling inventory control model.	225-233
Appendix-III:	List of publications	234-235
Appendix-IV:	Biographical profile of researcher	236

`

LIST OF FIGURES

Figure No.	Title	Page No.
1.1	Risk pooling in Supply Chain	3
1.2	Research Methodology	7
2.1	A typical use of stock	10
2.2	Classification of articles research technique wise	25
2.3	Research articles published year wise	42
2.4	Research articles published journal wise	43
3.1	Distribution network	55
3.2	Distribution network understudy	61
3.3	Alternative-12	83
3.4	Effect of demand uncertainty on ROP	84
3.5	Effect of demand uncertainty on SS	84
3.6	Effect of demand uncertainty on ESC	85
3.7	Effect of demand uncertainty on FR	85
3.8	Effect of demand uncertainty on AI	86
3.9	Effect of demand uncertainty on TC	86
3.10	Effect of supply uncertainty on ROP	87
3.11	Effect of supply uncertainty on SS	87
3.12	Effect of supply uncertainty on ESC	88
3.13	Effect of supply uncertainty on FR	88
3.14	Effect of supply uncertainty on AI	89
3.15	Effect of supply uncertainty on TC	89
3.16	Combined effect of both type of uncertainty on ROP	90
3.17	Combined effect of both type of uncertainty on SS	90
3.18	Combined effect of both type of uncertainty on ESC	91

`

Figure No.	Title	Page No.
3.19	Combined effect of both type of uncertainty on FR	91
3.20	Combined effect of both type of uncertainty on AI	92
3.21	Combined effect of both type of uncertainty on TC	92
4.1	Distribution network	97
4.2	Performance characteristics	111
4.3	Linguistic scale of weight of criteria	116
4.4	Linguistic scale of ratings of alternatives	116
4.5	Alternative-5	134
5.1	Distribution network of a sugar manufacturing unit	151
5.2	Alternative-5	159
5.3	Effect of demand uncertainty on ROP	160
5.4	Effect of demand uncertainty on SS	161
5.5	Effect of demand uncertainty on TC	161
5.6	Effect of supply uncertainty on ROP	162
5.7	Effect of supply uncertainty on SS	162
5.8	Effect of supply uncertainty on TC	163
5.9	Combined effect of both type of uncertainty on ROP	163
5.10	Combined effect of both type of uncertainty on SS	164
5.11	Combined effect of both type of uncertainty on TC	164
5.12	Milk distribution network for Jaipur Dairy	166
5.13	Final network for Jaipur Dairy	182

`

LIST OF TABLES

Table No.	Title	Page No.
2.1	Literature review of inventory control problems in supply chain	26-41
3.1	Some necessary parameters	61
3.2	Facility cost and operating cost	62
3.3	Distance of distributor from a retailer	62
3.4	Average demand of retailers and their standard deviations	63
3.5	Coefficient of correlation among the demand of retailers	63
3.6	Possible alternatives of distribution network	65
3.7	Solver result	66
3.8	Aggregated demand	67
3.9	Ordering quantity	68
3.10	Re-order point	69
3.11	Mean demand during lead time	70
3.12	Standard deviations for aggregated demand	71
3.13	Standard deviations for aggregated demand during lead time	72
3.14	Total Cost (TC) and product-miles for all alternatives	73
3.15	Safety stocks (SS)	74
3.16	Average Inventory (AI)	75
3.17	Inventory positions (IP)	76
3.18	Expected shortages per cycle (ESC)	77
3.19	Fill rates (FR)	78
3.20	Average criterion weight	81
3.21	Average ratings of alternatives	81
3.22	Closeness coefficient	82
4.1	Facility cost, operating cost and fixed ordering cost for all types	103

`

Table No.	Title	Page No.
	of products	
4.2	Inventory holding cost for all types of products and periods	104
4.3	Demand of all retailers for all types of products and periods	104
4.4	Standard deviations of demand of all retailers for all types of products and periods	104
4.5	Variables obtained for Alternative-1, Planning horizon-1, Product-1	105
4.6	Variables obtained for Alternative-1, Planning horizon-1, Product-2	105
4.7	Variables obtained for Alternative-1, Planning horizon-1, Product-3	105
4.8	Variables obtained for Alternative-1, Planning horizon-2, Product-1	106
4.9	Variables obtained for Alternative-1, Planning horizon-2, Product-2	106
4.10	Variables obtained for Alternative-1, Planning horizon-2, Product-3	106
4.11	Variables obtained for Alternative-1, Planning horizon-3, Product-1	106
4.12	Variables obtained for Alternative-1, Planning horizon-3, Product-2	107
4.13	Variables obtained for Alternative-1, Planning horizon-3, Product-3	107
4.14	Variables obtained for Alternative-1, Planning horizon-4, Product-1	107
4.15	Variables obtained for Alternative-1, Planning horizon-4, Product-2	107
4.16	Variables obtained for Alternative-1, Planning horizon-4, Product-3	108
4.17	Variables obtained for Alternative-1, Planning horizon-5, Product-1	108
4.18	Variables obtained for Alternative-1, Planning horizon-5, Product-2	108
4.19	Variables obtained for Alternative-1, Planning horizon-5, Product-3	108
4.20	Linguistic scale to evaluate the weight of criteria	117
4.21	Linguistic scale to evaluate the ratings of alternatives	117
4.22	Linguistic weight of criteria evaluated by DM	118
4.23	FTN of criteria evaluated by DM	118

`

Table No.	Title	Page No.
4.24	Aggregate FTN of weight of criteria	118
4.25	Linguistic ratings of alternatives evaluated by first DM	118
4.26	Linguistic ratings of alternatives evaluated by second DM	119
4.27	Linguistic ratings of alternatives evaluated by third DM	120
4.28	Linguistic ratings of alternatives evaluated by forth DM	121
4.29	Linguistic ratings of alternatives evaluated by fifth DM	121
4.30	FTN of ratings of alternative evaluated by first DM	122
4.31	FTN of ratings of alternative evaluated by second DM	123
4.32	FTN of ratings of alternative evaluated by third DM	124
4.33	FTN of ratings of alternative evaluated by forth DM	125
4.34	FTN of ratings of alternative evaluated by fifth DM	125
4.35	Aggregate FTN of ratings of alternatives	126
4.36	Normalized fuzzy decision matrix	127
4.37	Weighted normalized fuzzy decision matrix	128
4.38	Distances of ratings of each alternative from A^+	129
4.39	Distances of ratings of each alternative from A^-	130
4.40	Closeness coefficient (CC_i) for each alternative and each criteria	131
4.41	Out-ranking of alternatives	132
4.42	Alternative-5, Planning period-1, Product-1	134
4.43	Alternative-5, Planning period-1, Product-2	134
4.44	Alternative-5, Planning period-1, Product-3	134
4.45	Alternative-5, Planning period-2, Product-1	135
4.46	Alternative-5, Planning period-2, Product-2	135
4.47	Alternative-5, Planning period-2, Product-3	135
4.48	Alternative-5, Planning period-3, Product-1	135
4.49	Alternative-5, Planning period-3, Product-2	135
4.50	Alternative-5, Planning period-3, Product-3	136
4.51	Alternative-5, Planning period-4, Product-1	136
4.52	Alternative-5, Planning period-4, Product-2	136
4.53	Alternative-5, Planning period-4, Product-3	136

`

Table No.	Title	Page No.
4.54	Alternative-5, Planning period-5, Product-1	136
4.55	Alternative-5, Planning period-5, Product-2	137
4.56	Alternative-5, Planning period-5, Product-3	137
4.57	ROP vs. demand uncertainty levels in planning period-1	137
4.58	ESC vs. demand uncertainty levels in planning period-1	138
4.59	FR vs. demand uncertainty levels in planning period-1	138
4.60	SS vs. demand uncertainty levels in planning period-1	139
4.61	AI vs. demand uncertainty levels in planning period-1	139
4.62	TC vs. demand uncertainty levels in planning period-1	140
4.63	ROP vs. supply uncertainty levels in planning period-1	140
4.64	ESC vs. supply uncertainty levels in planning period-1	141
4.65	FR vs. supply uncertainty levels in planning period-1	141
4.66	SS vs. supply uncertainty levels in planning period-1	142
4.67	AI vs. supply uncertainty levels in planning period-1	142
4.68	TC vs. supply uncertainty levels in planning period-1	143
4.69	ROP vs. combined uncertainty levels in planning period-1	143
4.70	ESC vs. combined uncertainty levels in planning period-1	144
4.71	FR vs. combined uncertainty levels in planning period-1	144
4.72	SS vs. combined uncertainty levels in planning period-1	145
4.73	AI vs. combined uncertainty levels in planning period-1	145
4.74	TC vs. combined uncertainty levels in planning period-1	146
5.1	Possible alternatives of distribution network	152
5.2	Demand and standard deviations of demand	153
5.3	Distance of Rohana Kalan from Jaipur	153
5.4	Some other informations provided by company	153
5.5	Ordering quantity	154
5.6	Reorder point	154
5.7	Mean demand during lead time	155
5.8	Safety stocks	155
5.9	ESC	156

`

Table No.	Title	Page No.
5.10	Fill rates	156
5.11	Total cost vs. product-miles	157
5.12	Average criterion weight	158
5.13	Average ratings of alternatives	158
5.14	Closeness coefficients	159
5.15	Effect of demand uncertainty on ROP	160
5.16	Effect of demand uncertainty on SS	160
5.17	Effect of demand uncertainty on TC	161
5.18	Effect of supply uncertainty on ROP	161
5.19	Effect of supply uncertainty on SS	162
5.20	Effect of supply uncertainty on TC	162
5.21	Combined effect of both type of uncertainty on ROP	163
5.22	Combined effect of both type of uncertainty on SS	164
5.23	Combined effect of both type of uncertainty on TC	164
5.24	Possible distribution network	167
5.25	Morning Demand	168
5.26	Evening Demand	168
5.27	Standard deviations in morning demand	168
5.28	Standard deviations in evening demand	168
5.29	Facility cost for every type of product	169
5.30	Distance Matrix	169
5.31	Other information provided by industry experts	169
5.32	Ordering quantity for all alternatives	170
5.33	Reorder point for all alternatives	174
5.34	Product-miles for all alternatives	177
5.35	Total cost for all alternatives	181

`

ABBREVIATIONS

OQ	Ordering quantity
ROP	Re-order point
SS	Safety stocks
ESC	Expected shortages per cycle
FR	Fill Rates
AI	Average Inventory
IP	Inventory Positions
TC	Total Cost
PM	Product-miles
DM	Decision makers
FTN	Fuzzy Triangular Number
NDM	Normalized Decision Matrix

`

CHAPTER 1

INTRODUCTION

1.1 Introduction

Inventory comprises of twenty to sixty percent are the total assets of the most manufacturing firms. Inventory management policies prove to be critical in drawing the profits of such firms (Arnold, 1998) and their relevance clearly increases when supply chains (SCs) are considered. Inventory management is in fact a major issue in Supply Chain Management (SCM) (Christopher, 1992; Lee, Billington, 1992, Routroy and Kodali, 2005). In the current scenario, the trends of all type of businesses are highly dynamic and uncertain and in this uncertain environment the business firm requires more flexible and responsive structure (Agrawal, 2014). Under uncertain environment, the companies are bound to manage their competitive advantage by optimizing their business processes (Govindan et al., 2017). The business firm give the top priorities to the customer service level in terms of responsiveness (shorter lead time) and reduced cost under uncertain environment (Gaur and Ravindran, 2006). Most of the manufacturing units along with their distribution networks face dynamic challenges of uncertain demand and supply that require not only well planned inventory in the network, but also robust supply chain networks with tight coordination mechanisms (Lee and Billington., 1992). A small change in demand and supply of any product may lead to price fluctuations in various stages of a supply chain as a result overall cost of system may get affected (Angkiriwang et al., 2014). As it is clear that the inventory is involved at various stages of supply chain (Ganeshan, 1999) and higher inventory levels increase the responsiveness of supply chain but decreases its operational cost because of holding inventory at various stages (Etienne, 2005). In this situation, the business firm has to manage proper inventory levels by reduced cost and improved customer responsiveness in supply chain (Constantin, 2016). The effective supply chain inventory planning keeps not only the wheels of business moving but also increases the operational efficiency and provides competitive advantages (Jones and Riley, 1985).

It has been discussed above that inventory is involved in various stages of supply chain (Ganeshan, 1999), it is more appropriate to understand about the basic definition of supply chain.

`

A supply chain management is defined as a chain which links the entire element from the production process to the supply process and from raw materials to the end customer, encompassing several organizational boundaries (Scott and Westbrook, 1991 & New and Payne, 1995). Supply chain management focuses on how firms utilize their suppliers' processes, technology, and capability to enhance competitive advantage (Farley, 1997), and the coordination mechanism of the production, transport, and inventory management functions within an organization and in supply chain distribution (Lee and Billington, 1992). A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials in to intermediate products and the distribution of finished products to the customers. A Supply chain includes both the service and manufacturing organizations, although the complexity of chain may vary from industry to industry and firm to firm (Ganeshan and Harrison, 2002). The distribution refers to the steps taken to move and store a product from supplier stage to customer stage in supply chain. Distribution is a key driver of the overall profitability of the firm because it directly impacts both the supply chain costs and the customer experience. Good distribution can be used to achieve a variety of supply chain objectives ranging from low cost to high responsiveness. As a result companies in the same industry often select very different distribution network (Chopra, 2003) and the sustainability can be achieved through the redesigning of supply chain network (Ravert, 2013). Keskinocak and Kayur (2001) established that the primary goal of supply chain management is to deliver the required product at the correct place on the correct time while maintaining cost efficiencies.

1.2 Overview of research problem: Now - a -days, supply chains are highly sensitive to the uncertainty because of their increased globalization competition (Dilts, 2005). Disturbances like shipment delay or production delay affects the profitability of the firm (Hoffmann, 2005). In such situations the “Risk Pooling” is one of the most powerful tools to handle demand and lead time uncertainty (Simchi-Levi et al., 2008) and the risk pooling is an effective strategy for reducing safety stock under uncertain demand. Tagaras (1989) proved that pooling improves the customer service level and the total expected cost is lowered. Tagaras (1989) also proved that risk pooling is an effective means of improving customer service and reducing total cost of the system through lateral transshipment (Tagaras, 1999). Weng (1999) investigated that this approach is very effective under stochastic demand and he studied the effects of risk pooling

`

over demand uncertainty of multiple products sharing product modularity in a two echelon distribution system. Xu and Evers (2003) examined that sometimes partial risk pooling is favored over complete risk pooling. Eppen, (1979) investigated that the expected holding cost and penalty cost in a decentralized system exceeds those in centralized system and Caplin (1985) developed a general theory of aggregate implication of inventory policies. Chang and Lin (1991) also validated the statement of Eppen (1979) and also proved that a centralized system is more effective than the decentralized system. Risk pooling in supply chain can reduce the safety stock. The amount of safety stock depends on the level of pooling between the distributors and retailers.

From Figure 1.1, it is clear that for effective and efficient flow of products and information is disturbed by demand and lead time uncertainty. Risk pooling can mitigate these uncertainties. It can be implemented in between various stages of supply chains (Supplier, Manufacturer, and Wholesaler, distributors, central warehouse, retailers and customers)

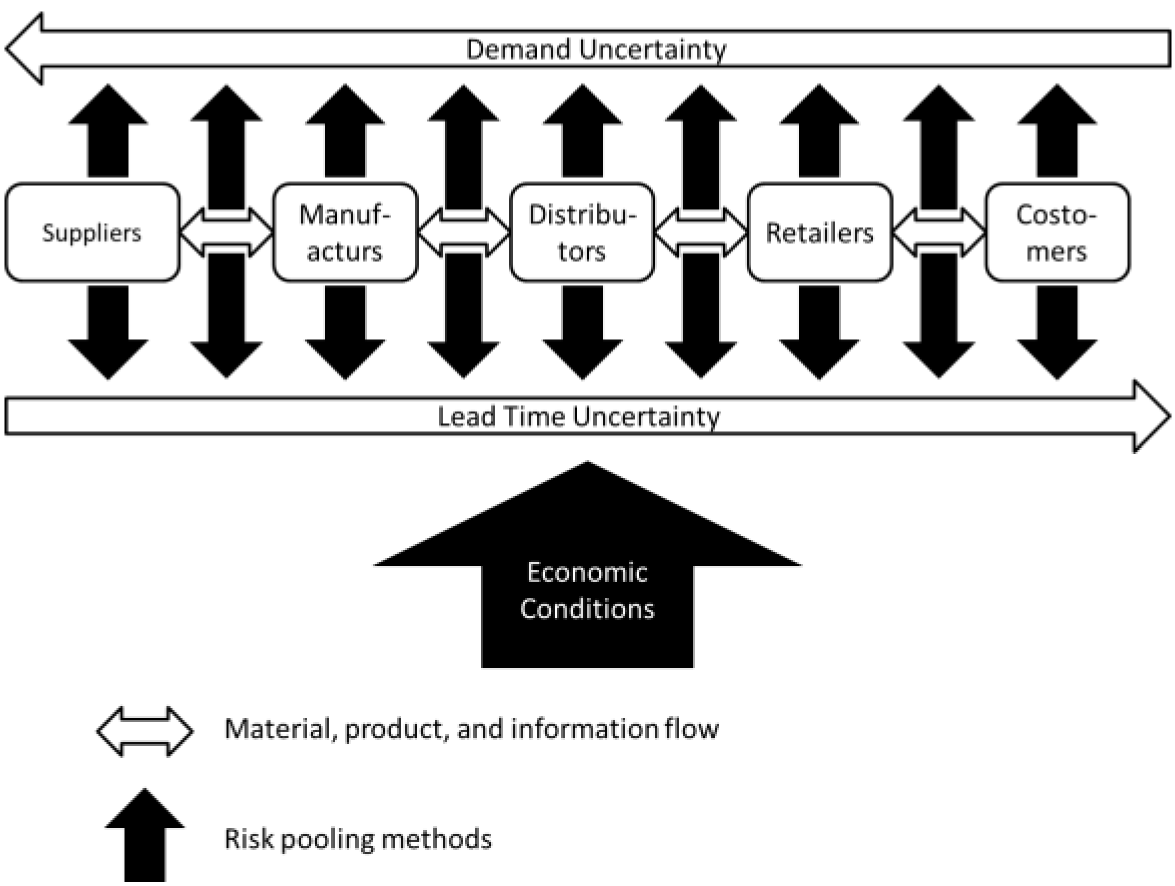


Figure 1.1: Risk pooling in supply chain (Adopted from Oesser, 2010)

`

Risk pooling can be implemented anywhere along the supply chain. In general, risk pooling may aggregate the inventories of different locations at various stages of the supply chain. As it is clear that risk pooling is an effective strategy to reduce the safety stocks and expected total cost under demand and lead time uncertainty. In this study the risk pooling has been implemented in between the two stages of supply chain as represented by distributors and retailers. The different level of risk pooling has also been generated for finding the best inventory policy.

1.3 Research Scope, Motivation and Objectives:

Supply, demand, information delays associated with manufacturing and distribution processes as well as inventory and backorder costs are usually uncertain (Verwijmeren et al.,1996; Evans, 1986; Park, 1987). Supply chain uncertainty governs the operational decisions in a supply chain. Uncertainties in supply, process and demand are recognized to have major impact on a supply chain. Uncertainty propagates throughout the network and leads to inefficient processing and non-value adding activities. Sales would deviate from the forecasted demand and in transit component would also be damaged if the plan mismatches to the fabrication which is regarded as the most common event being the direct result of uncertainty (Geary et al. 2002). It is very difficult to maintain proper inventory levels in supply chain under uncertain environment. In a supply chain, inventory decisions play very important roles (Christopher, 1992). The inventory decisions may be categorized into maintaining proper inventory level and customer responsiveness.

Risk pooling helps a company to cope up with demand and lead time uncertainty and thus to carry out these activities at a lower cost for a given service level, a higher service level for a given cost, or combination of both (Chopra and Meindl, 2007). Thus, it may increase expected profit (Porters, 1985) by reducing expected costs and or increasing expected revenues. There should be an optimum investment on the inventory. Excessive investment on inventory reduces the economic proficiency of the firm. On the other hand, inadequate investment on inventory may create the problem of stock out. This may lead to the interruption in production and sales damaging the reputation of the firm and shifting of customer to its competitor. While discussing the inventory decision problems, it is compulsory to maintain economic efficiency of the firm with the proper customer responsiveness. For maintaining economic efficiency a larger compromise cannot be done with the customer responsiveness. There should be an optimum

`

balance between the cost and responsiveness. These issues become more complex under uncertainty. From the previous discussion, it is clear that risk pooling can be implemented anywhere in supply chain to reduce the safety stock and total expected cost. A two stage supply chain network is considered for implementing risk pooling approach. This network is considered in between distributors and retailers. To minimize the expected total cost and to reduce safety stock, risk pooling is implemented at various levels among the retailers. This study addresses two of the above problems; one is that of total cost minimization and the other is to lower safety stock under fixed service level which provides best level of the risk pooling among the retailers.

For addressing the above problems, initially an inventory control model is developed using risk pooling approach under the uncertainty of lead time and demand in a supply chain distribution network with different combinations of risk pooling. Later, this model is extended to the multi-product and multi-period conditions. At the end, different case studies are performed for investigating the practical applicability and managerial implications of the above models. The specific objectives of this study are given as follows:

1. Develop an inventory control model for a distribution network under uncertain conditions of demand and lead time with risk pooling
2. Develop a model for multi-product and multi-period conditions for same set of distribution networks.
3. Selection of the best level of risk pooling to obtain an inventory policy with optimized cost and service level.
4. To investigate the practical applicability of the above models with the help of various real world case studies.

The major contribution of this research is an in-depth investigation of inventory control problems under uncertainty and risk pooling approach. The findings of this current research will be helpful for researcher and practitioners involved in strategic and tactical decision making in a supply chain distribution network. This study highlights the ways which can be used to embark the best level of pooling in distribution network. The models formed in this study are widely tested with numerical illustration and different case study from various firms. This study successfully demonstrates the selection of best alternative of distribution network among all the possible

`

alternatives. The eventual goal of this research is to create the opportunities for industrial managers and researchers to improve inventory decisions in supply chain distribution network.

1.4 Research Methodology

It describes the research plan, mathematical modeling and programming, solution approaches, numerical illustration and other procedures that are appropriate for achieving research objectives mentioned in the previous section. The overall research plan is illustrated in the Figure 1.2. The research methodology has been divided in to six subsections.

1.4.1 Literature Review: The first step of this study is to perform literature review. A systematic literature review has been presented here. It describes the importance of inventory control problems in supply chain distribution network under uncertain environment. The methodologies and other solution approaches to tackle the uncertainty for inventory control problems have also been discussed in this section. The key trend regarding inventory control problems under uncertainty, an optimization method are identified and it has provided the critical issues for inventory control in supply chain distribution network.

1.4.2 Finding Research Gaps: After systematic literature review some key issues and research gaps are identified.

1.4.3 Design and development of model: Designing of a research problem takes place and a mathematical model is developed for addressing the above research gaps. Numerous numerical test and case studies are carried out for solving this research problem.

1.4.4 Extension of Model: Initially, the research problem for inventory control is designed for single product and single periods and some tests are carried out to know the outcome of this study. In this section, the research problem is designed and extended for multi-product and multi-period conditions in order to test the applicability of this research in real world.

1.4.5 Numerical Test: After developing the model, several numerical tests are carried out to know the outcome of this research. The input data, result analysis are discussed here.

1.4.6 Case study: To check the real world applicability of this study, some case studies have been performed. The results of the case study and its suggestions are discussed in the case study section.

`

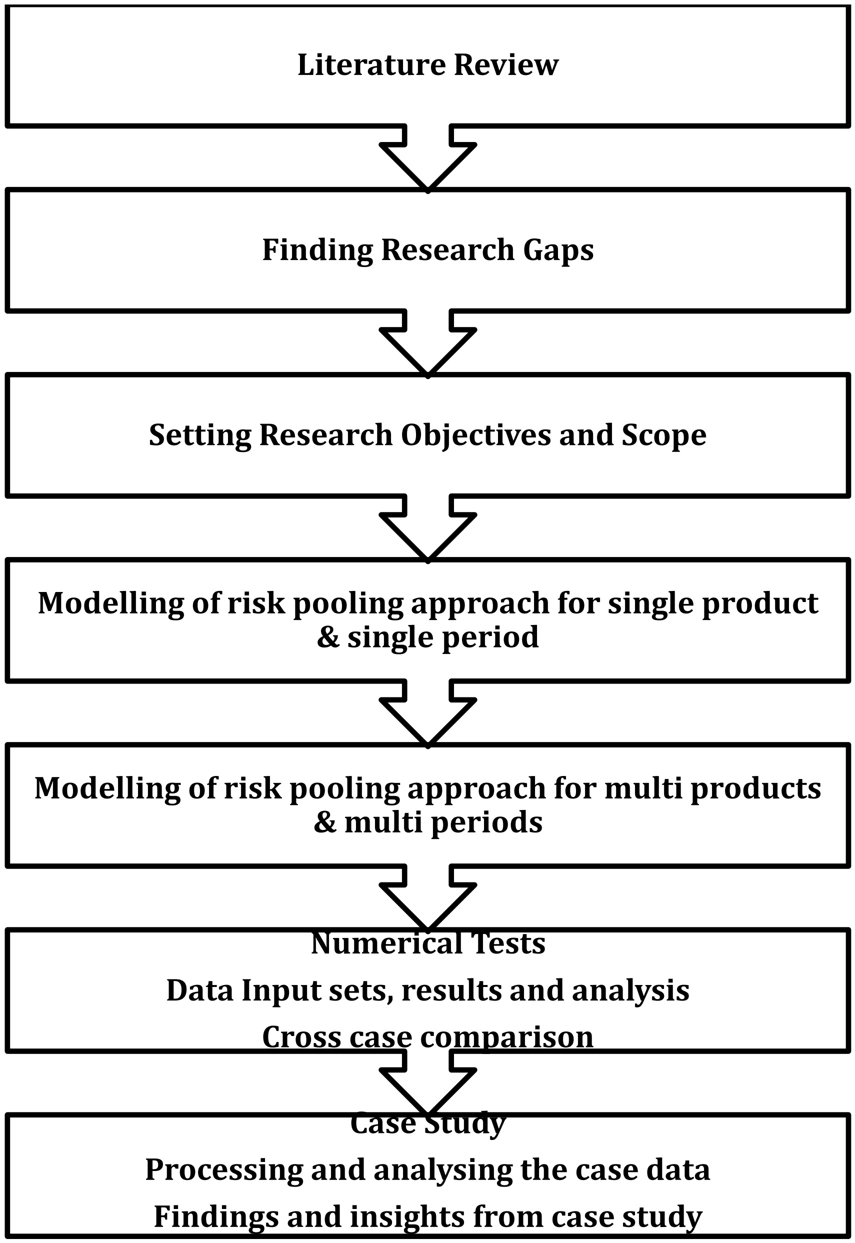


Figure 1.2: Research Methodology

1.5 Overview of Thesis:

This thesis consists of six chapters. The first chapter deals with the introduction. It includes background, overview, research scope, research objective and thesis outlines. Chapter 1 shows the importance of the problem taken in this study with the help of previous literature. A detailed literature review has been discussed in chapter 2. This literature review includes the inventory control problems using risk pooling approach, working under different types of uncertainty. A

`

summary of research gap is prepared based on the literature review. In chapter 3, a mathematical approach is developed using risk pooling approach. In this chapter, different scenarios of risk pooling are generated and the best level of risk pooling is also selected by using some multi-attribute decision making approach. The solution approach, numerical test and the result analysis are also presented in Chapter 3. In Chapter 4, the mathematical approach, which has been developed in Chapter 3, is extended for multi-product and multi-period scenarios. Two case studies are performed one is from sugar mill distribution and second is from milk distribution network in Chapter 5 for checking the practical applicability of proposed mathematical approach. Sugar and milk both are daily used products, which are highly demanded and fast moving items. A small change in demand can affect the total cost and responsiveness of the system. The summary, conclusion and further research directions of this study, have been discussed in the Chapter 6.

`

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

A literature review provides a detailed survey of books, scholarly articles, research papers and some other relevant resources related to a particular issue, field of research, or theory, and by doing so, it provides a description, summary, and critical evaluation of relevant field in order to investigate a new research problem. In this connection, a critical review of inventory control problems under uncertainty has been discussed. This chapter provides a state of art literature review of inventory control problems under demand and supply uncertainty and the inventory control problems handling through risk pooling approach. The basic aim of this chapter is to provide (i) a critical analysis of inventory control problems under uncertainty (ii) to discuss the solution approaches of inventory control problems under uncertainty (iii) to discuss the inventory control problem handling through risk pooling approach and finally, (iv) to identify the findings of literature review and to identify some research gaps. For this purpose, a detailed literature review has been presented in the preceding sections.

2.2 Definition of Inventory and inventory management:

A stock is defined as the storage of different items which are kept for the use in future and the list of items held in stock is termed as inventory (Walters, 2003). Employing the generic definition of inventory, a large spectrum of situations can be structured as inventory management problems. These includes (i) Raw material inventory as input to the manufacturing system, (ii) Brought out parts (BOP) inventory which directly go to assembly of product as it is, (iii) Work in progress (WIP) inventory or pipe line inventory, (iv) Finished goods inventory for supporting the distribution to the customers and (v) Maintenance, repair, and operating (MRO) supplies. These include spare parts, indirect materials, and all other sundry items required for production/service systems (Vrat, 2014).

As discussed in the previous chapter that inventory is also an important issue in supply chain (Christopher, 1992) and it is involved in each stage of supply chain (Ganeshan, 1999) and

`

inventory represents from twenty to sixty percent of most manufacturing firms' total assets (Arnold, 1998). Inventory act as bridge between demand and supply (Chopra and Meindl, 2001) and both demand and supply are uncertain in nature (He and Zhao, 2016) and in this situation inventory management is an extremely important function to any business, since inadequacies in control of inventory may cause some serious problems in the system. If inventories are managed in an inefficient manner, it is likely that delays in production, dissatisfied customers, or curtailment of working capital will result (Lancioni and Howard, 1974).

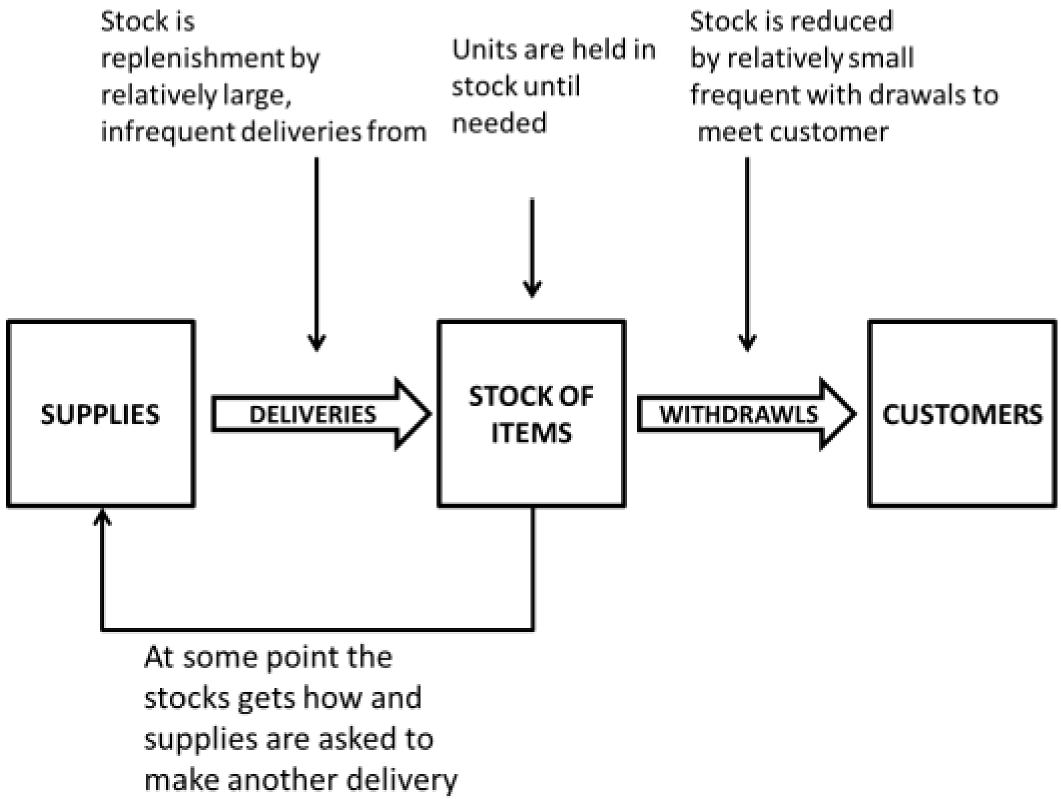


Figure 2.1: A typical use of stock (Walter, 2003)

Inventory management is more crucial under the uncertain environment of demand and supply and the risk pooling is an effective strategy to handle inventory control problems under uncertain environment of demand and supply (Amit and Foque, 2005). Therefore, there are the two most important selection criteria for the selection of research articles are; first one is the inventory control problem under the uncertain conditions of demand and supply and the second one is the inventory control problems handled by risk pooling approach. For designing a new suitable

`

research problem a detailed literature review has been performed by collecting a number of research articles from various databases. These research articles mainly consist of two types of research articles. First types of the research articles are the inventory control problem addressing demand and supply uncertainty in supply chain distribution network. The second type of research articles are related to the inventory control problems handled by risk pooling. It is to be noted here that, only those research articles are to be selected for the study in which inventory control problems are addressing the demand and supply uncertainty. The major resources for the collection of the research papers are, Elsevier, Emerald insight, Taylor and Francis, Springer, Inderscience and some other relevant resources from 1996 to 2018. Two main criteria of the selection of papers are

(i) Inventory control problems under demand and supply uncertainty in supply chain distribution

(ii) Handling of inventory control problems by risk pooling approach

At first, the focus of review is the Inventory control problems under demand and supply uncertainty in supply chain which has been described in the following section.

2.3 Inventory control problems under demand and supply uncertainty in supply chain distribution

Supply Chain Management (SCM) is an integrated approach to the production, planning, control of materials, the flow of information and materials from suppliers to consumers and vice versa, within the company (Minner, 2003) under the different functions. Recently, this area has attracted a great deal of attention of the various researchers and observed that it is as a tool to provide great competitiveness to the researchers (Routroy and Kodali, 2005). Supply chain management is a set of different functions and methods for effectively integrating the suppliers, manufacturers, warehouses, and stores so that goods are produced and distributed in the right amount, at the right place and at the right time, minimizing the system wise cost of meeting service level requirements (Routroy and Kodali, 2005 & Levi et al., 2000). Therefore, the supply chain consists of different members of stages. Supply chain is a system which may involve numerous of participants that’s why supply chain is dynamic, random and complex system (Routroy and Kodali, 2005). Most of the manufacturing firms may involve 20% to 60% of the

`

total assets on the inventory. Therefore, inventory management is the most important function of a supply chain system. Thus, inventory management policies are critical to determining the profitability of these companies (Arnold, 1998). Inventory management is more relevant, when the entire supply chain network is considered, for example, the activities of purchasing, transformation of raw material in to work in process (WIP) or finished products, and delivering firms which are considered in the network of supply chain. Inventory management is really one of the major issues in supply chain management, being an important part of the integrated approach in supply chain management (Routroy and Kodali, 2005 & Giannoccaro et al., 2003). Due to numbers of reasons, inventory exists in the entire supply in different forms (Ganeshan, 1999). The bullwhip effect, which is defined as “the amplification in the demand variability moving towards the upstream stages of supply chain”, is produced due to the lack of a coordinated inventory management. The bullwhip effect leads to the larger investment in the inventory, losses in the revenues, unreasonable capacity plans, inadequate transportation, mislaid production schedules and poor customer service (Giannoccaro et al., 2003). Many research scholars have investigated and studied these problems, as well as emphasized the necessity of integration among with the supply chain stages, so that make the supply chain may be effective and efficient to satisfy the customer demands (Towill, 1996). Beyond to the integration issues, uncertainties must be addressed in supply chain to determine effective supply chain inventory policies. In addition to supply uncertainties (e.g., delivery lead time), demand, information delays associated with manufacturing and distribution processes determine the characteristic of the supply chain (Giannoccaro et al., 2003).

Inventory management in Supply Chains is an important issue, because there are many elements that have to coordinate with each other. They must also arrange their inventories to coordinate. There are many factors that complicate successful inventory management, e.g. uncertain demands, lead times, production times, product prices, costs, etc., especially the uncertainty in demand and lead times wherein the inventory cannot be managed between echelons optimally.

In the current research, a detailed literature review is presented, addressing the inventory management in supply chain from 1996 to 2018. The review of the papers is against the uncertainty of demand and lead time. Initially, echelon concept and multi-echelon inventory management in supply chains are defined. Then, the literature review conducted from an

`

operational research point of view between 1996 and 2018, is presented. Finally, directions for future research are suggested.

2.3.1. Inventory management in supply chains

Most manufacturing enterprises are organized into networks of manufacturing and distribution sites that procure raw material, process them into finished goods, and distribute the finish goods to customers. The terms 'multi-echelon' or 'multilevel' production/ distribution networks are also similar to such networks (or supply chain), when an item moves through more than one step before reaching the final customer. Inventories exist throughout the supply chain in various forms for multiple reasons. At any manufacturing point, they may be as raw materials, work in progress, or finished goods. They exist at the distribution warehouses, and in-transit, or 'in the pipeline', on each path linking these facilities (Ganeshan, 1999). Manufacturers procure raw material from suppliers and process them into finished goods, sell the finished goods to distributors, and then to retailer and/ or customers. When an item moves through more than one stage before reaching the final customer, it forms a 'multi-echelon' inventory system (Rau et al. 2003). The echelon stock of a stock point equals all stock at this stock point, plus in-transit to or on-hand at any of its downstream stock points, minus the backorders at its downstream stock points (Diks and de Kok, 1998).

The analysis of multi-echelon inventory systems that pervades the business world has a long history (Chiang and Monahan, 2005). Multi-echelon inventory systems are widely employed to distribute products to customers over extensive geographical areas. Given the importance of these systems, many researchers have studied their operating characteristics under a variety of conditions and assumptions (Moinzadeh and Agrawal, 1997). Since the development of the economic order quantity (EOQ) formula by Harris (1913), researchers and practitioners have been actively concerned with the analysis and modeling of inventory systems under different operating parameters and modeling assumptions (Routroy and Kodali, 2005). Research on multi-echelon inventory models has gained importance over the last decade mainly because integrated control of supply chain consisting of several processing and distribution stages has become feasible through modern information technology (Rau et al. 2003, Diks and de Kok, 1998 and Kalchschmidt et al. 2003). Clark and Scarf (1960) were the first to study the two- echelon inventory model. They proved the optimality of a base-stock policy for the pure-serial inventory

`

system and developed an efficient decomposing method to compute the optimal base-stock ordering policy. Bessler and Veinott (1965) extended the Clark and Scarf (1960) model to include general arbore scent structures. The depot warehouse problem described above was addressed by Eppen and Schrage (1981) who analyses a model with a stockless central depot (Heijden, 1999).They derived a closed-form expression for the order-up-to-level under the equal fractile allocation assumption. Owing to the complexity and intractability of the multi-echelon problem, Hadley and Whitin (1963) recommend the adoption of single-location, single-echelon models for the inventory systems (Chiang and Monahan, 2005)

Sherbrooke (1968) considered an ordering policy of a two-echelon model for warehouse and retailer. It is assumed that stock outs at the retailers are completely backlogged (Rau et al. 2003). Also, Sherbrooke (1968) constructed the METRIC (multi-echelon technique for recoverable item control) model, which identifies the stock levels that minimize the expected number of backorders at the lower-echelon subject to a budget constraint. This model is the first multi-echelon inventory model for managing the inventory of service parts (Ganeshan, 1999, Chiang and Monahan, 2005, Sherbrooke, 1992). Thereafter, a large set of models, which generally seek to identify optimal lot sizes and safety stocks in a multi-echelon framework, were produced by many researchers (Axsater, 1990, Moinzadeh & Lee, 1986, Svoronos & Zipkin 1988, Nahmias & Smith 1994, Aggarwal & Moinzadeh 1994). In addition to analytical models, simulation models have also been developed to capture the complex interactions of the multi-echelon inventory problems (Chiang & Manmohan 2005, Pyke 1990, Dada 1992, Alfredsson & Verrijidt 1999). As far as, literature has devoted major attention to the forecasting of lumpy demand, and to the development of stock policies for multi-echelon supply chains (Clark and Scarf, 1960). Inventory control policy for multi-echelon systems with stochastic demand has been a widely researched. More papers have recently been covered by Silver and Pyke (1968). The advantage of centralized planning, available in periodic review policies, can be obtained in continuous review policies, by defining the reorder levels of different stages, in terms of echelon stock rather than installation stock (Mitra and Chatterjee, 2004 a)

2.3.2 Literature review: from 1996 to 2018

A detailed literature review, conducted from an operational research point of view, has been presented in this section. It addresses multi-echelon inventory management in supply chain from

`

1996 to 2018. The selection criteria of the papers that are reviewed are: using operational research techniques to overcome multi- echelon inventory management problems, and being demand and lead time sensitive (there are uncertain demand and lead times). Here, the behavior of the papers against demand and lead time uncertainty is emphasized. The papers reviewed here are categorized into groups on the basis of the research techniques in which they are used. These techniques can be grouped as:

(a) Mathematical modeling;

(b) Mathematical modeling and other techniques;

(c) METRIC modeling;

(d) Markov decision process;

(e) Simulation;

(f) Stackelberg game;

(g) Literature review;

(h) Other techniques (vari-METRIC method, heuristics, scenario analysis, fuzzy logic, etc.).

While the research techniques are common for papers that are grouped according to their research techniques, the number of echelons they consider, inventory/system policies, demand and lead time assumptions, the objectives, and the solutions' exactness may be different. Therefore, these factors are also analyzed.

2.3.2.1 Mathematical modeling technique

Rau et al. (2003), Diks and De Kok (1998), Dong and Lee (2003), Mitra and Chatterjee (2004 a), Hariga (1998), Chen (1999), Axsater and Zhang (1999), Nozick and Turnquist (2001), and So and Zheng (2003) use a mathematical modeling technique in their studies to manage multi-echelon inventory in supply chain. Diks and de Kok's study (1998) considers a divergent multi-echelon inventory system, such as a distribution system or a production system, and assumes that the order arrives after a fixed lead time. Hariga (1998) presents a stochastic model for a single-period production system composed of several assemblies/processing and storage facilities in

`

series. Chen (1999), Axsater and Zhang (1999), and Nozick and Turnquist (2001) consider a two-stage inventory system in their papers. Axsater and Zhang (1999) and Nozick and Turnquist (2001) assume that the retailers face stationary and independent Poisson demand. Mitra and Chatterjee (2004 a) examine De Bodt and Graves' model (1985), which they developed in their paper 'Continuous-review policies for a multi-echelon inventory problem with stochastic demand', for fast moving items from the implementation point of view. The proposed modification of the model can be extended to multi-stage serial and two-echelon assembly systems. In Rau et al.'s (2003) model, shortage is not allowed, lead time is assumed to be negligible, and demand rate and production rate is deterministic and constant. So and Zheng (2003) used an analytical model to analyze two important factors that can contribute to the high degree of order quantity variability experienced by semiconductor manufacturers: supplier's lead time and forecast demand updating. They assumed that the external demands faced by the retailer are correlated between two successive time periods and that the retailer uses the latest demand information to update its future demand forecasts. Furthermore, they considered that the supplier's delivery lead times are variable and are affected by the retailer's order quantities. Dong and Lee's (2003) revisits the serial multi-echelon inventory system of Clark and Scarf (1960) and develops three key results. First, they provided a simple lower-bound approximation to the optimal echelon inventory levels and an upper bound to the total system cost for the basic model of Clark and Scarf (1960). Second, they showed that the structure of the optimal stocking policy of Clark and Scarf (1960) holds under time-correlated demand processing using a Martingale model of forecast evolution. Third, they extended the approximation to the time-correlated demand process and study, in particular for an autoregressive demand model, the impact of lead times, and autocorrelation on the performance of the serial inventory system. After reviewing the literature about multi-echelon inventory management in supply chains using mathematical modeling technique, it can be said that, in summary, these papers consider two, three, or N-echelon systems with stochastic or deterministic demand. They assumed the lead times to be fixed, zero, constant, deterministic, or negligible. They gained exact or approximate solutions.

2.3.2.2 Mathematical modeling and other techniques Routroy and Kodali (2005), Ganeshan (1999), Bollapragada et al. (1998), van der Heijden (1999), Verrijdt and de Kok (1996), Parker and Kapuscinski (2004), Seferlis and Giannelos (2004), Axsater (2003), Forsberg (1996), Graves

`

(1996), Mohebbi and Posner (1998), Dekker et al. (1998), Korugan and Gupta (1998), van der Heijden et al. (1999), Andersson and Marklund (2004), Cachon and Fisher (2000), Axsater (2000), Axsater (2001 b), Tsiakis et al. (2001), Moinzadeh (2002), Tang and Grubbström (2003), Chiu and Huang (2003), Mitra and Chatterjee (2004 b), Chen and Lee (2004 b), Jalbar et al. (2005 a), Seifbarghy and Jokar (2005), and Han and Damrongwongsiri (2005) used mathematical modeling and other research techniques in their papers. Forsberg (1996), Graves (1996), Verrijdt and de Kok (1996), Bollapragada et al. (1998), Dekker et al. (1998), Korugan and Gupta (1998), van der Heijden et al. (1999), van der Heijden (1999), Andersson and Marklund (2000), Axsater (2000), Axsater (2001 b), Mitra and Chatterjee (2004 b), Seferlis and Giannelos (2004), Seifbarghy and Jokar (2005), Moinzadeh (2002), and Axsater (2003) considered a two-stage inventory system with stochastic demand in their papers, while Mohebbi and Posner (1998) considered only a single-echelon system with stochastic demand. Tang and Grubbström (2003) assumed the demand to be constant and deterministic. In all these papers, mathematical modeling and simulation techniques are used together in the same paper. Ganeshan (1999) considered a three-echelon inventory system with stochastic demand and lead times. Forsberg (1996) and Verrijdt and de Kok (1996) assumed that lead times were constant, and Mohebbi and Posner (1998) assumed stochastic lead times, while Graves (1996) assumed deterministic ones, and Bollapragada et al. (1998) assumed fixed lead times. Verrijdt and de Kok (1996) presented two adjustment methods that improved the service performance considerably in certain cases. The work by Bollapragada et al. (1998) was a generalization of earlier work by Eppen and Schrage (1981), to allow for non-identical warehouses. Dekker et al. (1998) analyzed the effect of the break quantity rule on the inventory costs. The break- quantity rule is to deliver large orders from the warehouse, and small orders from the nearest retailer, where a so-called break quantity determines whether an order is small or large. In most l-warehouse– N-retailers distribution systems, it is assumed that all customer demand takes place at the retailers (Eppen & Scharge, 1981, Federgruen & Zipkin, 1984, Jackson, 1988 and Axaster et al. 1994). However, it was shown by Dekker et al. (1995) that delivering large orders from the warehouse can lead to a considerable reduction in the retailer's inventory costs. In Dekker et al. (1998) the results of Dekker et al. (1995) were extended by also including the inventory costs at the warehouse. The study by Mohebbi and Posner's (1998) contains a cost analysis in the context of a continuous-review inventory system with replenishment orders and lost sales. The policy considered in the

`

paper by van der Heijden et al. (1999) is an echelon stock, periodic review, order- up-to (R,S) policy, under both stochastic demand and lead times. Andersson and Marklund's (2000) approach was based on an approximate cost-evaluation technique. Axsater (2000) presented a method for exact evaluation of control policies that provides the complete probability distributions of the retailer inventory levels. Mitra and Chatterjee (2004 b) examined the effect of utilizing demand information in a multi-echelon system. Seferlis and Giannelos (2004) presented an optimization-based control approach that applies multivariable model-predictive control principles to the entire network. The inventory system under Seifbarghy and Jokar's (2005) considered continuous review inventory policy (R, Q) and assumed constant lead times. In Moinzadeh's paper (2002), each retailer placed their order to the supplier according to the well-known 'Q, R' policy. It was assumed that the supplier had online information about the demand, as well as inventory activities of the product at each retailer, and uses this information when making order/replenishment decisions. Tang and Grubbström's (2003) general formulae are developed for solving the optimal planned lead times with the objective of minimizing total stock-out and inventory holding costs. Axsater (2003) assumes that the system is controlled by continuous review installation stock (R, Q) policies with given batch quantities and presents a simple technique for approximate optimization of the reorder points. Cachon and Fisher (2000) and Tsiakis et al. (2001) used mathematical modeling and scenario analysis in their studies. Cachon and Fisher (2000) considered a two-echelon inventory system with stochastic demand, while Tsiakis et al. (2001) assumed a four-echelon inventory system with time-invariant demand, differently from most studies. Cachon and Fisher (2000) study the value of sharing demand and inventory data in a two-echelon inventory system, while Tsiakis et al.'s (2001) objective was the minimization of the total annualized cost of the network Chiu and Huang (2003) use mathematical modeling and simulated annealing algorithm in their studies and consider an N-echelon serial supply chain. Their paper proposes a multi-echelon integrated just-in-time inventory (MEIJITI) model with random-delivery lead times for a serial supply chain in which members exchange information to make purchase, production, and delivery decisions jointly.

Parker and Kapuscinski (2004) used mathematical modeling and Markov decision processes in their paper, and consider a two-echelon inventor system with stochastic demand. Extending the Clark and Scarf (1960) model to include installations with production capacity limits, they

`

demonstrate that a modified echelon base-stock policy is optimal in a two- stage system when there is a smaller capacity at the downstream facility.

A multi-product, multi-stage, and multi-period production and distribution planning model was proposed in Chen and Lee (2004 b) to tackle the compromised sales prices and the total profit problem of a multi-echelon supply chain network with uncertain sales prices. They use mathematical modeling (mixed integer non-linear programming) and fuzzy optimization in their study. Jalbar et al. (2005 a) used mathematical modeling, Schwarz heuristic, Graves and Schwarz procedure, Muckstadt and Roundy approach, and O (N log N) heuristic in their study, and consider a two-echelon inventory system with one-warehouse and N-retailers. The goal was to determine single-cycle policies that minimize the average cost per unit time, that is, the sum of the average holding and set-up costs per unit time at the retailers and at the warehouse. In Routroy and Kodali's paper (2005) mathematical modeling and differential evolution algorithms were used. A three-echelon inventory system is considered consisting of a retailer, a warehouse, and a manufacturer.

Han and Damrongwongsiri's (2005) purpose was establishing a strategic resource allocation model to capture and encapsulate the complexity of the modern global supply chain management problem. A mathematical model was constructed to describe the stochastic multi-period two-echelon inventory with the many-to-many demand–supplier network problem. Genetic algorithm (GA) is applied to derive near optimal solutions through a two-stage optimization process. Demand in each period can be represented by the probability distribution, such as normal distribution or exponential distribution. Most of the papers reviewed here use simulation with mathematical modeling. They consider intensively two-echelon inventory system with stochastic demand, 1, 3, or N-echelon systems are rarely considered. They gain exact or approximate solutions. Scenario analysis, simulated annealing algorithm, Markov decision process, fuzzy optimization, heuristics, differential evolution algorithm, and GAs are used in addition to mathematical modeling in some of these papers. These techniques, however, are not used commonly in more than in a few papers, as they consider mostly two-echelon systems, but there are papers considering two-, three-, four-, or N-echelons. They usually assume stochastic demand and constant, fuzzy, or negligible lead times. With the exception of Parker and Kapuscinski (2004), they obtain approximate solutions.

`

2.3.2.3 METRIC modeling technique

Moinzadeh and Aggarwal (1997) use METRIC modeling and simulation techniques in their study, while Andersson and Melchiors (2001) and Wang et al. (2000) use METRIC modeling only. The three of them consider a two-echelon inventory system with stochastic demand, and obtain approximate solutions. Moinzadeh and Aggarwal (1997) study a (S-1,S)-type multi-echelon inventory system where all the stocking locations have the option to replenish their inventory through either a normal or a more expensive emergency resupply channel. Wang et al. (2000) study the impact of such center-dependent depot- replenishment lead times (DRLTs) on system performance. Andersson and Melchiors (2001) evaluate and optimize (S-1, S) policies for a two-echelon inventory system consisting of one central warehouse and an arbitrary number of retailers.

2.3.2.4 Markov decision process technique

Iida (2001), Chen and Song (2001), Chen et al. (2002), and Minner et al. (2003) use the Markov decision process in their studies, while Chiang and Monahan (2005) use Markov decision process and scenario analysis, and Johansen (2005) used Markov decision process, simulation, and Erlang's loss formula together. Iida (2001) and Chen and Song (2001) considered an N-echelon inventory system, but under stochastic demand in the first study and Markov-modulated demand in the second one, respectively. Chen et al. (2002), Minner et al. (2003), and Chiang and Monahan (2005) considered a two-echelon inventory system with stochastic demand. Johansen (2005) considered a single-item inventory system and a sequential supply system with stochastic demand. The main purpose of Iida's (2001) paper was to show that near-myopic policies were acceptable for a multi- echelon inventory problem. It is assumed that lead times at each echelon are constant. Chen and Song's (2001) objective was to minimize the long-run average costs in the system. In the system by Chen et al. (2002), each location employs a periodic-review (R, nQ), or lot-size reorder point inventory policy. They showed that each location's inventory positions were stationary and the stationary distribution is uniform and independent of any other. In the study by Minner et al. (2003), the impact of manufacturing flexibility on inventory investments in a distribution network consisting of a central depot and a number of local stock-points is investigated. Chiang and Monahan (2005) presented a two-echelon dual-channel inventory model in which stocks were kept in both a manufacturer warehouse (upper echelon)

`

and a retail store (lower echelon), and the product is available in two supply channels: a traditional retail store and an internet enabled direct channel. Johansen’s (2005) system was assumed to be controlled by a base-stock policy. The independent and stochastically dependent lead times were compared. To sum up, these papers consider two- or N-echelon inventory systems, with generally stochastic demand, except for one study that considers Markov-modulated demand (Chen and Song, 2001). They generally assumed constant lead time, but two of them accept it to be stochastic. They gained exact or approximate solutions.

2.3.2.5 Simulation

Tee and Rossetti (2002), Ng et al. (2003), Martel (2003), Kiesmüller et al.(2004), and Liberopoulos and Koukoumialos (2005) used simulation as a research technique in their studies about multi-echelon inventory management. Tee and Rossetti (2002) examined the robustness of a standard model of multi-echelon inventory systems, specifically the models discussed in Axsater (2000). Tee and Rossetti (2002), Liberopoulos and Koukoumialos (2005) considered a two-echelon inventory system, while Ng et al. (2003), Martel (2003), and Kiesmüller et al. (2004) considered N-echelon systems.

Tee and Rossetti’s (2002) study evaluated the behavior of a (R, Q) multi-echelon inventory model in predicting the total system cost under a non-stationary Poisson demand process. Also, here, it was assumed that the transport lead times were one day for all situations. Ng et al. (2003) use different inventory policies at the echelon level, and the demand and lead times were uncertain. Martel (2003) develops rolling, planning, horizon policies to manage material flows in multi- echelon supply–distribution networks with relatively general stochastic-demand processes and procurement, transportation, inventory, and shortage cost structures under (S-1, S) policy. Kiesmüller et al. (2004) assumed that all stock points are controlled by continuous review (s, nQ) installation stock policies with stochastic transportation times and compound renewal demand. Liberopoulos and Koukoumialos (2005) numerically investigated trade-offs between near-optimal base-stock levels, numbers of kanbans, and planned supply lead times in base-stock policies and hybrid base-stock/kanban policies with advance demand information used for the control of multi-stage production/inventory systems. In summary, all papers that were reviewed here gained approximate results. They usually present generalized models with N-echelon, and

`

solve a small example as two- or three-echelon. They assumed demand and lead times to be stochastic, uncertain, constant, or deterministic.

2.3.2.6 Stackelberg game

Axsater (2001) and Lau and Lau (2003) utilized the Stackelberg game in their papers. In this paper, a cost structure is provided that can be used to decentralize control of a multi-echelon inventory system consisting of a central depot and several retailers. It was assumed that the demand of retailers is derived from independent Poisson processes. Lau and Lau's paper (2003) applied different demand curve functions to a simple inventory/pricing model, and showed that while the common-wisdom implication was valid for a single-echelon system, assuming different demand curve functions can lead to very different results in a multi-echelon system.

2.3.2.7 Literature review technique

Minner (2003) and Thomas and Griffin (1996) reviewed the literature about multi-echelon inventory management in supply chains. Minner (2003) reviewed inventory models with multiple supply options and discussed their contribution to supply chain management in his paper. Further, related inventory problems from the fields of reverse logistics and multi-echelon systems were presented. Within the context of review, the studies that make deterministic and stochastic demand and lead time assumptions were placed. Thomas and Griffin (1996) reviewed the literature addressing coordinated planning between two or more stages of the supply chain, placing particular emphasis on models that would lend themselves to a total supply chain model.

2.3.2.8 Other techniques

In multi-echelon inventory management, there are some other research techniques used in literature, such as heuristics, vari-METRIC method, fuzzy sets, model predictive control, scenario analysis, statistical analysis, and GAs. These methods are used rarely and only by a few authors. Yoo et al. (1997) and Jalbar et al. (2005) used heuristics to multi-echelon inventory management in supply chain. Yoo et al. (1997) utilized from a heuristic method in their study, and they have made their experiment with various demand distributions, forecast error distributions, and lead times. Jalbar et al. (2005 b) used Raundy procedure and O (N log N) heuristics in their paper, and assumed that customer demand arrived at each retailer at a constant

`

rate and lead times were negligible. Liang and Huang (2005) and Köchel and Nielander (2005) used genetic algorithm. Additionally, Liang and Huang (2005) used an agent-based system, beer game, and statistical analysis to strengthen the solution methodology. Similarly, Köchel and Nielander (2005) used simulation for the same purpose. Liang and Huang's study (2005) developed a multi-agent system to simulate a supply chain, where agents operated these entities with different inventory systems. The demand is forecasted with a genetic algorithm and the ordering quantity was offered at each echelon incorporating the perspective of 'systems thinking'. Köchel and Nielander (2005) proposed the simulation optimization approach where a simulator was combined with an appropriate optimization tool. Here, the analyses were made according to zero and random lead-time situations, and infinite or finite (Poisson or arbitrary process) and constant- or random-demand characteristics situations. In Steptchenko et al.'s work (2002) the vari-METRIC method is used in multi-echelon, multi-indenture supply systems for repairable service parts with finite repair capacity. It was assumed here that demands occur according to stationary Poisson processes, independent of the number of items under repair.

Giannoccaro et al.'s paper (2003) presented a methodology to define a supply chain inventory management policy, which was based on the concept of echelon-stock and fuzzy-set theory. In particular, the echelon-stock concept is adopted to manage the supply chain inventory in an integrated manner, whereas fuzzy-set theory is used properly to model the uncertainty associated with both market demand and inventory costs. Finally, by adopting simulation, the performance of the three- stage supply chain is assessed and shown to be superior to that which the adoption of a local inventory management policy would guarantee. In this study, lead times were assumed to be deterministic and constant.

Kalchschmidt et al.'s work (2003) described an integrated system for managing inventories in a multi-echelon spare parts supply chain, in which customers of different sizes lay at the same level of the supply chain. Here, an algorithmic solution was provided through probabilistic forecasting and inventory management. The translation of the supply chain problem into a formulation amenable to model predictive control (MPC) implementation is initially developed for a single- product, two-node example. Insights gained from this problem are used to develop a partially decentralized model predictive control (MPC) implementation for a six-node, two-product, and three-echelon demand network problem developed by Intel Corporation that

`

consists of interconnected assembly/test, warehouse, and retailer entities in Braun et al.'s paper (2003). Lead times were estimated by facility personnel.

A multi-product, multi-stage, and multi-period scheduling model is proposed by Chen and Lee (2004 a) to deal with multiple incommensurable goals for a multi-echelon supply chain network with uncertain market demands and product prices. The uncertain market demands are modeled as a number of discrete scenarios with known probabilities, and the fuzzy sets are used for describing the sellers' and buyers' incompatible preference on product prices. The paper by Chandra and Grabis (2005) quantifies the bullwhip effect in the case of serially correlated external demand, if autoregressive models are applied to obtain multiple steps demand forecasts. Here, under autoregressive demand, inventory management of a two-echelon supply chain consisting of a retailer and a distributor is considered. It is assumed that the lead time is deterministic. The papers using the other techniques consider (one-, two-, three-, four-, five-, or N-echelon systems) assume stochastic, constant, fuzzy, or deterministic demand and lead times. All of them obtain approximate solutions. From the collected set of articles for review and on the basis of research technology, the above discussion can be represented by a simple diagram as:

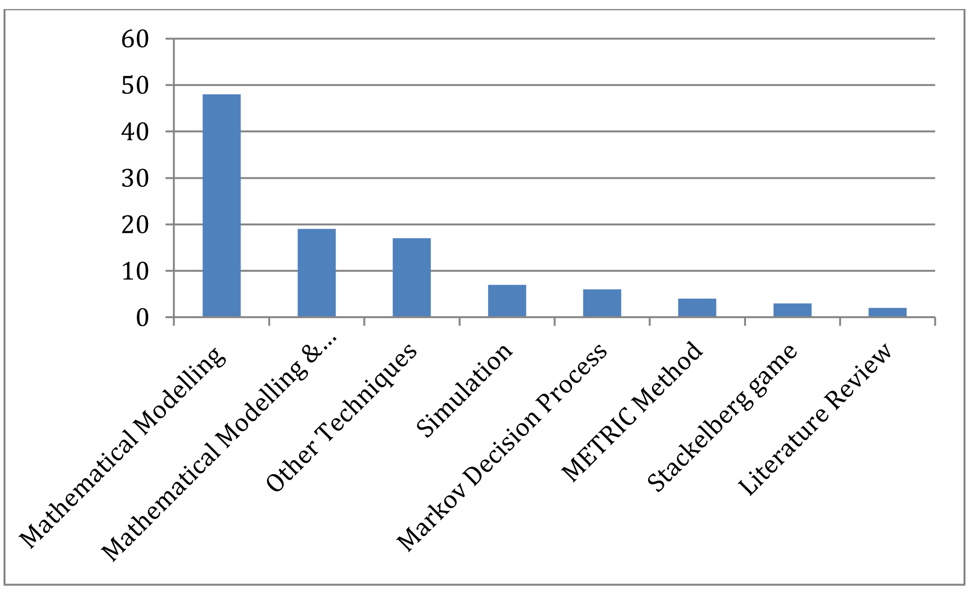


Figure 2.2: Classifications of articles research technique wise

The summary of literature review addressing inventory control problems under demand and supply uncertainty in supply chain can be represented in Table 2.1.

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Forsberg (1996)	Mathematical modeling and simulation	2 – one warehouse, *N* different retailers	Continuous review (R,Q)* policies	Stochastic – Poisson	Constant	Exact	To show how exactly to evaluate holding and shortage costs for a two-level inventory system
Graves (1996)	Mathematical modeling and simulation	2-one central warehouse, several retailers	Order up to policy	Stochastic	Deterministic	Approximate	To develop a new model for studying multi-echelon inventory system with stochastic demand
Verrijdt and DeKok (1996)	Mathematical modeling and simulation	2-one central depot, a number of end stock point	Order up to policy	Stochastic	Constant	Approximate	To present two adjustment methods that improve the service performance considerably in certain cases and to generalize the concept of imbalance
Thomas and Griffin (1996)	Literature review	2 or more stages	-	-	-	-	To review the literature addressing coordinated planning between two or more stages of supply chain
Yoo et al. (1997)	A heuristic method	2 – one central distribution center, *N* regional distribution center	Reorder point policy and fixed-order interval policy	Probabilistic	Constant	Approximate	To propose an improved DRP method to schedule multi-echelon distribution network
Moinzadeh and Aggarwal (1997)	METRIC Modeling and simulation	2-a warehouse and M retail centers	(S-1, S)*type inventory system	Random and stochastic-poisson	Deterministic	Approximate	To propose and test an order/expediting policy that use the information about the remaining lead times of the orders
Mohebbi and Posner (1998)	Mathematical modeling and simulation	1-two or more supplier	Continuous review inventory system	Stochastic-compound poisson	Stochastic exponentially distributed	Exact	To present an exact treatment of sole versus dual sourcing problem in the context of lost sales inventory system

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Bollapragda et al. (1998)	Mathematical modeling and simulation	2-a single depot that supplies several warehouse	Base stock policy-optimal allocation policy at the depot	Stochastic	Fixed	Approximate	To generalize the earlier work by Eppen and Scharge, to allow for non-identical warehouse
Dekker et al. (1998)	Mathematical modeling and simulation	2-one central warehouse, N retailers	Not specified	Stochastic	Fixed	Approximate	To provide insight in to effect of break quality rule on inventory holding cost
Diks and DeKok (1998)	Mathematical modeling	Divergent N-echelon	Periodic review Order up to policy	i.i.d^{+}	Fixed	Approximate	To minimize the expected holding and penalty costs per period
Hariga (1998)	Mathematical modeling	N-echelon	A composite strategy of the assemble to order and assemble in advance policies	Stochastic- Different demand distributions	Not specified	Approximate	To present a stochastic model for single period production system composed of several assembly/processing and storage facilities in series
Korugan and Gupta (1998)	Mathematical modeling and simulation	2 – a warehouse and N-retailers and a warehouse and M customer	A continuous (Q, r) policy	Demand rate is probabilistic	Not specified	Approximate	To model a two echelon inventory system by usage of an open queuing network with finite buffers
Ganeshan (1999)	Mathematical modeling and simulation	3 multiple retailers, one warehouse and multiple supplier	A near optimal (s, Q) type inventory policy	Stochastic	Stochastic	Approximate	To present a near optimal (s, Q) type inventory logistics cost minimization model for production and distribution network
Chen (1999)	Mathematical modeling	Multistage	Continuous review (R, Q)* inventory system	Deterministic	Zero	Approximate	To minimize the long run average total cost

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Van der Heijden et al. (1999)	Mathematical modeling and simulation	2-a central depot and multiple (non-identical) local warehouses	Periodic review, Order up to (R, S) policy	Stochastic	Stochastic	Approximate	To develop an algorithm to analyze multi echelon divergent networks with integral (R,S) inventory control under stochastic demand and lead time
Van der Heijden (1999)	Mathematical modeling and simulation	2-a central depot and multiple non identical local warehouses	Order up to (R, S) policy	Stochastic and stationary in time	Constant and deterministic	Approximate	To present a computational method to derive the control parameter in two echelon distribution system with different shipment frequencies at both levels
Axaster and Zhang (1999)	Mathematical modeling	2-a central warehouse and a number of identical retailers	Warehouses uses a regular installation stock batch ordering policy	Stochastic and stationary and independent Poisson demand	Constant	Exact	To show how the costs can be evaluated, and compare the policy both to an installation stock and to an echelon stock policy
Anderson and Marklund (2000)	Mathematical modeling and simulation	2-one central warehouse and N non-identical retailers	An installation Stock (R, Q)* policy	Stochastic normally distributed demand	Stochastic	Approximate	To analyse a conceptually quite simple model for highly decentralized control of two level distribution system
Wang et al. (2000)	METRIC modeling	A central repair depot and multiple inventory stocking centers	Continuous review, one for one policy	Stochastic Poisson	i.i.d[+]	Approximate	To find impact such center dependent depot replenishment lead times on system performance
Cachon and Fisher (2000)	Mathematical Modeling and scenario analysis	One supplier and N retailers	A reorder point (R, Q) policy	Stationary stochastic consumer demand	Constant	Approximate	To study the value of sharing information and develop a simulation based lower bound over all feasible policies
Axaster (2000)	Mathematical modeling and simulation	One central warehouse and N retailers	Continuous review installation (R, Q) policy	Stochastic Poisson	Constant	Exact	To present a method for exact evaluation of control policies that provides the complete probability distributions of retailer inventory level

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Lida (2001)	Markov Decision process	N-echelon serial inventory system	Near myopic policy	Stochastic non stationary	Constant	Approximate	To show the near myopic policies are acceptable for multi-echelon inventory problems
Axaster (2001 a)	Stackelberg game	A central warehouse and a number of retailers	S policies or (R, Q) policy	Stochastic-derived from independent poisson process	Constant	Approximate	To provide a cost structure that can be used for decentralized control for multi-echelon inventory system
Axaster (2001 b)	Mathematical modeling and simulation	2-echelon distribution inventory system	Continuous review (R, Q) policies	Stochastic	Constant	Approximate	To suggest and evaluate an approximate method for optimization of a two echelon inventory system
Chen and Song (2001)	Markov decision process	N- echelon serial inventory system	State dependent (s, S) policy	Markov modulated demand	Constant	Exact	To minimize the long run average costs in the system
Nozick and Turnquist (2001)	Mathematical modeling	Multi-product inventory system	Not specified	Stochastic-Poisson	Not specified	Exact	To present a model for optimizing the location of inventory for individual products and integration location analysis for distribution centers
Anderson and Melchiors (2001)	METRIC modeling	2-one central warehouse and an arbitrary numbers of retailers	(S-1, S) policies with continuous review	Stochastic-compound poisson demand	Constant	Approximate	To evaluate and optimize (S-1, S)-policies by a heuristic method for a warehouse, multiple retailers inventory system
Tsiakis et al. (2001)	Mathematical modeling (MILP) and scenario analysis	4- a number of manufacturing sites warehouses, distribution centers and customer zones	Not specified	Time invariant demand but possibly uncertain	Not specified	Approximate	To minimize the total annual cost of the network, taking in to account both infrastructure and operating costs

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Chen et al. (2002)	Markov decision process	2-one supplier and multiple retailers	A periodic review or lot sizing reorder point inventory policy	Stochastic-interdependent demand	Constant	Exact	To show that each location's inventory positions are stationary and the stationary distribution is uniform and independent of any other's
Steptchenko et al. (2002)	VARI-METRIC method	Multi-echelon, multi-indenture supply systems	(S-1, S) inventory policy	Stochastic-stationary poisson processes	Dependent – repair lead times	Approximate	To show that commonly used assumption of infinite capacity may seriously affect system performance and stock allocation decision repair shop utilization is relatively high
Moinzadeh (2002)	Mathematical modeling and simulation	2- one supplier and M identical retailers	(Q, R)* policy	Stochastic random but stationary	Constant	Exact	To propose a replenishment policy for the supplier and then to provide an exact analysis of the operating measures
Tee and Rossetti (2002)	Simulation	2-one warehouse, multiple retailers system	(R, Q)* inventory policy	Stochastic-non stationary Poisson demand process	One day for all situations	Approximate	To examine the robustness of a standard model of multi-echelon inventory systems, specifically the models discussed in Axaster (2000)
Rau et al. (2003)	Mathematical modeling	3-single supplier, single producer, and single retailer	Not specified	Demand rate is deterministic and constant	Negligible	Exact	To develop multi-echelon inventory model for a deteriorating item and to derive an optimal joint total cost from an integrated perspective among the supplier, the producer and the buyer
Minner (2003)	Literature review	-	-	-	-	-	To review the literature on inventory models with multiple suppliers and to discuss their potential contribution to SCM issue
Giannoccaro et al. (2003)	Simulation and Fuzzy set theory	N-stage serial system	Echelon periodic-review control policy	Fuzzy	Assumed to be deterministic and constant	Approximate	To present a methodology defining a supply chain inventory management policy, which is based on the concept of echelon stock and fuzzy set theory

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
So and Zheng (2003)	Mathematical modeling	2-a retailer and a supplier	Order-up –to policy	Stochastic – independent and identically distributed	Variable and affected by the retailer's order quantities	Approximate	To analyse two important factors that can contribute to the high degree of order quantity variability experienced by semiconductor's manufacturer supplier's lead time and forecast demand updating
Kalchschmidt et al. (2003)	An algorithm solution is provided through probabilistic forecasting and inventory management	1 and 2 central warehouse serves on one side, a one echelon chain and a two echelon supply chain	Order-up –to policy	Stochastic-variable and lumpy	Not specified	Approximate	To describe an integrated system for managing inventories in a multi-echelon spare parts supply chain in which customers of different size lay at the same level of the supply chain
Tang and Grubbström(2003)	Mathematical modeling and simulation	2- level assembly system	Lot for lot policy	Constant and deterministic	Stochastic	Approximate	To minimize total stock-out and inventory holding costs
Ng et al. (2003)	Simulation	N-echelon supply chains	Different inventory policies at echelon level	Uncertain	Uncertain	Approximate	To develop of a simulation workbench for modeling and analyzing multi-echelon supply chains
Martel (2003)	Simulation	N-and three echelon for numerical example	(S-1, S)* policy	Stochastic	A preplanned integer number	Approximate	To develop rolling planning horizon policies to manage material flows in multi-echelon supply distribution network
Dong and Lee (2003)	Stackelberg game	M-echelon serial periodic review inventory system and 3 echelon for numerical example	An echelon base stock inventory policy, order-up to S policy	An auto aggressive demand model	Variable to see the impact of lead times and auto correlation on the performance of the system	Approximate	To extend the approximation to the time correlated demand process of Clark and Scarf

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Lau and Lau (2003)	Stackelberg game	1- An integrated firm 2- The manufacturer and retailer 3- The manufacture, wholesaler and retailer	Not specified	Different demand curve functions	Not specified	Approximate	To apply different demand curve functions to a simple inventory/pricing model and show the different results gained
Axaster (2003)	Mathematical modeling and simulation	2-a central warehouse and a number of retailers	Continuous review installation stock (R, Q)* policies with given batch quantities	Stochastic	Constant and stochastic	Approximate	To present a simple technique for approximate optimization of the reorder points
Barun et al. (2003)	Model predictive control (MPC) and simulation	Six node, two product three echelon demand network	Not specified	Deterministic	Estimated by facility personnel	Approximate	To develop a partially decentralized MPC implementation
Minner et al. (2003)	Markov decision process	A central depot and a number of local stock point	Periodic review echelon order up to policy	Stochastic	Constant, may be different for different retailers	Approximate	To investigate the impact of manufacturing flexibility on inventory investments in a distribution network consisting of central depot and a number of local stock points
Chiu and Huang (2003)	Mathematical modeling and (MINLP) and simulated annealing algorithm	A serial N-echelon supply chain in which each echelon contains only one number	A time buffer and emergency borrowing policies	Demand rate of member 1 is known and of member I is related to production rate	A non-negative random variable following a probability distribution	Approximate	To purpose a multi echelon integrated just in time inventory model with random delivery lead times for a serial supply chain

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Mitra and Chatterjee (2004 a)	Mathematical modeling	2-stage 1 is facing demand and is supplied by stage 2 which in turn is supplied by an outside source	Continuous review (R, Q)* system under nested and echelon stock based policy	Stochastic	Joint and deterministic	Approximate	To examine de Bodt and Graves's (1985), model which they develop in their paper 'continuous review policies for multi echelon inventory problem with stochastic demand and suggest a modification
Mitra and Chatterjee (2004 b)	Mathematical modeling and simulation	2-one warehouse two retailer	Periodic review inventory policy	Stochastic-stationary independent and normally distributed	Deterministic	Approximate	To examine the effect of utilizing demand information in a multi echelon system
Parker and Kapuscinski (2004)	Mathematical modeling (Dynamic programming)	2-stage system	A modified echelon based stock policy	Stochastic and independent from period	Integer and lead time model follows MEBS policy	Exact	To demonstrate optimal policies for capacitated serial multi echelon production/inventory systems
Chen and Lee (2004 a)	Discrete scenario based approach, fuzzy logic and mathematical programming	A multi-product, multi stage, and multi period model	Not specified	Stochastic-different scenarios of demand are forecasted with known probabilities	Fuzzy	Compensatory	To deal with incommensurable goals for a multi-echelon supply chain network with uncertain market demands and product prices
Chen and Lee (2004 b)	Mathematical modeling and fuzzy optimization	A multi-product, multi stage, and multi period model	Not specified	Uncertain and stochastic	Deterministic	Approximate	To tackle the compromised sales prices and the total profit problem of multi-echelon supply chain network with uncertain sales prices
Kiesműller et al. (2004)	Simulation	N-echelon and two and three echelon for numerical example	Continuous review (s, nQ)* installation stock policy	Stochastic – compound renewal demand	Stochastic	Approximate	To derive analytical approximations for performance characteristics of a divergent multi-echelon distribution network

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Seferlis and Giannelos (2004)	Mathematical modeling and simulation	4-two production nodes, two warehouse nodes, 4 distribution centers and 16 retailers	A decentralized safety inventory control policy	Both stochastic and deterministic demand variations	Not specified	Approximate	To treat process uncertainty with in the deterministic supply chain network model, a rolling horizon, model perspective control approach is suggested
Chiang and Monahan (2005)	Markov decision process and scenario analysis	2-a manufacturer warehouse and retail store	One for one replenishment inventory control policy	Stochastic	Stochastic-independent exponential random variables	Exact	To analyze the impact of customer's search rates on the channel performance and to present a two echelon dual channel inventory model
Jalbar et al. (2005 a)	Mathematical modeling, Schwarz heuristic, Graves and Schwarz procedure, Muckstadt and Roundy approach	2-one warehouse and N retailers	Single cycle policy	Demand rates are assumed to be known and constant	Negligible	Approximate	To determine single cycle policies that minimize the average cost per unit time
Jalbar et al. (2005 b)	Roundy procedure and heuristics	2-one warehouse and N retailers	Nested policy	Arrives at each retailer at a constant rate	Negligible	Approximate	To propose a heuristic process to compute near optimal policies
Johansen (2005)	Markov decision process, simulation and Erlang's loss formula	Single item inventory system and a sequential supply chain	Base stock policy	Stochastic-Poisson	Stochastic-Erlangian	Approximate	To study how to compute the optimal base stock for a lost sales inventory model with sequential supply system and Erlalangian lead times
Chandra and Grabis (2005)	Simulation and statistical analysis	2- a retailer and a distributor	An order up to policy and MRP	Autoregressive demand	Deterministic	Approximate	To quantify the bullwhip effect in the case of serially correlated external demand

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Liberopoulous and Koukoumialos (2005)	Simulation	Single –stage and two stage production/inventory systems	Base stock policy and hybrid –base stock/kanban policy	Arrives randomly with constant demand lead time	A fixed lead time parameter and analytically obtained	Approximate	To investigate tradeoffs between near optimal base stock levels, number of Kanbans and planned supply lead times in base stock policies, and hybrid base stock/kanban policies
Liang and Huang (2005)	Agent based system, genetic algorithm, beer game and statistical analysis	4-supplier (P system), manufacturer (Q system), distributor (P system) and retailer (Optional system)	(P Q P O) inventory policy- periodic review, continuous review and optional system	Demand is forecasted with genetic algorithm	The lead time data are collected by the control agents	Approximate	To develop multi agent system to simulate supply chain
Seifbarghy and Jokar (2005)	Mathematical Modeling (deterministic) and simulation	2-one central ware house and many identical retailers	Continuous review inventory policy (R, Q)*	Stochastic independent Poisson demand	Constant	Approximate	To develop an approximate cost function to find optimal reorder points for given batch sizes in all installations
Routroy and Kodali (2005)	Mathematical modeling and Differential evolution algorithm	3-a retailer, a warehouse and a manufacturer	A continuous review policy (Q, r)*	Stochastic- normally distributed	Constant	Approximate	To minimize total system wide cost i.e. supply chain inventory capital, supply chain ordering/setup cost, supply chain inventory stock out cost
Köchel and Nielander (2005)	Simulation and genetic algorithm	5-factory depot, central stock, district warehouse, branch store and retailer outlet	Continuous- review, order point, order quantity strategy (s,Q)*	Stochastic- Poisson, constant or random	Zero or random	Approximate	To show that simulation optimization successes fully can be applied to define optimal policies in very general multi-echelon inventory systems
Han and Damrongwongsiri (2005)	Mathematical modeling and genetic algorithm	2-I number of warehouses and J number of markets	A (t,S)* control policy	Stochastic- represented by the probability distribution, (normal or exponential)	There is no lead time	Approximate	To establish a strategic resource allocation model to capture and encapsulate the complexity of modern global supply chain management problem

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Gaur & Ravindran (2006)	Non Linear programming and AHP	Warehouses and retailers (Consumer Zones)	Continuous review policy	Stochastic- Normally distributed	Stochastic- Normally distributed	Approximate	Total cost minimization and responsiveness maximization
Brandimarte (2006)	Mathematical Modeling Branch and bound method	Demand and supply nodes	Not Specified	Stochastic-tree based structure	Not specified	Exact	Cost Minimization
Boute et al (2007)	Mathematical Modeling and Markov decision processes	2 echelon supply chain, single retailer and customer	periodic review, base-stock, or order-up-to replenishment Policies.	i.i.d	Not specified	Approximate	Effect of lead time on retailer's safety stock
Feng & Viswanathan (2007)	Mathematical modeling	one vendor, multi-buyer supply chain	Not specified	Stochastic- Normally distributed	Not specified	Approximate	Impact of demand uncertainty on the effectiveness of coordinating such a supply chain
Lin (2008)	Fuzzy mathematical programming	Not specified	Periodic review	Not specified	Fuzzy Lead time	Approximate	To extend Ouyang, L. Y., Chuang, B. R. (2001).
Zhang (2008)	Mathematical modeling	supply-manufacturing two-tier supply chain	Not specified	Uncertain demand- normally distributed	Not specified	Iterative algorithm with GAMS and its MINLP solvers	To maximize the expected revenue of the manufacturer
Lau and Song (2008)	METRIC methodology	multi-echelon	(s – 1, s) inventory policy	Non stationary demand	Not specified	Approximate	To minimize life support cost
Seth and Pandey (2009)	Mathematical modeling	Not specified	Periodic review	Non stationary- normally distribute	Not specified	Approximate	new scheme to arrive at the inventory replenishment levels and tries to improve the pull in the system

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Babaï et al. (2009)	Empirical Investigation	Not specified	dynamic re-order point control policy	Non stationary demand	Uncertain demand	Approximate	The performance of this policy is assessed by means of empirical experimentation on a large demand data set from the pharmaceutical industry
Routroy and Maddala (2009)	Mathematical modeling and differential evolution	3-a retailer, a warehouse and a manufacturer	A continuous review policy (Q, r)*	Demand uncertainty-stochastic (Normally distributed)	Lead time uncertainty-stochastic (Normally distributed)	Approximate	To minimize total system wide cost while maintaining proper service level
Mahnam et al. (2009)	Mathematical modeling and PSO	Assembly supply chain network	Periodic review policy	Fuzzy demand	Deterministic	Approximate	External supplier's reliability has determined using a fuzzy expert system
Huang and Lin (2010)	Mathematical modeling and modified ant colony optimization	Not specified	Not specified	stochastic (Normally distributed)	Not specified	Approximate	To minimize total travel length, and incorporates the attraction of pheromone values that indicate the Stock-out costs on nodes.
Yang and Lo (2010)	Mathematical modeling	single vendor with multiple buyers for a single product	Continuous review policy	Uncertain-Normally distribute	Not specified	Approximate	To determine a suitable policy of inventory and purchasing management is investigated in this study, with the objective of minimizing total expected inventory costs with multiple partners
Schmitt et al. (2010)	Mathematical modeling	Supplier and its retailer	Optimal base stock policy	Stochastic-Normally distributed	Stochastic-Normally distributed	Exact and Approximate	To develop a closed-form approximate solution by focusing on a single stochastic period of demand or yield
Lam and Ip (2011)	Mathematical modeling	Not specified	Not specified	Not specified	Not specified	Approximate	To minimize inventory cost and to maximize CSI value.
Dey and Chakraborty (2011)	Mathematical Modeling	Not specified	Continuous review policy	Uniformly distributed	Not specified	Approximate	To minimize the total cost

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Taleizadeh et al. (2011)	Mathematical modeling and Harmony search algorithm	Not specified	Not specified	Stochastic – Poisson distribution	Zero lead time	Approximate	to maximize expected profit
Rajgopal et al. (2011)	Mathematical modeling and stochastic mixed integer programming	production–distribution system consisting of a set of geographical distributed facility locations	Periodic review policy	Stochastic – Poisson distribution	Not specified	Approximate	To minimize the fixed costs of facilities, plus the expected cost of raw materials, cutting and transportation, minus the value of the scrap that is salvaged, over all scenarios
Sajjadi and Cheraghi (2011)	Mathematical modeling an iterative heuristic algorithm	Not specified	Not specified	Stochastic Normally distributed	Not specified	Approximate	To minimize total network cost
Yeo and Yuan (2011)	Mathematical modeling	Not specified	Periodic review	Demand cancellation	Uncertain independent	Approximate	The convexity for the optimal cost is established and the optimal ordering level is derived.
Fernandes et al. (2011)	Mathematical modeling	Not specified	All the production and stock policy is make-to-stock	Stochastic-log normally distribute	Not specified	Approximate	To define of a key performance indicator, able to measure the impact of the demand uncertainty in a multi-stage supply chain inventory level
Dey and Chakraborty (2012)	Fuzzy mathematical modeling	Not specified	Periodic review	Fuzzy random demand	Not specified	Approximate	To minimize total cost
He and Zhao (2012)	Mathematical modeling	One raw-material supplier one manufacturer and one retailer.	Not specified	Uncertain-normal distribution function	Uncertain-follows cumulative distribution function	Approximate	To study the inventory, production, and contracting decisions of a multi-echelon supply chain with both demand and supply uncertainty

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Rahim and Aghezzaf (2012)	Mathematical modeling and the constrained vehicle routing problem is solved using a constructive local search procedure	The warehouse and retailers	Not specified	Stochastic Demand	Stochastic lead time	Approximate	To minimize the within-cluster travel costs and/or distances and are then replenished using an optimal direct shipping strategy satisfying some additional restrictions
Schmitt and Synder (2012)	Mathematical modeling	Two suppliers	periodic-review base-stock policy	Deterministic	Zero lead time	Approximate	To develop models for both cases to determine the optimal order and reserve quantities in two cases in which one is unreliable supplier and second is reliable supplier but expensive
Özen et al. (2012)	Mathematical modeling and two heuristics relaxation and approximation	Single stage inventory	Periodic review	Non stationary demand	Zero lead time	Approximate	To minimize the total expected cost
Taleizadeh et al .(2013)	Mathematical modeling and hybrid intelligent approach	Not specified	Replenishment lead time is independent random variable	Fuzzy numbers	Not specified	Approximate	To maximize the expected profit
Guo and Lee (2014)	Mathematical modeling and GA	Suppler warehouse and retailer	The continuous review(Q, R) policy	Demand occurs based on a Poisson process	Fixed	Approximate	To maximize the expected profit
Firouzi et al. (2014)	Mathematical Modeling	Not specified	Periodic review	i.i.d	Not specified	Approximate	To improve the accuracy of the forecast leads to making a better ordering decision and eliminating the negative effect of supply disruption on the total cost

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Wang et al. (2015)	Mathematical modeling	Not specified	Not specified	Uncertain random variables	Not specified	Approximate	To provide theoretical analysis of the models that attains optimality when demand information availability in subjective judgments leading to uncertainty along with random variation.
Pasandideh et al. (2015)	Mathematical modeling	Manufacturing plants, distribution centers (DCs), and customer nodes	Not specified	Stochastic-normally distributed	Not specified	Approximate	To determine the quantities of the products produced by the manufacturing plants in different periods, the number and locations of the warehouses, the quantities of products transported between the supply chain entities, the inventory of products in warehouses and plants, and the shortage of products in periods such that both the expected and the variance of the total cost are minimized
Lin and Song (2015)	Mathematical modeling and NSGA-II	Multi-layer multi-period stochastic inventory	(r, Q) policy	Stochastic	Not specified	Approximate	To minimize the expected cost and the risk measured by conditional value at risk.
Kumar and Dutta (2015)	Mathematical modeling and multi-objective fuzzy goal programming (MOFGP)	Not specified	Not specified	Demand is taken here as inversely related to the selling price of the product	Lead time is zero	Approximate	To maximize the total expected profit
Movahed and Zhang (2015)	Mathematical modeling	Manufacturer, distributor and retailer	Not specified	Demand is uncertain with known mean and variance	LT is uncertain with known mean and variance	Approximate	To minimize expectation and deviation of the total costs
Thorsen and Yao (2015)	Mathematical modeling	Not specified	Periodic review	Normally distributed	Normally distributed	Approximate	To develop an approach based on Benders' decomposition to compute optimal robust (i.e., best worst-case) policy parameters

`

Table 2.1: Literature review of inventory control problems in supply chains addressing demand and supply uncertainty

Author, year	Research technique	Number of echelons	Inventory system/policy	Demand assumption	Lead-time assumption	Exact/ approximate solution	Objective
Sadeghi and Niaki (2015)	Mathematical modeling and NRGA & NSGA	A central warehouse, a single vendor and multiple retailers	Not specified	trapezoidal fuzzy demand	Not specified	Approximate	To Minimize both the total inventory cost and the warehouse space
Chu et al.(2015)	Mathematical model and agent based simulation	Plant, distribution center and distributor	(r, Q) inventory policy	Random customer order demands	Not specified	Approximate	To minimize the inventory cost while maintaining acceptable service levels quantified by the fill rates.
Moghaddam and Raziei (2016)	Mathematical modeling	Two echelon distribution network	Not specified	Fuzzy	Not specified	Approximate	To minimize the cost and the shortages
Attar et al. (2016)	Mathematical modeling, simulation and optimization	Not specified	Continuous-review, base-stock inventory model	Poisson demand	Constant	Approximation	To minimize the cost by simulation process and to get optimized inventory policy
Puga and Tancrez (2017)	Mathematical modeling and heuristics algorithm	A central plant to retailers, passing through Distribution Centers		Normally distributed	Normally distributed	Approximate	To minimize the inventory cost while maintaining acceptable safety stocks to avoid shortages
Qiu et al. (2017)	Mathematical modeling	Not specified	periodic-review inventory management	ellipsoid uncertainty sets to model demand distribution uncertainty	Not specified	Approximate	To Build robust dynamic programming models to prove the optimality of a (s, S) policy
Aref et al. (2018)	Mathematical modeling and average approximation approach	Multi echelon supply chain	reorder point order-up-to-level (s, S) policy	Uncertain	Not specified	Approximate	to maximize the profit by a two-stage stochastic mathematical model

`

The collected set of research articles year wise can be classified as:

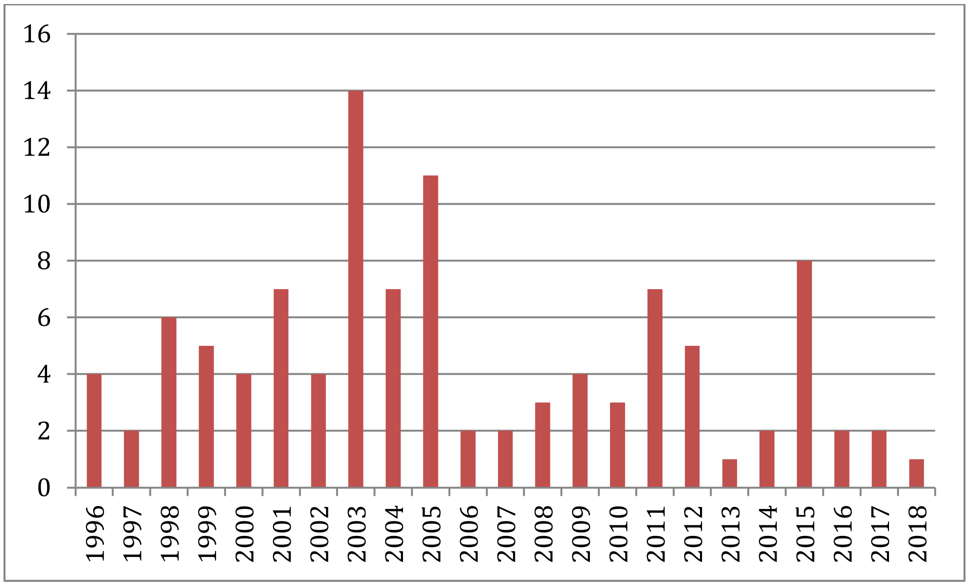


Figure 2.3: Research articles published year wise

The collected set of research articles journal wise can be classified as:

`

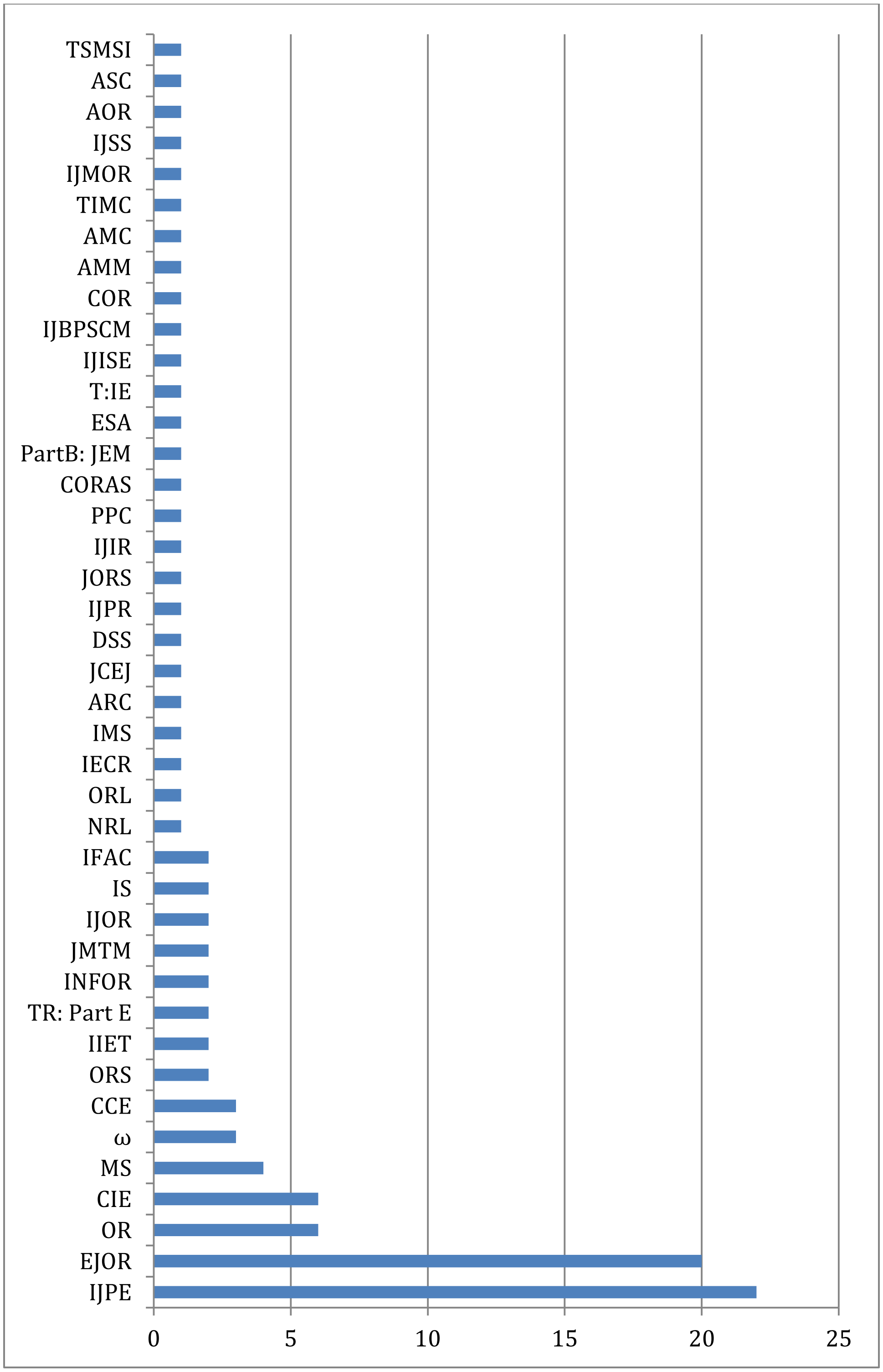


Figure 2.4: Research articles published Journal wise

`

2.4 Inventory control problems handling by risk pooling approach

The literature offers various confusing definitions about the term variability, variance, or volatility (Hubbard, 2009) uncertainty (Knight, 2005). Lead time and demand uncertainty may arise from lead time and demand variability or incomplete knowledge. “Uncertainty is the inability to determine the true state of affairs of system”. “Uncertainty caused by variability is a result of inherent fluctuations or differences in the quality of concern. More precisely, variability occurs when the quantity of concern is not a specific value but rather a population of values (Hamies, 2009).

Supply and demand uncertainty may lead to economic risk (Bowersox et al., 1986), the possibility of a negative deviation from expected values or desired targets. The corporate target is expected profit; the difference expected revenue and expected cost. The possibility of a positive deviation from an expected value constitutes a chance. Despite of risk pooling entails (Kim and Benjaafar, 2002), it may reduce variability and thus uncertainty and expected (ordering, inventory holding, stock out and backorder) cost (Eppen, 1979) and or increase expected revenue (product availability, fill rate, service level) (Chen and Lin, 1989) and thus expected profit (Anupindi and Bassok, 1999).

We define risk pooling in supply chain as consolidating individual variabilities (measured with standard deviation) (Sussams, 1986) of demand or supply in order to reduce the total variability (Chopra and Meindl, 2007) they form and thus uncertainty and risk (Pishchulov, 2008). The individual variabilities are consolidated by aggregating (Anupindi et al. 2006) demands (Gerchak and Mossaman, 1992) (demand pooling) (Evers 1997) and or lead time (Thomas and Tyworth, 2006) (lead time pooling). Consolidating and aggregating means “combining several different in elements in to a whole” (Soanes and Hawker, 2008).

As individual variabilities (Gerchak and He, 2003) and not individual risk (Anupindi and Bassok, 1999) are pooled, the term risk pooling is misleading. Nonetheless, we use it, because it is conventional. Risk pooling is described as the hedge the uncertainty so that, the firm is in better position to mitigate the consequences of uncertainty (Cachon and Terwiesch 2009) and enables to avoid uncertainty (Pishchulov 2008), or removes some of uncertainty involved in the planning of stock levels (Jackson and Muckstadt, 1989). Risk pooling is also considered a form

`

of operational hedging. “Hedging is the action of a decision maker to mitigate a particular risk exposure. Operational hedging is the mitigation of risk using operational instruments (Mieghem, 2007) e.g. pure diversification of demand pooling.

2.4.1 Characteristics of risk pooling:

Now turned to describing the five important characteristics common to risk pooling methods:

2.4.1.1 Increasing returns

The benefit of or return on risk pooling is variability reduction and thus enable inventory reduction for a given service level or in increase level for a given inventory. The risk pooling returns generally (in the following cases) the increasing application or the number of participants:

It augments with the number of

- Participating stock keeping locations in inventory pooling (Lin et al. 2001) time and logistics postponement (Zinn and Bowersox, 1988), transshipment (Jonsson and Silver, 1987) and cross filling (Ballou and Burnetas, 2000)

- Substitutable, products and degree of substitution in product substitution, demand reshape (Eynan and Fouque, 2003, 2005) and resource flexibility (Tomlin and Wang, 2005)

- Products in product line (Zinn, 1990) or products being postponed (Graman and Magazine, 2006) in manufacturing postponement,

- Products sharing components (increasing commonality) in component commonality (Srinivasan et al 1992), as well as

- Retailers in pooling over the out-side supplier lead time or centralized ordering (Jackson and Muckstadt, 1989) and risk pooling by drop shipping or virtual pooling (Netessine and Rudi, 2006)

`

2.4.1.2 Diminishing marginal returns with increasing application as well as increasing benefit

However, the marginal profit of risk pooling commonly decreases with each additional decrease in risk pooling participant. There appears to be diminishing marginal returns to risk pooling (Campbell et al. 2001) e.g. to cross filling (Ballou and Burnetas 2000) and transshipments with increasing number of locations transshipping (Evers, 1997) or increasing even allocation of demand across locations (Evers 1996) to virtual pooling by drop shipping with increasing number of retailers (Netessine and Rudi, 2006), to component commonality (Chopra and Meindl, 2007), postponement (Zinn, 1990) product substitution (Eynan and Fouque, 2003), capacity pooling and location pooling (Cachon and Terwiesch 2009).

The marginal benefit of location pooling decreases with increasing number of pooled locations, so that, the main benefit is gained by consolidating few locations and it might not be necessary to pool all locations. The same applies to transshipments (Tagaras 1989). Capacity pooling with a little bit flexibility as long as it is designed with long chains almost has the same benefit as full flexibility. Companies can benefit from any amount of risk pooling as long as they implemented it appropriately and the demand is not perfectly positive correlated.

Graman and Magazine (2006) similarly observe pertaining to postponement "that it is only necessary to postpone a portion of a product to realize most of the benefits of such as a strategy". "The mathematical inventory model shows that almost all the positive benefits of postponement (such as lower inventories) can be achieved with a partial (low capacity) postponement scenario". Cachon and Terwiesch (2009) show diminishing marginal returns of location-pooling with increasing mean for independent Poisson demand. This must not necessarily be hold for other distributions such as normal one.

2.4.1.3 Increasing demand variability

The (demand) risk pooling effect decreases with increasing demand correlation (Eppen 1979) and any magnitude (relative sizes of standard deviations of demand), and increases with decreasing correlation and Magnitude (Zinn et al. 1989). Likewise, the value of lead time risk pooling increase with decreasing correlation of replenishment lead times.

`

Therefore, many companies attempt to reduce inventory and manufacturing costs by manipulating correlated demand resources, or including demand patterns that balance each other in an average sense i.e. by risk pooling while maintaining sales (Gerchak and Gupta, 1991). The bullwhip effect can be reduced by risk pooling (effects) (Hartman and Dror, 2003), production smoothening and seasonality (Cachon et al. 2007), so that it may be overestimated. Evers and Beier (1993) and Evers (1995) debatably show that there are no further savings in safety stocks after first consolidation because of arising perfectly positive correlation. Tyagi and Das (1999) showed that if the demands are correlated and one appropriately takes advantage of their characteristics, a partial may be more cost-efficient than complete pooling of customers. On the other hand, Xu and Evers (2003) claim that partial pooling can never outperform complete pooling based only on demand correlation. Examples, suggesting that one should prefer partial to complete aggregation were based on inconsistent correlation matrices.

2.4.1.4 Decreasing demand correlation

The benefits of risk pooling generally increase with a structured increase in demand variability (Eppen 1979), if the lead time are exogenous. Gerchak and He (2003) provide a news vendor counter example with convex ordering where increased variability of two individual demands reduces the benefit of risk pooling. If supply lead times are endogenous in multi-item finite capacity production-inventory systems, both higher demand variability (Benjaafar and Kim, 2001) and capacity utilization and asymmetric backordering or holding costs make risk (inventory) pooling less valuable.

2.4.1.5 Decreasing concentration of uncertainty

Assuming a multi-variate normal distribution and uncorrelated demands, the value of pooling is lower when, uncertainty is more concentrated, i.e. the less uniform or more dispersed the values of the standard deviations of the demand rate are (Alfaro and Corbett, 2003). Correspondingly, the portfolio effect will be larger when variances are of similar rather highly varying magnitude (Tyagi and Das, 1998)

2.4.2 Inventory Pooling: Inventory pooling is the combination of inventories and satisfying various demands from it in order to reduce inventory holding and shortage costs through risk pooling (Benjaafar and Kim, 2001). It is also called vendor pooling (Monezka and Carter, 1976)

`

product consolidation (Benjaafar et al. 2004), demand pooling (Cachon and Terweisch, 2009) distributor integration (Simchi-Levi et al 2008), location pooling (Simchi-Levi et al 2008), or inventory centralization (Monezka and Carter, 1976). It can be achieved by inventory (Benjaafar et al. 2004), or warehouse (system) centralization (Eppen, 1979) or selective stock keeping respectively specialization (Anupindi et al. 2006). The latter strives to reduce inventory carrying cost treating products differently without reducing the service level substantially. For example, products with a low turnover might be stocked only a few locations due to cost considerations.

Inventory pooling is considered in location and allocation decisions (Teo et al. 2001) satisfying both on-line and store demand (Bendoly, 2004), speculative online exchanges (Milner and Kouvelis, 2007), airline revenue management (Zhang and Cooper, 2005) for spare parts of airlines (Hearn, 2007) and of U.S. manufacturing companies (Carter and Monczka, 1978). Inventory pooling is a 1/0 – risk pooling method: Inventories are consolidated and stochastic demands aggregated, as they are satisfied from the consolidated inventory (Pisholuv, 2008). Thus, demand variabilities may balance each other (demand pooling). The lead times to the separate inventories are pooled to the lead time to the consolidated stock according to Cachon and Terwiesch's (2009) notion of lead time pooling, but this actually is demand pooling during the replenishment lead time.

Inventory pooling does not pool lead times, so that their variabilities may balance each other according to Evers (1997, 1999) and Wanke and Saliby (2009). Inventory centralization can reduce expected costs in a cost minimization model (Eppen, 1979 and Chen & Lin, 1989) or increase profits and service level (Eynan, 1999) in a profit maximization model (Lin et al. 2001). Centralization (decreasing the number of warehouses) generally reduces inbound transportation costs from the supplier to the warehouses because of a higher utilization of transportation means (economies of density) Caves et al. (1984), possibly the distances between the production facility and the central warehouse(s) (Fawcett et al. 1992) and the unit costs in the warehouse system due to higher utilization (economies of scale). The system wide total throughput is distributed over a smaller number of warehouses and thus the average throughput per warehouse increases. The warehouse costs per item (unit costs for space, product handling, and personnel) decrease through the economies of scale (McKinnon, 1989).

`

The fixed costs per order and per unit holding costs usually diminish with centralization. Fixed warehouse costs, especially personnel costs, are distributed over a larger warehouse throughput so that, the unit costs decrease with increasing utilization and the return on rationalization investments in more productive low-cost methods increases (economies of scale).

Centralization can lead to economies of massed reserves, if it is used to lower system wide safety stock through risk is pooling and thus unit holding costs (Bowersox et al. 1986). The latter are further reduced by less inventory loss and thus, there are lower risk costs in the centralized system. Economies of massed reserves are cost savings due to centralized reserve holding with increasing firm size (Scherer and Ross, 1999). Among others Bowersox et al. (1986), Fawcett et al. (1992), argue that in spite of constant basic inventory and increasing in transit inventory total inventory is lowered by reducing safety stock in the centralized system.

Higher utilization of inbound transportation means with centralization decreases the number of stock placements and removals (Schulte, 1999). Therefore setups are reduced and larger loading and unloading equipment can be used saving time in the transportation system. Reducing idle time leads to a higher utilization in time and therefore to economies of density using larger means of transportation.

On the other hand, in a centralized logistics system outbound transportation costs may be higher because of lower utilization (Fawcett et al. 1992) and longer distances from the central warehouse(s) to customers (Ballou, 2005) and thus perhaps the service level might be lower (Schulte, 1999). Centralization may entail diseconomies of scale mostly in organization (Chandler 1990). With increasing size there are no gains from rationalization anymore, but increasing transaction and coordination costs and thus increasing warehouse unit costs in big warehouse systems. Das and Tyagi (1997) develop an optimization model for determining the optimal degree of centralization as a tradeoff between inventory and transportation costs.

Inventory centralization games optimize and allocate the savings from a centralized inventory, so that the participants' cooperation is maintained (Hartman et al. 2000). The risk pooling effect on (safety) stock levels evoked by inventory pooling or centralization can be quantified with the square root law (SRL), portfolio effect (PE), and inventory turnover curve. Maister's (1976) SRL states that the total system wide stock of n decentralized warehouses is

`

equal to that of a single centralized one multiplied with the square root of the number of warehouses. There is some confusion about which part of inventory (Heskett et al. 1974) it can be applied to and about its underlying assumptions (Sussams, 1986). Nevertheless, it applies to regular stock (Ballou, 2004), if an economic order quantity (EOQ) order policy is followed, the fixed cost per order and the per unit stock holding cost, demand at every location, and total system demand is the same both before and after centralization.

It holds for safety stock, if demands at the decentralized locations are uncorrelated, the variability (standard deviation) of demand at each decentralized location, the safety factor (safety stock multiple) (Maister, 1976) and average lead time are the same at all locations both before and after consolidation, average total system demand remains the same after consolidation (Evers and Beier, 1993), no transshipments occur (Zinn et al. 1989), lead times and demands are independent and identically distributed random variables and independent of each other (Evers and Beier, 1993), the variances of lead time are zero (Zinn et al. 1989), and the safety-factor ($k\sigma$) approach is used to set safety stock for all facilities both before and after consolidation (Maister, 1976). It applies to total inventory, the sum of regular and safety stock (Ballou, 2004), if the aforementioned assumptions of the SRL as applied to regular and safety stock hold collectively.

The savings in regular stock measured by the SRL stem from the assumption of constant fixed costs per order, holding costs, and total demand for all locations before and after centralization. Thus, if an EOQ policy is followed, the total order fixed cost usually is lower because of less orders and the inventory holding cost is lower due to a smaller total EOQ in the centralized than in the decentralized system. Savings in safety stock stem from the sub additively of the square root (the square root of the sum of the individual demand variances of the decentralized locations is usually less than the sum of the standard deviations of demand of the decentralized locations), i. e. from balancing demand variabilities (demand risk pooling). Schwarz (1981), Zinn et al. (1989), and Evers (1995) identify this as the portfolio effect.

Total cost savings from centralizing safety stock are probably larger than the ones from cycle stock centralization (Maister, 1976 & Evers 1995), because centralized cycle stocks have to be transported to the customer eventually, so that extra transportation costs are probably high. Centralized safety stocks only have to be transported to the customer in the less frequent case of a stock out, so that additional expenses for premium transportation are relatively small. If both

`

cycle and safety stocks are centralized some warehouses might be closed and fixed costs saved (Maister, 1976 & Evers 1995). It is difficult to isolate the effect of inventory centralization. The SRL shows the inventory necessary for a given service level in dependence of the number of stocking locations, if its underlying assumptions are fulfilled. This does not mean that reducing the number of warehouses automatically reduces inventory.

2.5 Findings of literature review

The findings of literature review have been summarized as:

Limited echelons of a multi-echelon inventory system are usually considered in the literature. They rarely generalize their models to N-echelon. Similarly, they usually consider serial systems, instead of a tree conformation. The authors generally assume demand and lead times to be stochastic, deterministic, constant, or negligible.

None of the aforementioned sources states whether the assumptions to the square root law (SRL) were fulfilled. Therefore actually no statements can be made about the accuracy of the square root law results. However, it seems that the square root law “tends to over-predict” (McKinnon, 1997) actual inventory savings, as it assumptions are not fulfilled.

The inventory required for a particular service level depends on the number of inventory stocking locations if inventory assumptions are satisfied. This does not mean that reducing the number of warehouses will automatically reduce inventory.

Heuristics, fuzzy logic, GA and other computational techniques are using for solving the different inventory control models. These techniques are not examined adequately yet in inventory management in multi-echelon supply chain. In addition, most of the research papers present mostly approximate models. There are small numbers of papers that give exact solutions.

There are some possibilities to explore inventory control problems with multi-criteria inventory problem. Risk pooling approach may be explored in multi-product multi-periods inventory control models. Product modularity concept is not very much explored with risk pooling approach. There is a possibility to integrate this approach with product modularity.

There are some chances to design inventory optimization problems along with multi-criteria inventory control problem. Inventory control models using risk pooling approach may be explored with multi-criteria inventory problems. There may be some opportunity to handle efficiently multi-criteria based network scenario selection problem while simultaneously considering inventory optimization problem using MADM approaches.

`

CHAPTER 3

SINGLE PRODUCT, SINGLE PERIOD, RISK POOLING INVENTORY CONTROL MODEL IN A DISTRIBUTION NETWORK

3.1 Introduction

Risk pooling is an effective approach in supply chain for controlling the inventory under uncertain demand (Gaur and Ravindran, 2006). This approach suggests that demand variability and the safety stock can be reduced, if a supply chain manager aggregates the demand across the different locations (Simchi-Levi, 2013). Inventories are obligatory realities in supply chain and it may play a significant role in supply chain efficiency (Monthatipkul & Chumpal 2008). The supply chain efficiency means that the operating cost should be low and the responsiveness should be high as much as possible (Minnich and Maier, 2007). Overall cost minimization cannot be the sole objective of a business firm, as it has to maintain adequate responsiveness as well in the supply chain.

The supply chain cost is a common term and every supply chain professional or academician can understand it, however it is important to explain the concept of responsiveness. Chopra and Meindl (2004) define supply chain responsiveness as; “Supply chain responsiveness includes a supply chain’s ability to do the following; Respond to wide ranges of quantities demanded; Meet short lead times; Handle a large variety of products; Build highly innovative products; Meet a very high service level; Handle supply uncertainty”. According to Holweg (2005), responsiveness can be defined as the “ability to react purposefully and within an appropriate time-scale to customer demand or changes in the marketplace, to bring about or maintain competitive advantage”. A responsive supply chain, in contrast, requires an information flow and policies from the market place to supply chain members in order to hedge inventory and available production capacity against uncertain demand (Fisher, 1997). According to Minnich (2007) responsiveness could be defined as “to raise the inventory levels of finished goods or components, which would allow more flexibility for reactions to changes in customer demand.” Responsiveness is designed to react quickly to satisfy customer demand.

`

Some of the previous researchers used responsiveness in terms of product miles and it is defined as "Product miles are the total distance produce is transported from its place of growth or production to the place of consumption. The concept of product miles is widely used in food production where there is a push by consumers for locally produced products." Gaur and Ravindran (2006) defined responsiveness in terms of product mils. According to them "It is the product (Multiplication) of total numbers of units of an item shipped from a distributor to retailer and the distance from distributor to retailer. Sirieix et al. (2007) defined it as "Food Miles" as the distance that foodstuff travels between the production location and the consumption marketplace.

The main objective of all the inventory managers is to reduce the inventory level to decrease the supply chain costs while maintaining even higher level of responsiveness. The main objective in front of a supply chain manager is to decide the optimal level of inventory while minimum supply chain costs, at the same time with higher level of responsiveness (Liao et al., 2011) and the most effective way for reducing the safety stock in supply chain is the risk pooling (Collier 1982). The risk pooling approach may be helpful to reduce the safety stocks in supply chain network, as a result the total expected inventory holding costs may be reduced (Eppen 1979). Safety stocks are maintained at various stages of supply chain for avoiding the stock out and these safety stocks improve the responsiveness but, they have adverse effect on the operational efficiency of the supply chain (Benton, 1991) due to the increment in inventory holding costs in supply chains (Gaur and Ravindran, 2006). Therefore, naturally there is tradeoff between cost and responsiveness. Sometimes it becomes very difficult to maintain the appropriate balance between cost and responsiveness in a supply chain distribution for a supply chain manager.

It may be stated that cost as well as responsiveness are important criteria's for deciding the supply chain efficiency and risk pooling approach is an effective way to reduce the safety stocks in supply chain network. The purpose of this study is to design a research problem using risk pooling for controlling inventory in supply chain network. A mathematical model is formulated to depict this situation. This model is tested with some numerical illustrations and input data. Current study provides the best level of risk pooling in a distribution network, best inventory policy (Ordering quantity and reorder point) and the optimized cost along with appropriate level of responsiveness in a distribution network.

`

3.2 Single product, single period, inventory control model using risk pooling approach in a distribution network

For including the criteria of cost and responsiveness, a mathematical model is formulated by using risk pooling approach. This model includes mainly two types of decision problems. First, it should provide the best inventory policy (Ordering quantity and reorder point) and minimum total cost. Secondly decision problem provides best level of aggregation among the various facilities along with appropriate responsiveness. For developing this model, the first objective function is formed for the total cost of the system and it includes the total inventory carrying costs, total cost for setting a distributor, total operating cost of a distributor and total cost of shipment from distributor to retailer.

Another term is also introduced here which is called as "Product Miles" and may be defined as the product of distance travelled by units from distributor to retailer and the demand fulfilled by distributor of retailer. Minimization of product miles is considered as maximization of responsiveness (Gaur and Ravindran, 2006). It means that lower demand can be fulfilled for those retailers which lie comparatively at a lower distance. In this case, lead time will be comparatively lower, because the distance between distributor and retailer is low. Larger demand can be fulfilled for those retailers which exist comparatively at a larger distance. Lead time will be higher due to larger distance in this concern.

For formulating a mathematical model for single product, single period risk pooling inventory control model for continuous review policy, a supply chain distribution network is considered between retailers and distributors. Let, there be 'm' distributors and 'n' retailers in a supply chain network. A specific distributor can fulfill the demand of various specific retailers, but a specific retailer can fulfill its demand by a specific single distributor only. It is assumed that all the distributors are capable of fulfilling the demand of all retailers situated in the supply chain network. This supply chain network is shown in Figure 3.1. Demand of retailers and lead time for replenishment are uncertain parameters and assumed to be normally distributed.

`

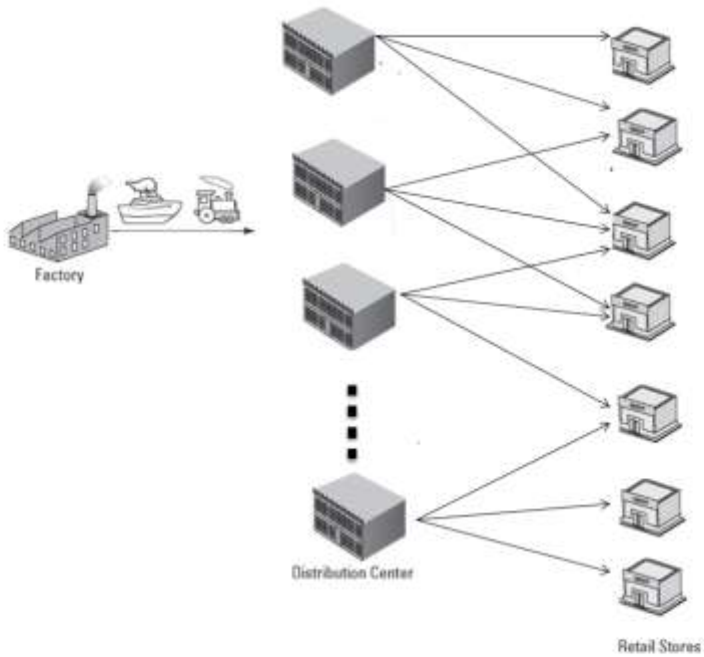


Figure 3.1: Distribution Network

Following assumptions considered taken for developing the model.

Assumptions:

1. Continuous review policy is adopted for review of inventory system.

2. Demand and lead time are independent random variables and assumed to be normally distributed.

3. The fill rate is assumed to be the same for all the distributors and defined as part of demand that is fulfilled from the immediate stock available without any backorder and lost sales.

4. The capacity of distributor to serve a retailer is predefined and considered as a parameter.

5. Single distributor can ship more than one retailer, but single retailer cannot be shipped by more than a single distributor. Partial fulfillment of orders is not allowed.

6. A particular distributor can fulfill the demand of some specific retailers only.

7. Back orders and lost sales are not allowed.

Sets:

Distributors, j: where j ε J, [J=1, 2…..j]

Retailer, k: where k ε K, [K=1, 2…...k]

`

Parameters:

1. F_j: Cost of establishment, it is the cost of establishing distributors in specific locations.
2. OP_j: Operating cost of distributor, it is the cost of running facility of distributor for each product
3. $T_{j,k}$: Cost of shipment, it is the cost for the transportation of one unit of product per kilometer from distributor to retailer
4. C: Cost of carrying the inventory, it is the cost of carrying the inventory at a distributor i per year
5. FO_j: Fixed order cost at distributor j
6. R_j: Average aggregated demands for a distributor j.
7. D_k: Demand of retailer k that has to be fulfilled by the distributor in supply chain
8. X_{j-k} : The distance between the distributor j and retailer k
9. μ_{xj}: Demand during lead time
10. $\rho_{r,s}$: The correlation coefficients between the demands of retailer 'r' and retailer's'.

Decision Variables:

1. Q_j: It shows the economic order quantity. It is the optimal order quantity, that is ordered by the retailer to the distributor in every order
2. Ro_j: It represents the reorder point of the distributor j.

Objective Function:

I. Minimize total anticipated cost = cost of establishment + fixed order cost + operating cost of distributor + cost of shipment + cost of carrying the inventory

Total anticipated cost:

1. **Cost of establishment:** It is the summation of all costs of establishment of all the distributors situated at different locations.

$$\text{Total operating cost of distributor} = \sum_j F_j$$

Where, F_j is the cost of establishment of a distributor.

2. **Operating cost of distributor:** It is the cost involved for running a distributor. It is equal to the product of operating cost per unit and the average demand of any item. It can be expressed mathematically as:

`

Total operating cost of distributor $= \sum_{j} \sum_{k} OP_j * R_j$

Where OP_j is the operating cost of distributor and R is the average daily demands for a distributor j

3. **Cost of shipment:** It is the cost of transportation involved for the units shipped from distributor j to retailer k and it is equal to the multiplication of three parameters. These three parameters are transportation cost per unit of item, distance from distributor to retailer and the demand of all retailers. It is expressed mathematically as

Total cost of shipment $= \sum_{j} \sum_{k} T_{j,k} * x_{j\text{-}k} * D_k$

Where, $T_{j,k}$ is the cost involved for the transportation of per unit per distance of product from distributor to retailer, $x_{j\text{-}k}$ is the distance between the distributor i and retailer j and D_k represents the demand of retailer k.

4. **Ordering Cost:** This is the cost involved when a distributor receives an order from retailers. It is defined as the product of number of orders and the ordering cost per unit.

The Number of order given per day by a distributor = Average daily demand/ordering quantity= R_j / Q

Where R_j = average daily demand of distributor, Q_j = economic order quantity.

Total number of annual orders = 365* R_j / Q_j

Total ordering cost$= \sum_{j} FO_j (365 * R_j / Q_j)$

Where FO_j is the fixed order cost for a distributor j.

1. **Cost of carrying the inventory:** It is the cost involved for maintain inventory in supply chain. It is the product of number of units hold in inventory and the inventory carrying cost per unit item.

5. Average cycle inventory = $Q_j/2$

Safety stock = $(Ro_j - \mu_{xj})$

Total Inventory in hand = $Q/2 + (Ro_j - \mu_{xj})$

Total Cost of carrying the inventory $= \sum_{j} \left\{ Q/2 + (Ro_j - \mu_{xj}) \right\} * C * v$

`

Where Ro_j is the reorder point, μ_{xj} is the average demand during lead time for a distributor j, C is the cost of carrying the inventory per unit item per year (in rupees) and v is the price of product..

Average demand for lead time for distributor (Chopra and Meindl, 2013): Suppose, that the lead time and the demand are independent unexpected variables,

L_j = average lead time from distributor to retailer

D_j= average demand for distributor j per unit time

μ_{ltd} = average demand for lead time

σ_{ltd} = standard deviation of demand for lead time

σ = standard deviation of demand

σ_L = standard deviation of lead time

The average of demand and the standard deviation of demand for lead time can be calculated as follows:

$\mu_{ltd} = D_j \text{ x } L_j$

$\sigma_{ltd} = (\sigma^2 \text{ x } L_j + \sigma_L^2 \text{ x } D_j^{\ 2})^{1/2}$

When a distributor is shipping more than one retailer, the average of aggregated demand and the standard deviation of aggregated demand are given as follows:

Let B_j is the set of all retailers served by distributor j

D_k = demands of retailers' k served by distributor

σ_r = standard deviation of demand of region r

σ_s = standard deviation of demand of region s

$\rho_{r,\, s}$ = Coefficient of correlation between the demand of retailers

R_j = Average Daily demands for a distributor i

$$R_j = \sum_{J \in B_i} D_k$$

σ_j = Standard deviation of aggregated demand

$\sigma_j = \sum_{(r,s)\varepsilon Bj}(\sigma_r^2 + \sigma_s^2 + 2\, \rho_{r,s}\, \sigma_r\, \sigma_s)^{1/2}$

If, a distributor is shipping more than one retailer and the demand of all retailers are aggregated, then the demand during lead may be represented as:

$\mu_{xj} = R_j \text{ x } L_j$

Where R_j is average aggregated demand for distributor i

`

The expression for Standard deviation during lead time is given below

$\sigma_{xj} = (\sigma_j^2 \text{ x } L_j + \sigma_L^2 \text{ x } R_j^{\,2})^{1/2}$

Where σ_{xj} is the aggregated demand's standard deviation for distributor j

Constraints:

1. The order quantity is always a non-negative quantity. The ordering quantity cannot be negative, it will be positive.

$Q_j \geq 0$ for all j

2. As, the demand uncertainty is considered in the model, so a safety stock is maintained in the system. This safety stock cannot be negative. Therefore, the reorder point should be higher than the average demand during lead time.

$Ro_j \geq \mu_{xj}$ for all j;

Here μ_{xj} is the average demand during the replenishment lead time (L) for a distributor 'j'.

3. Expected shortages in one cycle of order (ESC) are the average units of demand which is not satisfied from inventory in stock per replenishment cycle.

ESC= Expected shortfalls in one cycle of order (in between two orders) and it can be find out as

ESC = -SS*[1-NORMDIST (SS/σ_L, 0, 1, 1) + σ_L *NORMDIST (SS/σ_L, 0, 1, 0)

4. Fill rate may be defined as the fraction of demand of retailer satisfied by the distributor in a cycle of inventory.

Fill Rate (FR) = {1-ESC/Q}

Where Q = Economic order quantity

5. Expected shortfalls in one cycle of order (ESC) is always less than intended shortfalls in one cycle of order (in between two orders)

ESC ≤ ISC for all distributors

Where, ISC = (1-FR) x Q

6. Cycle service level is defined as the chance of not stocking out in a cycle and it is fixed as CSL≥0.95:

$$\text{CSL} = \text{NORMDIST}\left(Ro_j\ ,\ \mu_j\ ,\ \sigma_{Lj}\ ,\ 1\right)$$

`

Objective Function:

The main objective function of inventory control problem involves the various costs functions like ordering cost, cost of carrying the inventory, cost of shipments, cost of establishment of distributor and the operating cost of distributor.

Total inventory cost within the distribution network = Ordering cost + Cost of carrying the inventory + Cost of shipment of inventory from distributor to retailer + Cost of establishment of distributor + Operating cost of distributor.

Mathematical Representation of Model:

The first objective function of total anticipated cost (Z_1) =

$$\sum_j FO_i(365^* R_j/Q_j)+\sum_j \left\{Q_j/2+(Ro_j-\mu_{xj})\right\}^*C^*v+\sum_j \left(\sum_k 365^*D_k{}^*T_{j,k}{}^*x_{j-k}\right)+\sum_j F_j+\sum_j OP_j{}^*R_j)$$

Subjected to constraints:

$Q_j \geq 1$

$Ro_j \geq \mu_{xj}$

$ESC \leq ISC$

Where, $ISC = (1\text{-}FR) \times Q_j$

Fill Rate $(FR) = \{1\text{-}ESC/Q_j\}$

$CSL \geq 0.95$

Product miles: The product-miles may be defined as the product of demand and distance. If $X_{j\text{-}k}$ is the distance from distributor j to retailer k, than

Product miles or responsiveness = $\left(\sum_k \sum_j X_{j-k} * D_k\right)$

3.3 Numerical illustration:

For solving above risk pooling problem in a distribution network, a numerical problem is illustrated. In this problem, 5 distributors and 7 retailers are considered in a network (Figure 3.2). It is assumed that these distributors are capable of fulfilling the demand of all 7 retailers. The fill rate is assumed to be the same for all distributors. The capacity of the distributor is a parameter. A condition is imposed in the network, that a fixed distributor can fulfill the demand of some specific retailers only. This numerical illustration provides decision on mainly two problems, first one is to provide the numbers distributors are that actually working in the network after

`

applying risk pooling approach for fulfilling the demands of all retailers and the second one provides the best inventory policy and minimum cost with appropriate responsiveness. As discussed earlier that, 5 distributors and 7 retailers are considered in the network for numerical illustration. Let, these distributors are denoted by D1, D2, D3, D4 and D5. The 7 retailers are shown by, R1, R2, R3, R4, R5, R6 and R7 as shown in Figure 3.2

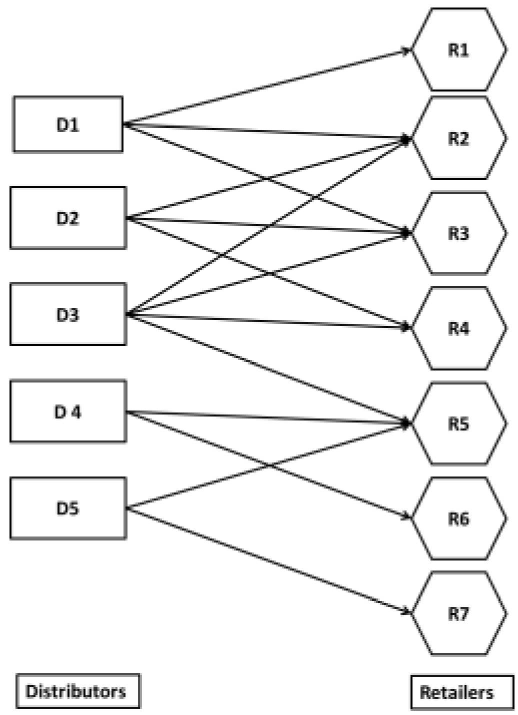


Figure 3.2: Distribution network understudy

Some numerical values are required for performing the illustration. Primarily, important data are shown in Table 3.1 which provides fixed ordering cost, cost of shipment from distributor to retailer (Transportation cost from distributor to retailer), inventory carrying cost (inventory holding cost), value of each item, and the lead time of shipment and the standard deviation of lead time.

Table 3.1: Some necessary parameters

Sr. No.	Parameters	Values
1	Fixed ordering cost (Fo_j)	Rs. 650 per/order
2	Cost of shipment (T)	Rs. 0.45 per unit per kilometer

`

Sr. No.	Parameters	Values
3	Opportunity cost of carrying the inventory (C)	Rs.0.25 per unit
4	Price of each item (v)	Rs.55 per unit
5	Lead time	4 hours
6	Standard deviation of lead time	0.2 hours

Table 3.2 provides the cost of setting of distributor (Facility cost) and the cost of operation of all the 5 distributors.

Table 3.2: Facility cost and operating cost

Distributor	**Cost of establishment (F_j) (Rs)**	**Operating cost of distributor (OP_j) (Rs)**
1	50000	6
2	35000	7
3	85000	5
4	35000	5
5	40000	6

Table 3.3 represents the distance (KM) from distributor i to retailer j. It show the distances from a particular distributor to some specific retailers. The values of the distances shown in the Table 3.3 are according to the network (Figure 3.2), which actually represents how many distributor and retailers are working in network.

Table 3.3: Distance of a distributor from a retailer

Distributor	**Retailers**						
	1	**2**	**3**	**4**	**5**	**6**	**7**
1	45	55	65				
2		35	40	55			
3			65	35	35	65	
4					55	35	
5					85		70

`

Table 3.4 shows the monthly average demand and the standard deviations of demand for all 7 retailers. It is assumed that the demands of retailers are random variables and are normally distributed. Hence, Table 3.4 show demands of retailer and the standard deviations of demands.

Table 3.4: Average demand of retailers and their standard deviations

Retailer	Average Demand (per month)	Standard deviation of demand
1	2500	75
2	3500	50
3	2500	50
4	3000	75
5	2500	100
6	4000	100
7	3500	75

Table 3.5 represents the coefficient of correlation among the demand of retailers, this shows that the demand of a retailer is dependent on the demand of other retailers and this may dependent on the relative distances among the retailers.

Table 3.5: Coefficient of correlations among the demand of retailers

Retailers	1	2	3	4	5	6	7
1	1	0.3	0.1	0.5	0.9	.1	0.4
2	0.3	1	0	2	0	.5	0.3
3	0.1	0	1	0.1	0.4	.7	0.2
4	0.5	1	0.1	1	0.3	.25	0.5
5	0.9	0	0.4	0.3	1	0	0.2
6	0.1	0.5	0.5	0.25	0.3	1	0.1
7	0.4	0.3	0.2	0.5	0.4	0.6	1

3.4. Solution procedure: For solving the above problem a distribution network is considered in a supply chain as shown in Figure 3.2. It is assumed that there are 5 distributors and 7 retailers in the network. These five distributors can be denoted by D1, D2, D3, D4 and D5 and the retailers in the given network are denoted by R1, R2, R3, R4, R5, R6 and R7 as discussed above. These 5

`

distributors are capable to fulfill the demand of all 7 retailers. According to this network, (Figure 3.2) all possible options of demand fulfillment of all 7 retailers may be that the retailers R1, R2 and R3 can be served by distributor D1. Retailers R2, R3 and R4 can be served by distributor D2. Retailers R3, R4, R5 and R6 can be served by distributor D3. Retailers R5 and R6 can be served by distributor D4. Retailers R5 and R7 can be served by distributor D5. Here, a condition is imposed in the network; a fixed distributor can fulfill the demand of some specific retailers only and a particular retailer can fulfill its demand by a single and a specific distributor or partial fulfillment for retailers is not allowed. Therefore, by imposing the above condition in the network, a series of all possible alternative networks may be generated for all probable options of demand fulfillment for all retailers (Table 3.6). Total 20 possible options are formed for the fulfillment of all 7 retailers by these 5 distributors. In Alternative-1, the demands of retailers R1 and R2 can be fulfilled by distributor D1, demand of retailer R3 can be fulfilled by distributor D2, demands of retailer R4, R5 and R6 can be fulfilled by distributor D3, D4 is inactive and demand of retailer R7 can be fulfilled by D5. In this case the demand of all 7 retailers can be fulfilled by distributors D1, D2, D3 and D5.

Similarly, all the possible combinations of the supply chain network are made by imposing the above given condition, in this problem, at this stage; the concept of risk pooling is introduced. If, more than one retailer is fulfilled by a distributor, than risk pooling approach is applied to the set of all retailers. For alternative 1, risk pooling is applied on retailers R1 and R2 for distributor D1; similarly risk pooling is applied for retailers R4, R5 and R6 with distributor D3. For distributor D2 and D4, no risk pooling is applied, because distributor D2 is serving for single retailer R3 and D4 is inactive in this option, as all the retailers are served by distributors D1, D2, D3 and D5. In the same way, all possible alternatives and the risk pooling is applied to all the alternatives. Following the same procedure for all 20 possible alternatives, the risk pooling approach is introduced in each alternative. Table 3.6 shows all the 20 alternatives and the options of demand fulfillment of all 7 retailers by 5 distributors.

`

Table 3.6: Possible alternatives of distribution networks

Alternatives	Retailers served by distributor D1	Retailers served by distributor D2	Retailers served by distributor D3	Retailers served by distributor D4	Retailers served by distributor D5
Alternative-1	R1, R2	R3	R4, R5, R6	-	R7
Alternative-2	R1, R2	R3	R4, R5	R6	R7
Alternative-3	R1, R2	R3	R4	R5, R6	R7
Alternative-4	R1, R2	-	R3, R4, R5, R6	-	R7
Alternative-5	R1, R2	-	R3, R4, R5	R6	R7
Alternative-6	R1, R2	-	R3, R4	R5, R6	R7
Alternative-7	R1, R2	-	R3, R4	R6	R5, R7
Alternative-8	R1, R3	R2	R4, R5, R6	-	R7
Alternative-9	R1, R3	R2	R4, R5	R6	R7
Alternative-10	R1, R3	R2	R4	R5, R6	R7
Alternative-11	R1, R3	R2	R4	R6	R5, R7
Alternative-12	R1, R2, R3	-	R4, R5, R6	-	R7
Alternative-13	R1, R2, R3	-	R4, R5	R6	R7
Alternative-14	R1, R2, R3	-	R4	R5, R6	R7
Alternative-15	R1, R2, R3	-	R4	R6	R5, R7
Alternative-16	R1	R2, R3	R4, R5, R6	-	R7
Alternative-17	R1	R2	R3, R4, R5, R6	-	R7
Alternative-18	R1	R2	R3, R4, R5	R6	R7
Alternative-19	R1	R2	R3, R4	R5, R6	R7
Alternative-20	R1	R2	R3, R4	R6	R5, R7

For solving the above problems, nonlinear programming is solved in AIMMS tool, because the main objective function is nonlinear and this problem is solved in CONPOT solver on an Intel (R) Core (TM)2 Duo CPU T 6570 with memory 4GB for all twenty alternatives of demand fulfillment. The outcomes of the optimization are total cost, product miles, reorder point,

`

ordering quantity for all distributors. The Table 3.6 shows all possible combinations of demand fulfillment of all 7 retailers.

3.5 Results of optimization: The problem is NLP type and solved through CONPOT solver in AIMMS software on an Intel (R) Core (TM)2 Duo CPU T 6570 with memory 4GB for total cost minimization. This problem contains 221 constraints and 221 variables and the solver results are shown in Table 3.7.

Table 3.7: Solver result

Constraints	221
Variables	221
None zero	522
Model type	NLP
Direction	Minimize
Solver	CONPOT 3.14V
Phase	3
Iteration	4
Max Gradient	9.09e-0.13
Objective	95426243
Best solution	95426243
Solving time	0.12 sec
Program Status	Locally optimal
Solver status	Normal completion

And the results provided by the software are shown in the following tables. Table 3.8 shows the aggregated demand of all 5 distributors.

`

Table 3.8: Aggregated Demand

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	6000	2500	9500		3500
Alternative-2	6000	2500	5500	4000	3500
Alternative-3	6000	2500	5500	4000	3500
Alternative-4	6000		12000		3500
Alternative-5	6000		8000	4000	3500
Alternative-6	6000		5500	6500	3500
Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-7	6000		5500	4000	6000
Alternative-8	5000	3500	9500		3500
Alternative-9	5000	3500	5500	4000	3500
Alternative-10	5000	7000	3000	6500	3500
Alternative-11	5000	3500	3000	4000	6000
Alternative-12	8500		9500		3500
Alternative-13	8500		5500	4000	3500
Alternative-14	8500		3000	6500	3500
Alternative-15	8500		3000	4000	6000
Alternative-16	2500	6000	9500		3500
Alternative-17	2500	3500	12000		3500
Alternative-18	2500	3500	8000	4000	3500
Alternative-19	2500	3500	5500	6500	3500
Alternative-20	2500	3500	5500	4000	6000

Table 3.9 shows the ordering quantities of all 5 distributors. This is the minimum ordering quantity given by every distributor in each alternative.

`

Table 3.9: Ordering Quantities (Q)

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	9674	6244	12173		7389
Alternative-2	9674	6244	9262	7899	7389
Alternative-3	9674	6244	6841	10070	7389
Alternative-4	9674		13682		7389
Alternative-5	9674		11171	7899	7389
Alternative-6	9674		9262	7899	9674
Alternative-7	9674		9262	7899	9674
Alternative-8	8831	7389	12173		7389
Alternative-9	8831	7389	9262	7899	7389
Alternative-10	8831	10449	6841	10069	7389
Alternative-11	8831	7389	6841	7899	9674
Alternative-12	11515		12173		7389
Alternative-13	11515		9262	7899	7389
Alternative-14	11515		6841	10069	7389
Alternative-15	11515		6841	7899	9674
Alternative-16	6245	9674	12173		7389
Alternative-17	6245	7389	13682		7389
Alternative-18	6245	7389	11171	7899	7389
Alternative-19	6245	7389	9262	10069	7389
Alternative-20	6245	7389	9263	7899	9674

Table 3.10 shows the reorder point of all 5 distributors. This is the minimum quantity of inventory in hand, when a distributor replaces a new order.

`

Table 3.10: Re-order Point (Ro)

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	965	407	1519		573
Alternative-2	965	407	915	664	573
Alternative-3	965	407	498	1065	473
Alternative-4	965		1936		573
Alternative-5	965		1305	664	573
Alternative-6	965		885	1065	573
Alternative-7	965		885	664	987
Alternative-8	810	557	1543		573
Alternative-9	810	557	915	664	573
Alternative-10	810	1082	498	1065	573
Alternative-11	810	557	498	664	987
Alternative-12	1349		1542		573
Alternative-13	1349		915	664	573
Alternative-14	1349		498	1065	573
Alternative-15	1349		498	664	987
Alternative-16	423	945	1542		573
Alternative-17	423	557	1936		573
Alternative-18	423	557	1305	664	573
Alternative-19	423	557	885	1065	573
Alternative-20	923	557	885	664	987

Table 3.11 shows the mean demand during lead time for all 5 distributors. It is obtained by the product of demand and Lead time ($\mu = D * L$)

`

Table 3.11: Mean Demand during Lead time (μ)

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	900	375	1425		525
Alternative-2	900	375	825	600	525
Alternative-3	900	375	450	975	525
Alternative-4	900		1800		525
Alternative-5	900		1200	600	525
Alternative-6	900		825	975	525
Alternative-7	900		825	600	900
Alternative-8	750	525	1425		525
Alternative-9	750	525	825	600	525
Alternative-10	750	1050	450	975	525
Alternative-11	750	525	450	600	900
Alternative-12	1275		1425		525
Alternative-13	1275		825	600	525
Alternative-14	1275		450	975	525
Alternative-15	1275		450	600	900
Alternative-16	375	900	1425		525
Alternative-17	375	525	1800		525
Alternative-18	375	525	1200	600	525
Alternative-19	375	525	825	975	525
Alternative-20	375	525	825	600	900

Table 3.12 shows standard deviations of aggregated demand for all alternatives and the standard deviations for aggregated demand is given by $\sigma_j = \sum_{(r,s)\varepsilon Bj}(\sigma_r^2 + \sigma_s^2 + 2\, \rho_{r\text{-}s}\, \sigma_r\, \sigma_s)^{1/2}$

`

Table 3.12: Standard deviations for aggregated demand (σ_j)

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	101.857	50	147.05		75
Alternative-2	101.8577	50	141.8626	100	75
Alternative-3	101.8577	50	75	141.421	75
Alternative-4	101.8577	0	213	0	75
Alternative-5	101.8577		165.27	100	75
Alternative-6	101.8577		94.28	141.421	75
Alternative-7	101.8577		94.2	100	136.473
Alternative-8	94.2	50	184.501		75
Alternative-9	94.2	50	141.862	100	75
Alternative-10	94.2	50	75	141.421	75
Alternative-11	94.2	50	75	100	136.473
Alternative-12	116.72		184.051		75
Alternative-13	116.72		141.862	100	75
Alternative-14	116.72		75	141.421	75
Alternative-15	116.72		75	100	136.473
Alternative-16	75	70.71	184.051		75
Alternative-17	75	50	213		75
Alternative-18	75	50	165.27	100	75
Alternative-19	75	50	94.28	141.421	75
Alternative-20	75	50	94.28	100	136.473

Table 3.13 shows standard deviations of aggregated demand during the lead time for all alternatives and it may be find as, $\sigma_{xj} = (\sigma_j^2 \text{ x } L + \sigma_L^2 \text{ x } \overline{D}^2)^{1/2}$

`

Table 3.13: Standard deviations for aggregated demand during Lead Time (σ_{xj})

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	302.582596	126.491106	478.402086		177.394335
Alternative-2	302.582631	126.491106	280.434929	203.715488	177.394335
Alternative-3	302.582631	126.491106	152.786616	329.583047	177.394335
Alternative-4	302.582631		605.644574		177.394335
Alternative-5	302.582631		405.089035	203.715488	177.394335
Alternative-6	302.582631		277.413604	329.583047	177.394335
Alternative-7	302.582631		277.409528	203.715488	304.620636
Alternative-8	252.648068	176.068169	480.344765		177.394335
Alternative-9	252.648068	176.068169	280.434884	203.715488	177.394335
Alternative-10	252.648068	350.535305	152.786616	329.583047	177.394335
Alternative-11	252.648068	176.068169	152.786616	203.715488	304.620636
Alternative-12	427.397396		480.318869		177.394335
Alternative-13	427.397396		280.434884	203.715488	177.394335
Alternative-14	427.397396		152.786616	329.583047	177.394335
Alternative-15	427.397396		152.786616	203.715488	304.620636
Alternative-16	128.330628	301.247383	480.318869		177.394335
Alternative-17	128.330628	176.068169	605.644574		177.394335
Alternative-18	128.330628	176.068169	405.089035	203.715488	177.394335
Alternative-19	128.330628	176.068169	277.413604	329.583047	177.394335
Alternative-20	128.330628	176.068169	277.413604	203.715488	304.620636

Table 3.14 provides the total cost and product-miles for all 20 alternatives.

`

Table 3.14: Total cost (TC) and product-miles (PM) for all alternatives

Alternatives	Product Miles	Total Cost	Alternatives	Product Miles	Total Cost
Alternative 1	1102500	14976667	Alternative 11	1100000	15030178
Alternative 2	982500	13576954	Alternative 12	1165000	15661694
Alternative 3	1032500	14175109	Alternative 13	1045000	14261981
Alternative 4	1165000	6341245	Alternative 14	1095000	14860129
Alternative 5	1045000	14232483	Alternative 15	1170000	15790978
Alternative 6	1095000	14835709	Alternative 16	1032500	14178674
Alternative 7	1170000	15765256	Alternative 17	1095000	14862950
Alternative 8	1095000	14900894	Alternative 18	975000	13466187
Alternative 9	975000	13501181	Alternative 19	1025000	14068113
Alternative 10	1147500	15885898	Alternative 20	1100000	14998967

At this stage, some other variables are also calculated with the help of excel solver for all the twenty possible alternatives. These variables are Safety stock, Average Inventory and Inventory Positions. The methods for finding, Safety Stock (SS), Average Inventory (AI) and Inventory Positions (IP) have been discussed below (Chopra and Meindl, 2013):

Safety Stocks (SS): Safety stocks is a term used by inventory managers to describe a level of extra stock which is maintained to mitigate risk of shortfalls (stock outs) due to uncertainties in demand and supply. The method of finding of safety stock is shown below

Safety Stock (SS) = Reorder point- Mean demand during lead time = Ro-μ = z* σ*SQRT (Lead Time)

Average Inventory (AI): Average inventory is defined as the summation of cycle inventory and the safety stocks and it is found out as:

Average Inventory (AI) = Cycle Inventory + Safety stock= Q/2 + SS

Inventory Positions (IP): Inventory positions is defined as the summation of ordering quantity and the reorder point and it is found out as

Inventory Positions (IP) = Ordering quantity + Reorder point= Q + Ro

`

All these parameters SS, AI and IP are treated as the performance parameters for an inventory system. The following tables depict these parameters. Table 3.15 shows the Cycle Service level for all alternatives.

Table 3.15 shows the safety stocks for all alternatives.

Table 3.15: Safety Stocks (SS)

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	65	32	94	0	48
Alternative-2	65	32	90	64	48
Alternative-3	65	32	48	90	-52
Alternative-4	65	0	136	0	48
Alternative-5	65	0	105	64	48
Alternative-6	65	0	60	90	48
Alternative-7	65	0	60	64	87
Alternative-8	60	32	118	0	48
Alternative-9	60	32	90	64	48
Alternative-10	60	32	48	90	48
Alternative-11	60	32	48	64	87
Alternative-12	74	0	117	0	48
Alternative-13	74	0	90	64	48
Alternative-14	74	0	48	90	48
Alternative-15	74	0	48	64	87
Alternative-16	48	45	117	0	48
Alternative-17	48	32	136	0	48
Alternative-18	48	32	105	64	48
Alternative-19	48	32	60	90	48
Alternative-20	548	32	60	64	87

Table 3.16 shows the Average Inventory

`

Table 3.16: Average Inventory (AI)

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	4902	3154	6181	0	3743
Alternative-2	4902	3154	4721	4014	3743
Alternative-3	4902	3154	3469	5125	3643
Alternative-4	4902	0	6977	0	3743
Alternative-5	4902	0	5691	4014	3743
Alternative-6	4902	0	4691	4040	4885
Alternative-7	4902	0	4691	4014	4924
Alternative-8	4476	3727	6205	0	3743
Alternative-9	4476	3727	4721	4014	3743
Alternative-10	4476	5257	3469	5125	3743
Alternative-11	4476	3727	3469	4014	4924
Alternative-12	5832	0	6203.5	0	3743
Alternative-13	5832	0	4721	4014	3743
Alternative-14	5832	0	3469	5125	3743
Alternative-15	5832	0	3469	4014	4924
Alternative-16	3171	4882	6204	0	3743
Alternative-17	3171	3727	6977	0	3743
Alternative-18	3171	3727	5691	4014	3743
Alternative-19	3171	3727	4691	5125	3743
Alternative-20	3671	3727	4692	4014	4924

Table 3.17 shows the Inventory Positions (IP)

`

Table 3.17: Inventory Positions (IP)

Alternatives	**Distributor-1**	**Distributor-2**	**Distributor-3**	**Distributor-4**	**Distributor-5**
Alternative-1	10639	6651	13692	0	7962
Alternative-2	10639	6651	10177	8563	7962
Alternative-3	10639	6651	7339	11135	7862
Alternative-4	10639	0	15618	0	7962
Alternative-5	10639	0	12476	8563	7962
Alternative-6	10639	0	10147	8964	10247
Alternative-7	10639	0	10147	8563	10661
Alternative-8	9641	7946	13716	0	7962
Alternative-9	9641	7946	10177	8563	7962
Alternative-10	9641	11531	7339	11134	7962
Alternative-11	9641	7946	7339	8563	10661
Alternative-12	12864	0	13715	0	7962
Alternative-13	12864	0	10177	8563	7962
Alternative-14	12864	0	7339	11134	7962
Alternative-15	12864	0	7339	8563	10661
Alternative-16	6668	10619	13715	0	7962
Alternative-17	6668	7946	15618	0	7962
Alternative-18	6668	7946	12476	8563	7962
Alternative-19	6668	7946	10147	11134	7962
Alternative-20	7168	7946	10148	8563	10661

Table 3.18 shows expected shortages per cycle (ESC)

`

Table 3.18: Expected Shortages per Cycle (ESC)

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	0.82407	0.4599	1.1899		0.606886
Alternative-2	0.82413	0.40459	1.147925	0.809181	0.606886
Alternative-3	0.82413	0.40459	0.606886	1.44352	0.606886
Alternative-4	0.82413		1.723555		0.606886
Alternative-5	0.82413		1.337330	0.809181	0.606886
Alternative-6	0.82413		0.762896	1.143520	0.606886
Alternative-7	0.82413		0.762248	0.809181	1.104313
Alternative-8	0.762248	0.40459	1.492940		0.606886
Alternative-9	0.762248	0.40459	1.14792	0.809181	0.606886
Alternative-10	0.762248	0.40459	0.606886	1.144352	0.606886
Alternative-11	0.762248	0.40459	0.606886	0.809181	1.104313
Alternative-12	0.9444716		1.489305		0.606886
Alternative-13	0.944476		1.14792	0.809181	0.606886
Alternative-14	0.944476		0.606886	1.144352	0.606886
Alternative-15	0.944476		0.606886	0.809181	1.104313
Alternative-16	0.606886	0.572172	1.489305		0.606886
Alternative-17	0.606886	0.40459	1.723555		0.606886
Alternative-18	0.606886	0.40459	1.337333	0.809181	0.606886
Alternative-19	0.606886	0.40459	0.762896	1.44352	0.606886
Alternative-20	0.606886	0.40459	0.762896	0.809181	1.104313

Table 3.19 shows the fill rate (FR)

`

Table 3.19: Fill Rate (FR)

Alternatives	Distributor-1	Distributor-2	Distributor-3	Distributor-4	Distributor-5
Alternative-1	0.999915	0.999935	0.999902		0.999918
Alternative-2	0.999915	0.999935	0.999876	0.999898	0.999918
Alternative-3	0.999915	0.999935	0.999911	0.999886	0.999918
Alternative-4	0.999915		0.999874		0.999918
Alternative-5	0.999915		0.99988	0.999898	0.999918
Alternative-6	0.999915		0.999918	0.999855	0.999937
Alternative-7	0.999915		0.999918	0.999898	0.999886
Alternative-8	0.999914	0.999945	0.999877		0.999918
Alternative-9	0.999914	0.999945	0.999876	0.999898	0.999918
Alternative-10	0.999914	0.999961	0.999911	0.999886	0.999918
Alternative-11	0.999914	0.999945	0.999911	0.999898	0.999886
Alternative-12	0.999918		0.999878		0.999918
Alternative-13	0.999918		0.999876	0.999898	0.999918
Alternative-14	0.999918		0.999911	0.999886	0.999918
Alternative-15	0.999918		0.999911	0.999898	0.999886
Alternative-16	0.999903	0.999941	0.999878		0.999918
Alternative-17	0.999903	0.999945	0.999874		0.999918
Alternative-18	0.999903	0.999945	0.99988	0.999898	0.999918
Alternative-19	0.999903	0.999945	0.999918	0.999886	0.999918
Alternative-20	0.999903	0.999945	0.999918	0.999898	0.999886

3.6 TOPSIS approach for selecting best inventory policy:

There are mainly two criteria's named "total expected cost and service level measures" against twenty alternatives of distribution network. Again service level can be divided into fill rate, cycle

`

service level and product-miles. Here, it is implicit to select the best inventory policy among all scenarios and the main objective is to select the best inventory policy against given performance parameters among all the alternatives. The results obtained through the first stage are quite close and similar. Due to the similarity of solutions in the first stage results, the TOPSIS approach is endorsed here for selecting the best inventory policy. TOPSIS is the acronym for the “Technique for order preference by similarity to ideal solution” and this approach has been suggested by (Hwang et al., 1981). A group decision environment is investigated in TOPSIS (technique for order performance by similarity to ideal solution) approach. TOPSIS is a useful and practical technique for ranking and selecting a number of externally identified alternatives through distance measurements. Normalization, weighted normalization, positive and negative ideal solution, distance metrics, and closeness coefficient are some operational steps in TOPSIS approach which provide the broad understanding of this technique. In addition, the preferences of more than one decision maker are internally aggregated into the TOPSIS procedure. It is readily applicable to many real-world decision making situations without increasing the computational burden. This approach is widely accepted in various multi-attribute decision-making problems. The Mathematical steps involved in TOPSIS methods are discussed here (Moghassem, 2010):

Step 1: **To establish the decision matrix:**

First of all weights of criteria weights are evaluated and a decision matrix is constructed between the criteria's and scenarios. The 4 criteria's are denoted by $C_{1,}$ $C_{2,}$ C_3 and C_n and the 'm' alternatives and the decision matrix is denoted by DM.

$$\begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_n \end{array} \\ DM= & \begin{pmatrix} x_{11} & x_{12}\cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & x_{m2}\cdots & x_{mn} \end{pmatrix} \end{array}$$

Step 2: To calculate a normalized decision matrix

In this step normalized decision matrix (NDM) is found out. The mathematical expression for finding normalized decision matrix (NDM) can be shown as:

`

$$r_{ij} = \frac{x_{ij}}{\sqrt{(\sum_{i=1}^{m} x_{ij})^{1/2}}}$$

Step 3: To determine the weighted normalized decision matrix

The weighted normalized decision matrix is found by multiplying the weights of criteria and the normalized decision matrix.

$$V_{ij} = w_{ij} x r_i$$

Where

$$\sum w_{ij} = 1$$

Step 4: To determine the Positive and Negative Ideal Solution

The positive ideal (A^+) and the negative ideal (A^-) solutions are defined according to the weighted decision matrix

$$PIS = A^+ = \{V_1^{\ +}, V_2^{\ +} \ldots, V_n^{\ +}\}, \text{ where: } V_j^{\ +} = \{(\max (V_{ij}) \text{ if } j \in J); (\min V_{ij} \text{ if } j \in J')\}$$

$$NIS = A- = \{V_1^{\ -}, V_2^{\ -} \ldots, V_n^{\ -}\}, \text{ where: } V_j^{\ -} = \{(\min (V_{ij}) \text{ if } j \in J); (\max V_{ij} \text{ if } j \in J')\}$$

Where, J is associated with the beneficial criteria and J' is associated with the cost criteria.

Step 5: To calculate the separation distance of each competitive scenario from the positive and negative ideal solution.

$$SM^+ = \sqrt{\sum_{j=1}^{n} \left(V_{ij} - V_j^{+}\right)^2}$$

$$SM^- = \sqrt{\sum_{j=1}^{n} \left(V_{ij} - V_j^{-}\right)^2}$$

Step 6: To measure the relative closeness of each location to the ideal solution.

The relative closeness with respect to the ideal solution for each scenario is evaluated:

$$C_i = \frac{SM^-}{SM^- + SM^+}$$

Step 7: Rank the preference order: The highest value of C_i shows that this solution is at the farthest distance from negative ideal solution (NIS) which is the desired alternative. For this, the rank is provided to the all alternatives in the decreasing order of the relative closeness from ideal solution. The alternative having least rank is the probable desired answer.

`

For implementing TOPSIS approach, total five responses have been collected. Three responses are collected from industrial professionals and two from academia which are shown in the following Tables. Table 3.20 shows the average criteria weights and Table 3.21 shows the average ratings of scenarios with respect to these 4 criteria's.

Table 3.20: Average criteria's weight

	Total cost (C1)	**Responsive ness (C2)**	**Fill Rate (C3)**	**Service level (C4)**
Criteria weights	0.4	0.2	0.2	0.2

Table 3.21: Average ratings of alternatives

	Total cost (C1)	**Responsive ness (C2)**	**Fill Rate (C3)**	**Service level (C4)**
Alternative-1	9	2	2	1
Alternative-2	8	2	2	1
Alternative-3	7	2	2	1
Alternative-4	8	2	2	1
Alternative-5	9	2	2	1
Alternative-6	9	2	2	1
Alternative-7	8	2	2	1
Alternative-8	7	2	2	1
Alternative-9	9	2	2	1
Alternative-10	9	2	2	1
Alternative-11	5	2	2	1
Alternative-12	2	1.5	1.5	1
Alternative-13	15	1.5	1.5	1
Alternative-14	8	1.5	1.5	1
Alternative-15	8	1.5	1.5	1
Alternative-16	8	3	2	2
Alternative-17	9	3	2	2
Alternative-18	8	3	2	2
Alternative-19	8	3	2	2
Alternative-20	8	3	2	2

`

From the above two matrices (Table 3.20 and Table 3.21), normalized decision matrix, weighted normalized decision matrix, positive ideal solution, negative ideal solution, measure of separation and closeness coefficient are calculated. At the end, rank is provided to the closeness coefficient and best scenario of inventory policy is selected. The closeness coefficients for all scenarios are shown in Table 3.22.

Table 3.22: Closeness Coefficient

Alternatives	Closeness Coefficient
Alternative-1	0.082965
Alternative-2	0.701563
Alternative-3	0.760868
Alternative-4	0.760868
Alternative-5	0.701563
Alternative-6	0.670640
Alternative-7	0.701563
Alternative-8	0.760868
Alternative-9	0.731162
Alternative-10	0.670640
Alternative-11	0.621025
Alternative-12	0.800829
Alternative-13	0.647527
Alternative-14	0.492086
Alternative-15	0.721292
Alternative-16	0.751116
Alternative-17	0.747815
Alternative-18	0.747815
Alternative-19	0.784731
Alternative-20	0.784731

From Table 3.22 it is clear that, alternative 12 has highest closeness coefficient which is equal to 0.800829. This alternative provides the 95% service level and more than 99 % fill rate with

`

appropriate weights to the performance criterions like, total cost, service level, fill rate and product-miles.

3.7 Discussion

From the previous section, alternative 12 achieved the highest value of closeness coefficient (CC_i) which is 0.800829. So it is at the farthest distance from negative ideal solution. Therefore, only alternative 12 needs to be considered for result analysis. The alternative 12 is shown in Figure 3.3.

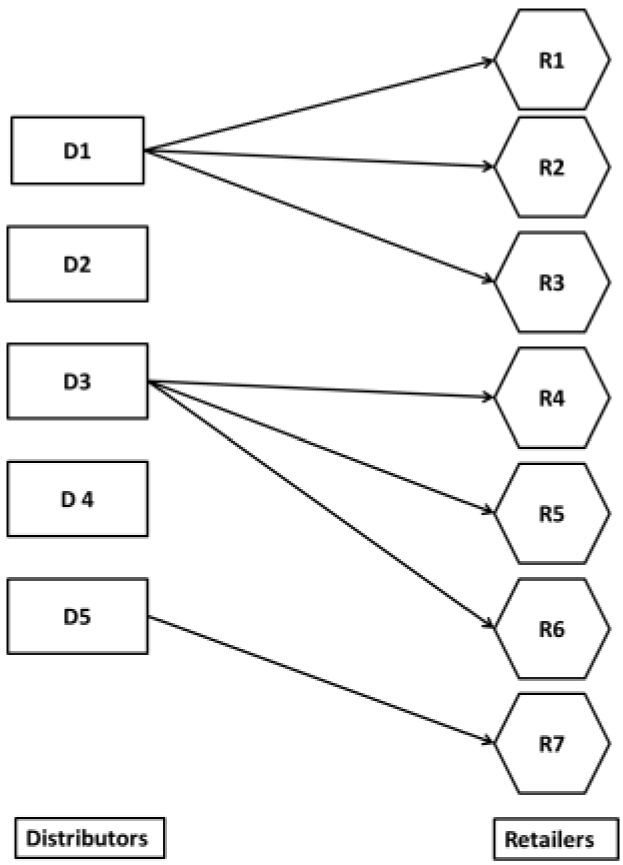


Figure 3.3: Alternative 12

In this alternative, retailers R1, R2 and R3 are pooled for distributor D1. Retailers R4, R5, and R6 are pooled for D3 and R7 are only for D5. For analysis of this alternative, four levels of uncertainty are considered. In this section, the effect of demand uncertainty, the effect of supply uncertainty and the combined effect of both types of uncertainty are examined on performance parameters.

3.7.1 Effect of demand uncertainty on performance parameters

For evaluating the effect of demand uncertainty on performance parameters, four levels of uncertainty are considered. The levels of uncertainties are 0.2, 0.4, 0.6 and 0.8. The parameters considered for analysis are reordering point, safety stock, anticipated shortages per cycle; fill rate, average inventory and the expected total cost. The effects of four levels of demand uncertainty on performance parameters are shown in the following Figures.

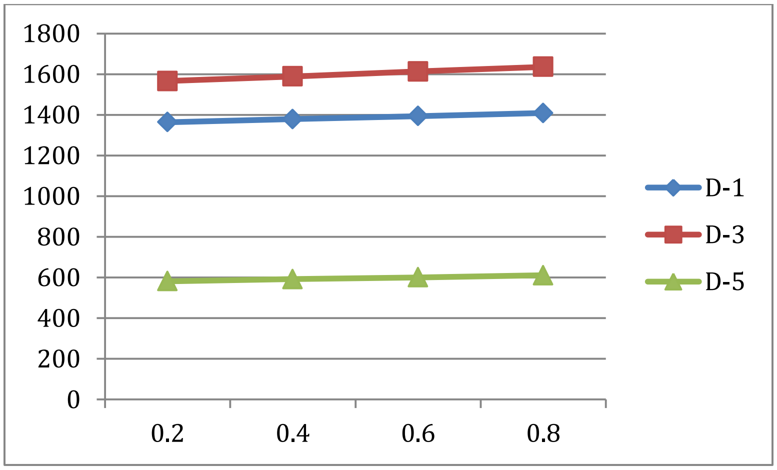


Figure 3.4: Effect of demand uncertainty on ROP

From Figure 3.4, it is clear that there is a very nominal increment in the reorder point in accordance to the demand uncertainty level which increases from 0.2 to 0.4 as it seems almost horizontal line.

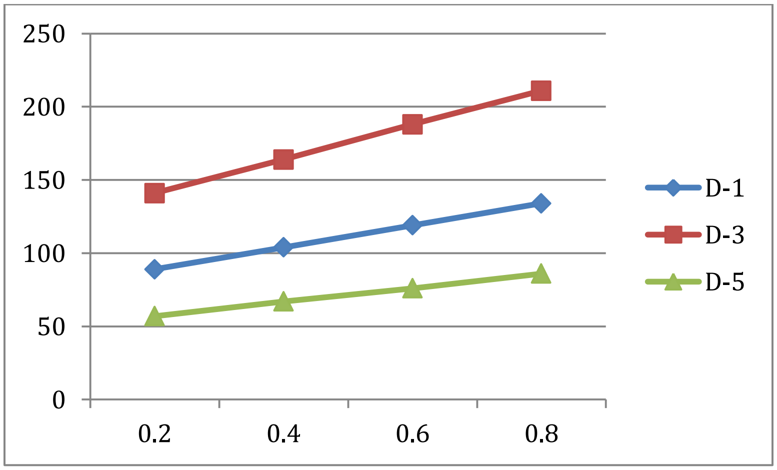


Figure 3.5: Effect of demand uncertainty on SS

Figure 3.5 depicts that there are rapid increments in the safety stocks under various demand uncertainty levels. Due to the severity of uncertainty in the system only safety stock has to be

`

maintained for avoiding the situations of stock outs. Inventory holding costs are increased because of these safety stocks and as a result the total cost of the system slightly increases.

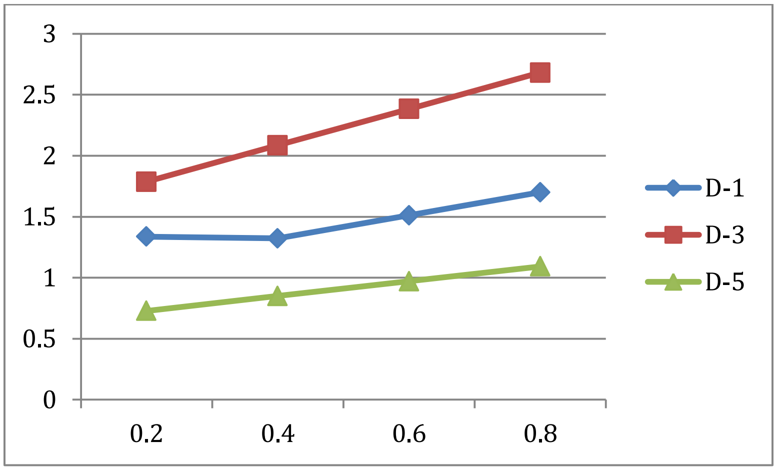


Figure 3.6: Effect of demand uncertainty on ESC

According to the Figure 3.6, the expected shortages per cycle are increasing as the demand uncertainty levels are increasing. The expected shortages per cycle are defined as the average units of demand that are not satisfied from the inventory stock in replenishment cycle. From Figure 3.6, it may be concluded that average units of not satisfying the demands are increasing, as the demand uncertainty level increases, that's why a larger safety stock is maintained to avoid the situations of stock outs at uncertainty level and its increase.

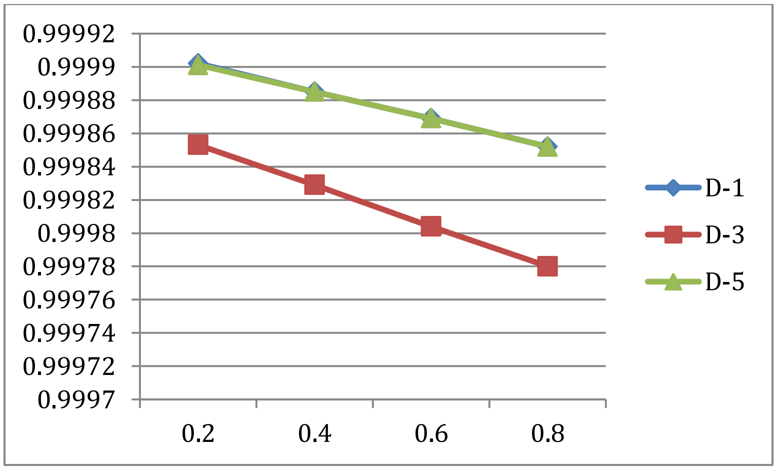


Figure 3.7: Effect of demand uncertainty on FR

Figure 3.7 shows that the fill rate decreases, as the demand uncertainty level increases but the main point to notice is that the fill rate is always greater than 99%. The fill rates of distributor D-1 and D-5 are almost same, because in the following plot (Figure 3.6) D-5 line has overlapped

`

with the D-1 line. At this point, it may be concluded that this model is capable to provide fill rates more than 99% whatever be the level of uncertainty.

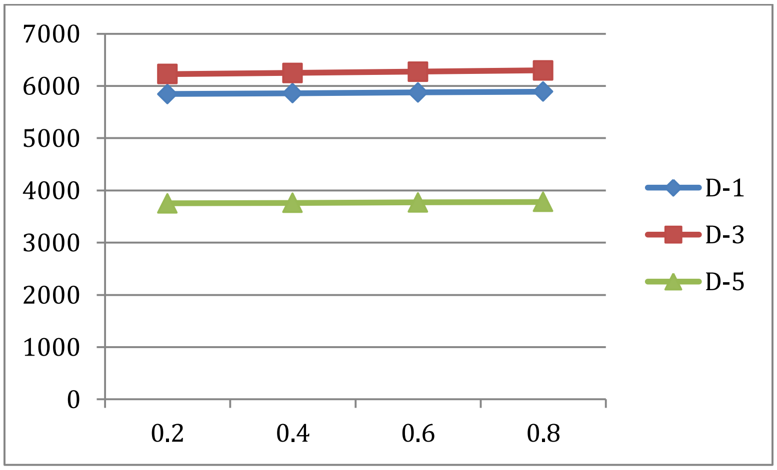


Figure 3.8: Effect of demand uncertainty on AI

The average inventory is the sum of half of ordering quantity and the safety stocks (Q/2 + SS). From Figure 3.8, it is observed that the average inventory is almost same as the demand uncertainty levels are increasing. As it is discussed earlier that safety stocks are maintained to avoid the situations of stock out under uncertainty. In this case, ordering quantity is almost the same only safety stocks are increasing. In the average inventory terms (Q/2 + SS), ordering quantity (Q) is assumed to be constant while safety stocks are increasing only. Due to this reason the average inventory is very slightly increasing as the uncertainty level increasing.

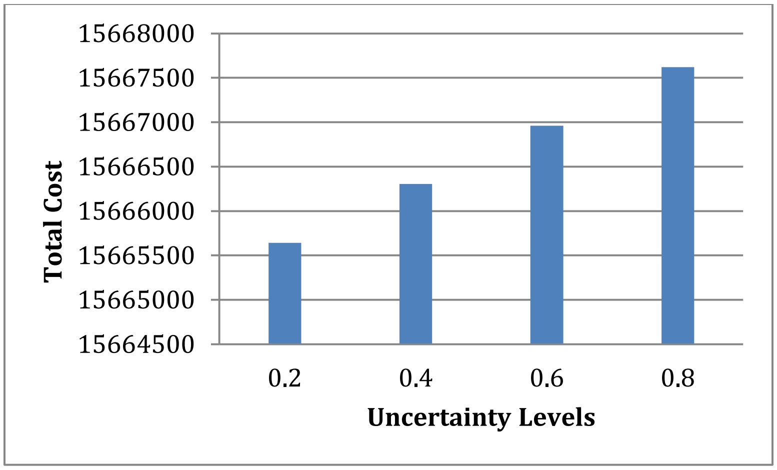


Figure 3.9: Effect of demand uncertainty on TC

`

It is observed from Figure 3.9, that total costs of the system increases as the uncertainty level increases. This is due to increments in the inventory holding cost, total cost of the system also increases.

3.7.2 Effect of supply uncertainty on performance parameters

For examining the effect of supply uncertainty on performance parameters, same levels of uncertainty (0.2, 0.4, 0.6 and 0.8) and same parameters (reorder point, safety stocks, expected shortages per cycle, fill rates, average inventory and total cost) are considered as discussed in the previous case of demand uncertainty. The effects of four levels of supply uncertainty on performance parameters are shown in following Figures.

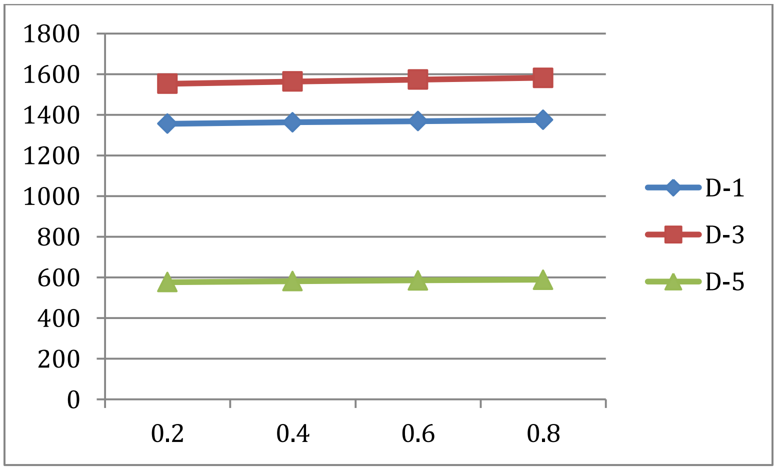


Figure 3.10: Effect of supply uncertainty on ROP

From Figure 3.10, it is observed that there is a very nominal increment in the reorder point in accordance to the supply uncertainty level increases from 0.2 to 0.8 as it seems almost horizontal line.

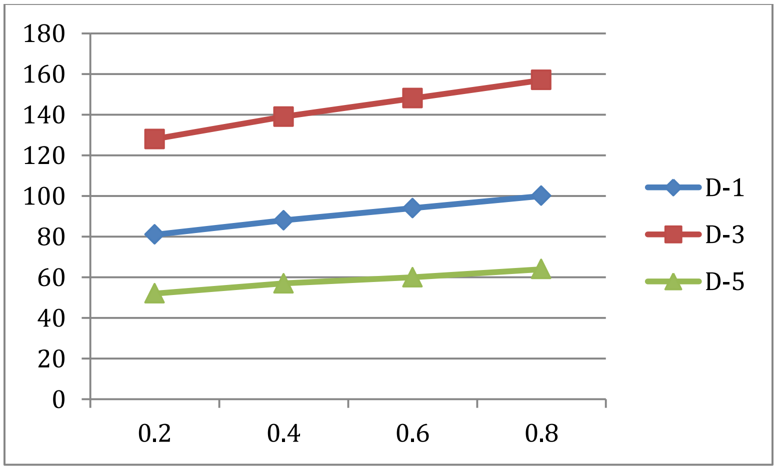


Figure 3.11: Effect of supply uncertainty on SS

`

Figure 3.11 shows that there are also fast increments in the safety stocks under various supply uncertainty levels. In case of supply uncertainty safety stock are also maintained for avoiding the situations of stock outs as maintained in case of demand uncertainty. Due to this safety stock, inventory holding costs are increased as a result total cost of the system also increase very minutely.

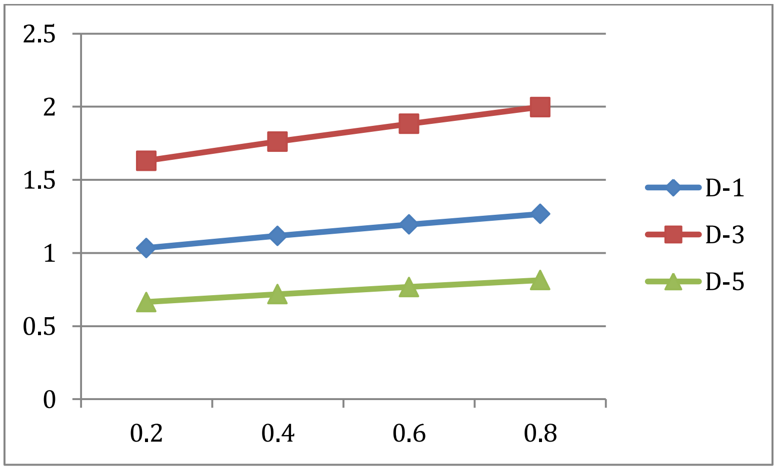


Figure 3.12: Effect of supply uncertainty on ESC

According to the Figure 3.12, the expected shortages per cycle are increasing as the supply uncertainty levels are increasing. From Figure 3.12, it may be concluded that ESC is increasing as the supply uncertainty increases.

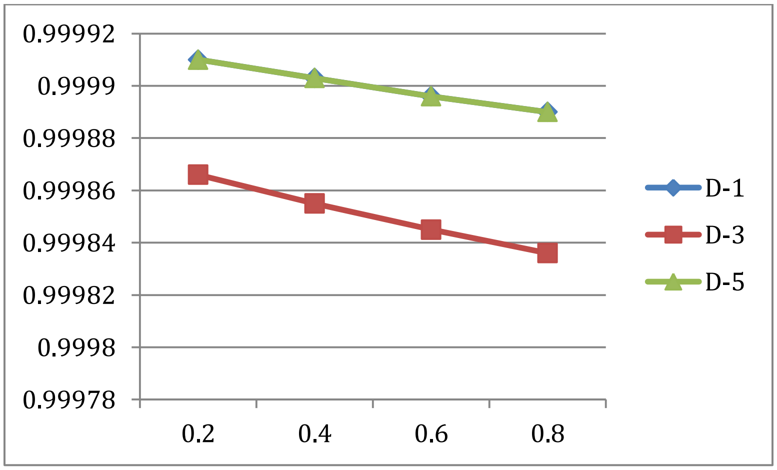


Figure 3.13: Effect of supply uncertainty on FR

Figure 3.13 shows that the pattern of decreasing the fill rate under supply uncertainty is quite similar to that of demand uncertainty. In this case, the fill rate is always greater than 99% as well. The fill rates of distributor D-1 and D-5 are almost same, because in this plot (Figure 3.13)

`

D-5 line overlaps with the D-1 line. Under supply uncertainty this model is also capable to provide fill rates more than 99%.

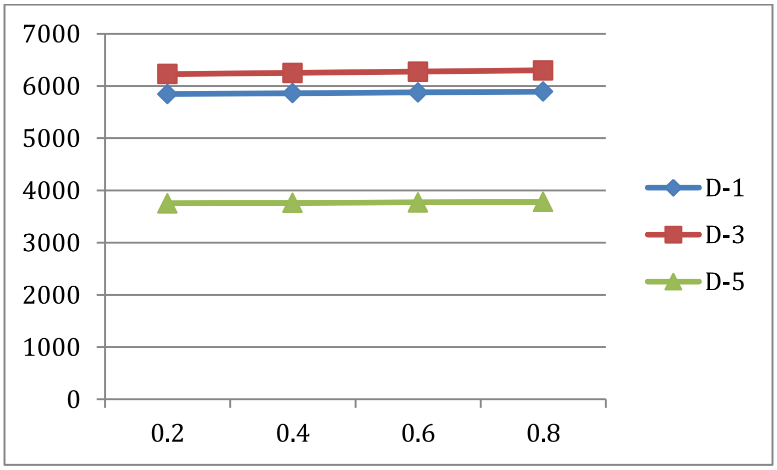


Figure 3.14: Effect of supply uncertainty on AI

As discussed earlier that, the average inventory is the sum of half of ordering quantity and the safety stocks (Q/2 + SS). From Figure 3.14, it is observed that the pattern of average inventory under supply uncertainty is almost similar to the pattern under demand uncertainty. Due to maintenance of safety stock, the average inventory is slightly increases with the increase in the uncertainty level.

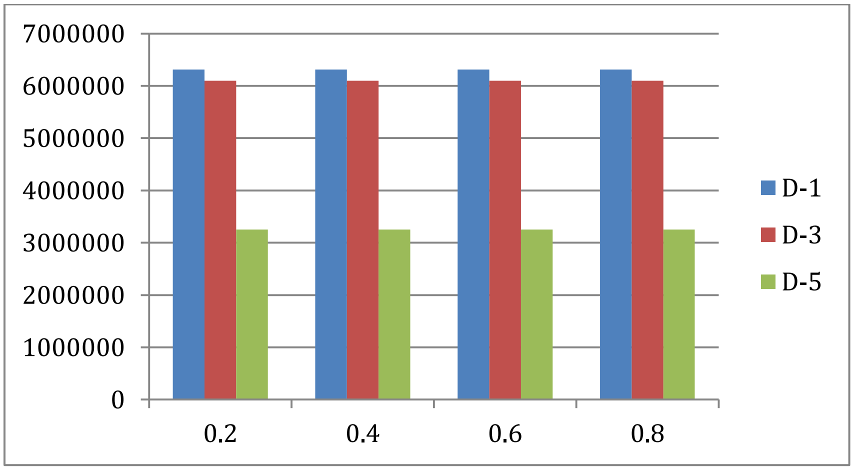


Figure 3.15: Effect of supply uncertainty on TC

The pattern of Figure 3.15 shows that the total costs of the system is increasing very slowly as the supply uncertainty levels are increasing. This increment is due to increments in the inventory holding cost as a result, total cost of the system is increased.

`

3.7.3 Combined effect of demand and supply uncertainties on performance parameters

Same levels of uncertainty (0.2, 0.4, 0.6 and 0.8) and the same performance parameters ((reorder point, safety stocks, expected shortages per cycle, fill rates, average inventory and total cost) are taken for evaluating the combined effect of both types of uncertainty (demand and supply) on performance parameters. The main observation is that there is a large increment in the performance parameters as compared to the individual demand and supply uncertainty. The combined effect of both types of uncertainty on performance parameters is shown in following Figures.

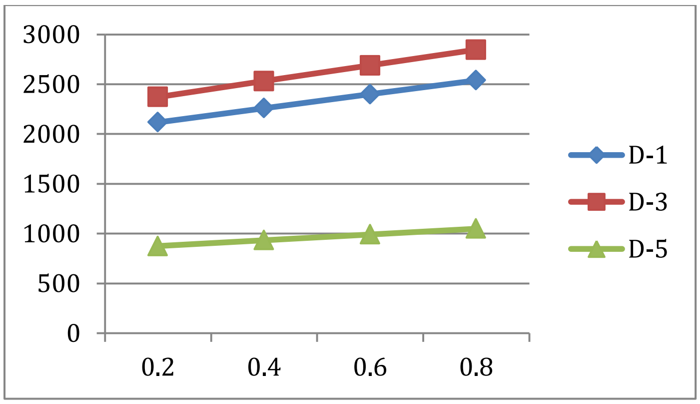


Figure 3.16: Combined effect of both type of uncertainty on ROP

For individual demand and supply uncertainty level, the increments in the reorder point is very much stable, while considering the combined effect of demand and supply uncertainty, the reorder point increases rapidly as compared to the individual type of uncertainty.

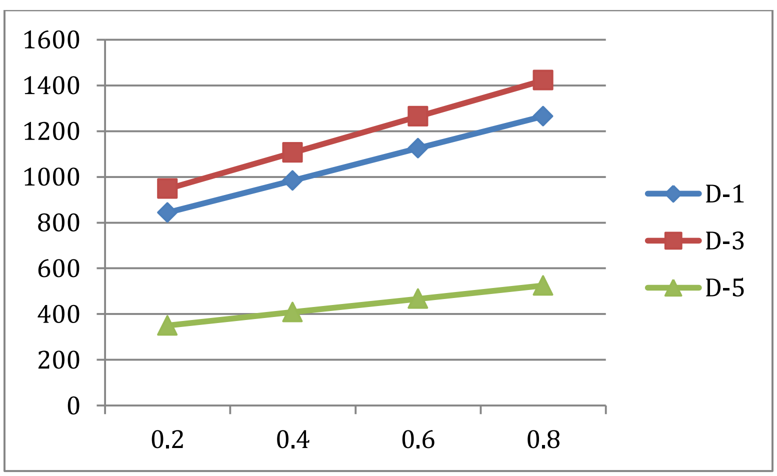


Figure 3.17: Combined effect of both type of uncertainty on SS

`

Safety stocks are kept in a system when demand and supply mismatches. While considering both type of uncertainty the safety stock required is very much high as compared to demand and supply considered individually.

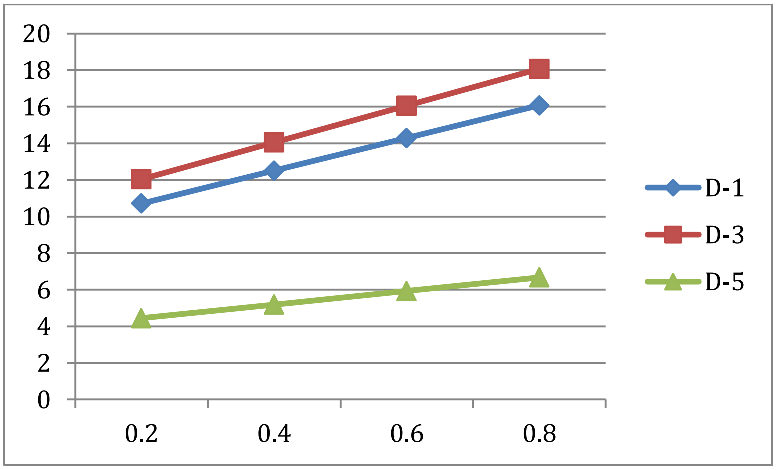


Figure 3.18: Combined effect of both type of uncertainty on ESC

According to the Figure 3.18, the expected shortages per cycle are increasing continuously as observed in the case of demand and supply uncertainty. From Figure 3.18, it is reported that the trend of increasing of ESC is very much faster in combined effect of uncertainty (demand and supply) as compared to the individual uncertainty of demand or supply.

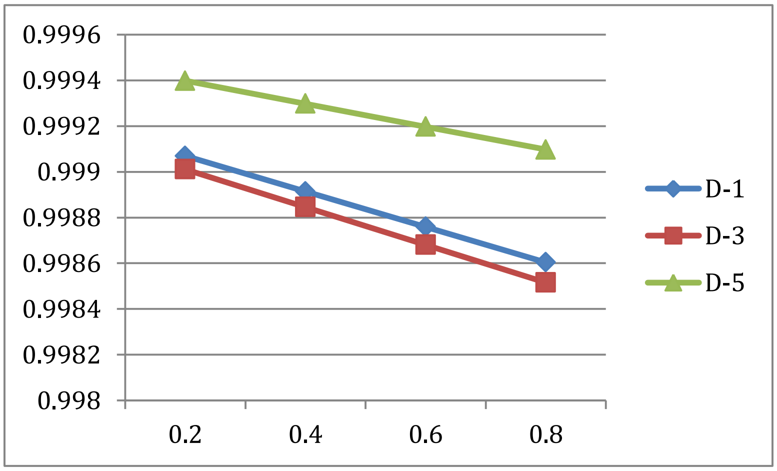


Figure 3.19: Combined effect of both type of uncertainty on FR

`

The fill rate decreases very rapidly under the combined effect of both type of uncertainty, as compared to the demand and supply uncertainty individually. In this case, the model is providing fill rate of more than 99%.

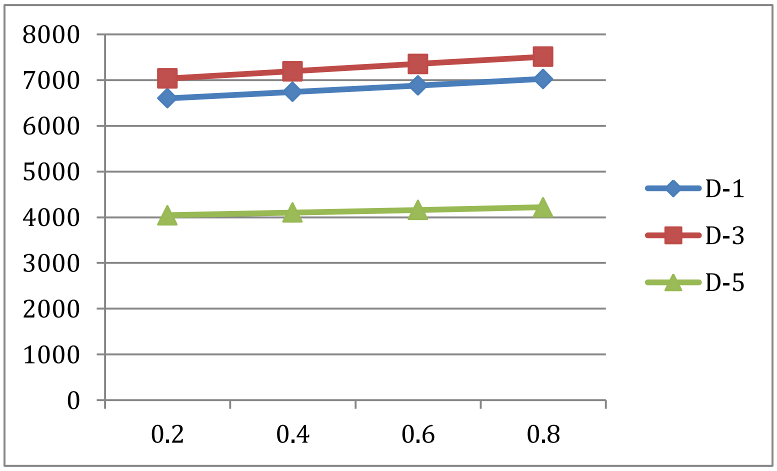


Figure 3.20: Combined effect of both type of uncertainty on AI

It is observed previously, that the average inventory is increasing as the demand and supply uncertainty levels are increasing. But the pattern of Figure 3.20 shows that, this increment in average inventory is much more in case of combined effect of uncertainty. It has been discussed previously that safety stocks are maintained to avoid the situations of stock out under uncertainty; therefore the average inventory is incrementally increasing as the uncertainty level is increasing.

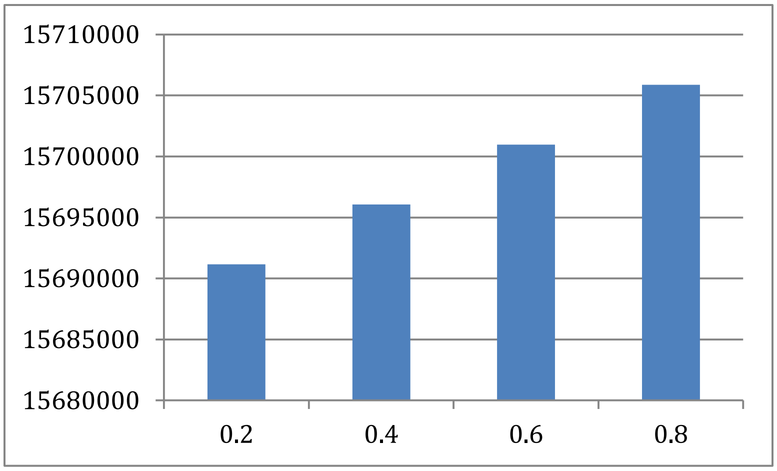


Figure 3.21: Combined effect of both type of uncertainty on TC

`

Figure 3.21 depicts that the total cost of the system increases rapidly as compared to the individual uncertainty of demand and supply. Here effect of demand uncertainty, supply uncertainty and both types of uncertainties are evaluated on various performance parameters. From the above Figure it is clear that, as the uncertainty levels increase, various performance parameters also increase except that of fill rate. Only fill rates are nominally decreased as uncertainty levels are decreased. In the combined effect of demand and supply uncertainty there is a much larger increment in the performance parameters as compared to the effect of individual demand and supply uncertainty. The comparison table between various variables and uncertainty levels for different distributors are given below:

Uncertainty levels		0.2			0.4			0.6			0.8		
Type of uncertainty		D	S	C	D	S	C	D	S	C	D	S	C
Ro	D-1	1364	1356	2119	1379	1363	2259	1394	1369	2400	1409	1375	2540
	D-2	1566	1553	2373	1589	1564	2531	1613	1573	2689	1636	1582	2847
	D-3	582	577	875	592	582	934	601	586	992	611	589	1050
SS	D-1	89	81	844	104	88	984	119	94	1125	134	100	1265
	D-2	141	128	948	164	139	1106	188	148	1264	211	157	1422
	D-3	57	52	350	67	57	409	76	60	467	86	64	525
ESC	D-1	1.337	1.03462	10.71552	1.32227	1.11752	12.50143	1.5116	1.19468	14.2874	1.70006	1.26715	16.0733
	D-2	1.78717	1.63145	12.04234	2.08503	1.76217	14.0494	2.38289	1.88384	16.0564	2.68075	1.99811	18.0635
	D-3	0.72826	0.66481	4.447551	0.84969	0.71808	5.1881	0.97102	0.76766	5.93007	1.09239	0.81422	6.67133
FR	D-1	0.999902	0.99991	0.999069	0.999885	0.9999	0.998914	0.999869	0.999896	0.99876	0.999852	0.99989	0.9986
	D-2	0.999853	0.999866	0.999011	0.999829	0.99986	0.998846	0.999804	0.999845	0.99868	0.99978	0.999836	0.99852
	D-3	0.999901	0.99991	0.999398	0.999885	0.9999	0.999298	0.999869	0.999896	0.9992	0.999852	0.99989	0.9991
AI	D-1	5847	5839	6601	5862	5845	6742	5876	5852	6882	5891	5857	7022
	D-2	6227	6215	7035	6251	6225	7193	6274	6235	7351	6298	6244	7509
	D-3	3752	3747	4045	3761	3751	4103	3770	3755	4161	3781	3759	4220
TC	D-1	6313650	6313543	6324023	6313855	6313633	6325956	6314059	6313717	6327890	6314264	6313795	6329823
	D-2	6100211	6100043	6111313	6100534	6100184	6113485	6100856	6100316	6115658	6101179	6100440	6117830
	D-3	3251782	3251714	3255809	3251914	3251771	3256611	3252045	3251825	3257413	3252177	3251875	3258216

D = Demand Uncertainty,

C = Combined level of uncertainty

Ro = Reorder point

SS = Safety stocks

ESC = Expected shortages per cycle

FR = Fill rates

AI = Average Inventory

TC = Total cost

S – Supply uncertainty

3.8 Summary of the chapter

In this chapter a mathematical model for single product, single period is developed for inventory control in a distribution network by incorporating risk pooling approach. The main objective of this model is to minimize the cost which includes facility cost, operating cost of facility, transportation cost, ordering cost and inventory holding costs. The second objective is to find the best level of risk pooling in the distribution network. The third objective is to maintain the proper balance between total cost and service level measures. For numerical illustration, 5 distributors and 7 retailers are considered in the network. An important condition is imposed in the network; a fixed distributor can fulfill the demand of some specific retailers only and a particular retailer can fulfill its demand by a single and a specific distributor. Therefore by imposing the above condition in the network, total 20 possible options are generated for the fulfillment of the demands of all retailers. All the variables and parameters like reorder point, ordering quantity, product-miles, total cost, expected shortages per cycle, intended shortages per cycle, fill rate, safety stocks, inventory positions and average inventory etc. are evaluated for each alternative. At the end, TOPSIS approach is implemented for finding the best alternative of distribution network and the best level of risk pooling. For this total cost, fill rates, cycle service level and product-miles are considered as the criteria and twenty options of distribution network are treated as alternatives. Alternative 12 of distribution network has been found to be the best option after implementing TOPSIS approach. The analysis and discussion of the results has been found suitable and logical for alternative 12. The effect of uncertainty levels on each performance parameters is evaluated for alternative 12.

CHAPTER 4

MULTI-PRODUCT MULTI-PERIOD RISK POOLING INVENTORY CONTROL MODEL IN DISTRIBUTION NETWORK

4.1 Introduction

The previous chapter represents the formulation of single product, single period mathematical model for inventory control incorporating risk pooling approach within the different combinations of distribution network. But in actual practice there are the rare chances of single product and single period situation. Generally in practice, there exist the situations of multi-products and multi-periods scenarios. Therefore there is a need to develop the previous formulation to multi product and multi period situations. In this continuation, firstly this chapter aims to achieve the one of the most important objective of this research, to incorporate multi-product, multi-period situations in the previous model. Secondly it provides the best level of risk pooling and best combination of distribution network. Finally, it provides the best inventory policy for the network obtained.

4.2 Multi- product, multi-period inventory control model using risk approach

Same conditions, alternatives and scenarios of distribution network are considered for developing the multi-product, multi-period inventory control model using risk pooling within the distribution network. The difference is only that, here multi-product and multi-period situations are considered for the formulation. The different alternatives of distribution networks and assumptions are similar to the previous chapter.

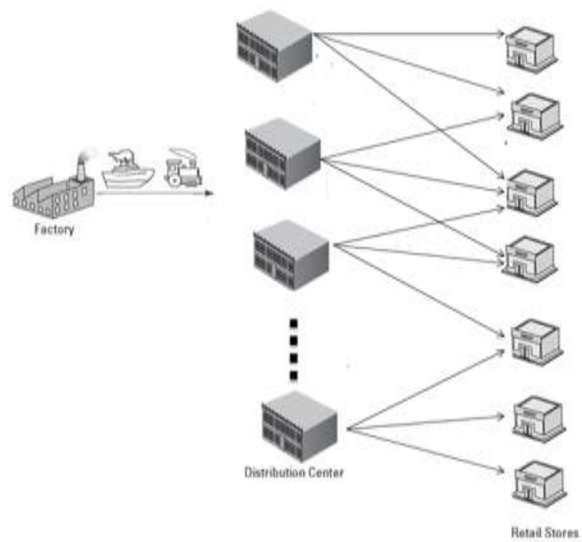


Figure 4.1: Distribution Network

Assumptions:

1. Continuous Review Policy is adopted for review of inventory system.

2. Demand and Lead time are independent random variable and assumed to be normally distributed.

3. The fill rate for all the distributors is assumed to be the same.

4. The capacity of distributor to serve a retailer is predefined and considered as parameter.

5. Backordering and lost sales are not allowed.

6. Single distributor can ship more than one retailer, but single retailer cannot be shipped by more than a single distributor. Partial fulfillment is not allowed.

7. A fixed distributor can fulfill the demand of some specific retailers only.

8. It is assumed that all products are shipping from distributor to retailers

9. It is assumed that transportation cost per unit is same for all types of products.

The notations and symbols are as follows:

Sets:

Distributor, i, $i \in I$, where I = [1,2,3.....i];
Retailer, j, $j \in J$, where J = [1,2,3.....j];
Product, k, $k \in K$, where K = [1,2,3.....k];
Period, l, $l \in L$, where L = [1,2,3.....l];

Parameters:

1. $F^{l}_{i,k}$: Facility Cost (Cost of establishment of a facility), it is the cost of establishing of distributors in specific locations for i^{th} distributor, k^{th} product and for l^{th} period
2. $OP^{l}_{i,k}$: Operating cost of distributor, it is the cost of running i^{th} distributor, k^{th} product and for l^{th} period
3. $T_{i,j}$: Cost of Shipment, it is the cost involved for the transportation of one unit per kilometer of product from distributor to retailer. It is assumed that it is same for all i^{th} distributor, k^{th} product and for l^{th} period
4. $C^{l}_{i,k}$: Cost of carrying the inventory, it is the cost of carrying the inventory for i^{th} distributor, k^{th} product and for l^{th} period
5. $FO^{l}_{i,k}$: Fixed order cost for i^{th} distributor, k^{th} product and for l^{th} period
6. $R^{l}_{i,k}$: Average Daily demands for i^{th} distributor, k^{th} product and for l^{th} period
7. $D^{l}_{j,k}$:Demand of retailer j that has to be fulfilled by the i^{th} distributor, k^{th} product and for l^{th} period
8. $X_{i,j}$: It is the distance between the distributor i and retailer j
9. μ_{xi}: Mean demand during lead time

Decision Variables:

1. $Q^{l}_{i,k}$: It shows the economic order quantity for i^{th} distributor, k^{th} product and for l^{th} period. It is the minimum order quantity, which is ordered by the retailer to the distributor at every order
2. $Ro^{l}_{i,k}$: It represents the reorder point for i^{th} distributor, k^{th} product and for l^{th} period.

Objective Function:

Minimize total cost = cost of establishment + fixed order cost + operating cost of distributor + cost of shipment + cost of carrying the inventory

Total cost:

2. **Cost of establishment:** It is the summation of all costs of establishment of all the distributors situated at different locations.

   Total Cost of establishment $= \sum_{l} \sum_{k} \sum_{i} F^{l}_{i,k}$

   Where, $F^{l}_{i,k}$ is the cost of establishment of a distributor i.

3. **Operating cost of distributor:** It is the cost of running the facility of a distributor. It is equal to the product of operating cost per unit and the average demand of any item. Mathematically it can be expressed as

   Total operating cost of distributor $= \sum_{l} \sum_{k} \sum_{i} OP^{l}_{i,k} * R^{l}_{i,k}$

   Where $OP^{l}_{i,k}$ is the operating cost of distributor and $R^{l}_{i,k}$ is the average daily demands for a distributor i

4. **Cost of shipment:** It is the cost of transportation of the units shipped from distributor i to retailer j and it is equal to the multiplication of three parameters. These three parameters are transportation cost per unit of item, distance from distributor to retailer and the demand of all retailers. It is expressed mathematically as

   Total Cost of shipment $= \sum_{l} \sum_{k} \sum_{j} \sum_{i} T_{i,j} * x_{i,j} * D_{j}$

   Where, $T_{i,j}$ is the cost involved for the transportation of per unit per kilometer of product from distributor to retailer, $X_{i,j}$ is the distance between the distributor i and retailer j and D_{j} represents the demand of retailer j.

5. **Ordering Cost:** This is the cost involved when a distributor receives an order from retailers. It is defined as the product of number of orders and the ordering cost per unit.

   The Number of order given per day by a distributor = Average daily demand/ordering quantity= $R^{l}_{i,k} / Q^{l}_{i,k}$

   Where $R^{l}_{i,k}$ = average daily demand of distributor,

$Q^l_{i,k}$ = economic order quantity.

Total number of annual orders = $365^* R^l_{i,k} / Q^l_{i,k}$

Total Ordering cost= $\sum_l \sum_k \sum_i FO_i(365^* R^l_{i,k} /Q^l_{i,k})$

Where Fo_i is the fixed ordering cost per order for a distributor i.

6. **Cost of carrying the inventory:** It is the cost involved for maintain inventory in supply chain. It is the product of number of units hold in inventory and the inventory carrying cost per unit item.

Average cycle inventory = $Q^l_{i,k} /2$

Safety stock $=\left(Ro^l_{i,k} - \mu^l_{i,k}\right)$

Total Inventory in hand $=Q^l_{i,k} /2+ \left(Ro^l_{i,k} - \mu^l_{i,k}\right)$

Total Cost of carrying the inventory $=\sum_l \sum_k \sum_i \left\{Q^l_{i,k} /2+ \left(Ro^l_{i,k} - \mu^l_{i,k}\right)\right\}^* C^l_{i,k} {}^* v$

Where $Ro^l_{i,k}$ is the reorder point, $\mu^l_{i,k}$ is the average demand during lead time for a distributor i, $C^l_{i,k}$ is the cost of carrying the inventory per unit item per year (in rupees) and v is the price of particular product.

Average demand for lead time of distributor (Chopra and Meindl, 2013): Suppose that the lead time and the demand are independent unexpected variables,

$L^l_{i,j}$ = Average lead time from i^{th} distributor to j^{th} retailer

$D^l_{j,k}$ = average demand per unit time for j^{th} retailer

$\mu^l_{i,j}$ = average demand during lead time from i^{th} distributor to j^{th} retailer

$\sigma^l_{j,k}$ = standard deviation of demand for j^{th} retailer

σ_{ltd} = standard deviation of demand for lead time

S_L = standard deviation of lead time

The average of demand and the standard deviation of demand for lead time can be calculated as follows:

$$\mu^l_{i,j} = D^l_{j,k} {}^* L^l_{i,j}$$

$$\sigma_{ltd} = \left(\sigma^{l}{}_{j,k}{}^{2} x\, L^{l}_{i,j} + S_{L}{}^{2} x\, D^{l}{}_{j,k}{}^{2}\right)^{1/2}$$

When a distributor is shipping to more than one retailer, the average of aggregated demand and the standard deviation of aggregated demand are given as follows:

Let B_i is the set of all retailers served by distributor i

$D^{l}{}_{j,k}$ = demands of retailers' j served by distributor

σ_r = standard deviation of demand of region r

σ_s = standard deviation of demand of region s

$\rho_{r,s}$ = Coefficient of correlation between the demand of retailers

$R^{l}{}_{i,k}$ = Average Daily demands for a distributor i

$$R^{l}{}_{i,k} = \sum_{J \in B_i} D^{l}{}_{j,k}$$

σ_j = Standard deviation of aggregated demand

$$\sigma_j = (\sigma_r{}^2 + \sigma_s{}^2 + 2\, \rho_{r,s} \sigma_r \sigma_s)^{1/2}$$

If a distributor is shipping more than one retailer and the demand of all retailers are aggregated, then the demand during lead may be represented as

$$\mu_{xi} = R^{l}{}_{i,k} x\, L^{l}{}_{i,k}$$

$$\text{CSL} = \text{NORMDIST}\left(Ro^{l}{}_{i,k}, \mu^{l}{}_{i,k}, \sigma^{l}{}_{i,k}, 1\right)$$

Where $R^{l}{}_{i,k}$ is average aggregated demand for distributor i

The expression for standard deviation during lead time is given below

$$\sigma_{xi} = (\sigma_j{}^2 x\, L^{l}_{i,k} + S_L{}^2\, R^{l}{}_{i,k}{}^{2})^{1/2}$$

Where σ_{xi} is the aggregated demand's standard deviation for distributor i

Constraints:

1. The order quantity is always a nonnegative quantity. The ordering quantity cannot be negative, it will be positive.

   $Q^{l}_{i,k} \geq 0$ for all i

2. As the demand uncertainty is considered in the model, so a safety stock is maintained in the system. This safety stock cannot be negative. Therefore the reorder point should be higher than the average demand during lead time.

$Ro^{l}_{i,k} \geq \mu^{l}_{i,k}$ for all;

Here $\mu^{l}_{i,k}$ is the average demand during the replenishment lead time for a distributor 'i'.

3. Expected shortfalls in one cycle of order (ESC) is the average units of demand that are not satisfied from inventory in stock per replenishment cycle.

   ESC = Expected shortages in one cycle of order.

   ESC = -SS*[1-NORMDIST (SS/σ_L, 0, 1, 1) + σ_L *NORMDIST (SS/σ_L, 0, 1, 0)

4. Fill rate may be defined as the fraction of demand of retailer satisfied by the distributor in a cycle of inventory.

   Fill Rate (FR) = Max (1-ESC/Q)

   Where Q = Economic order quantity

   Expected shortfalls in one cycle of order (ESC) is always less than intended shortfalls in one cycle of order (in between two orders)

   ESC $\leq$ ISC

   Where ISC = (1-FR)*Q

5. Cycle service level which is defined as the probability of not stocking and CSL $\geq$ 0.95

   Where, $\text{CSL} = \text{NORMDIST}\left(Ro_{i,k}{}^{1}, \mu_{i,k}{}^{1}, \sigma_{Li,k}{}^{1}, 1\right)$

Objective Function:

The prime objective function of inventory control problem involves the various costs functions like ordering cost, cost of carrying the inventory, cost of shipments, cost of establishment of distributor and the operating cost of distributor.

Total inventory cost with in the distribution network = Ordering cost + Cost of carrying the inventory + Cost of shipment of inventory from distributor to retailer + Cost of establishment of distributor + Operating cost of distributor.

Mathematical representation of model:

The first objective function of total anticipated cost:

$$Z1=\text{MINMIZATION}[\sum_{l}\sum_{k}\sum_{i}F^{l}_{i,k}+\sum_{l}\sum_{k}\sum_{i}OP^{l}_{i,k}*R^{l}_{i,k}+\sum_{l}\sum_{k}\sum_{j}\sum_{i}T_{i,j}*x_{i,j}*D_{j}$$

$$+\sum_{l}\sum_{k}\sum_{i}FO_{i}(365*R^{l}_{i,k}/Q^{l}_{i,k})+\sum_{l}\sum_{k}\sum_{i}\left\{Q^{l}_{i,k}/2+\left(Ro^{l}_{i,k}-\mu^{l}_{i,k}\right)\right\}*C^{l}_{i,k}*v\}]$$

Subjected to constraints:

$Q^{l}_{i,k} \geq 0$; for all i and k

$Ro^{l}_{i,k} \geq \mu^{l}_{i,k}$ for all i and k

$CSL \geq 0.95$

$CSL = NORMDIST\left(Ro_{i,k}{}^{l}, \mu_{i,k}{}^{l}, \sigma_{Li,k}{}^{l}, 1\right)$

Fill Rate (FR) = Max (1-ESC/Q)

ESC≤ISC

Where ISC = (1-FR)*Q

Product miles: The product-miles may be defined as the product of demand and distance. If $X_{j\text{-}k}$ is the distance from distributor j to retailer k, than

$$\text{Product-miles} = \left(\sum_{l}\sum_{i}\sum_{k}\sum_{j} X_{j\text{-}k} * D_{k}\right)$$

4.3 Numerical Illustration and solution procedure:

For solving the above model, same alternatives of distribution networks are considered as discussed in the previous chapter. The solution procedure is same as mentioned in the former chapter. It is known that, 5 distributors, 7 retailers and 20 possible alternatives of distribution networks are considered for numerical illustrations in the last chapter. For solving multi-products and multi-periods risk pooling inventory control model, three different types of products and 5 planning periods are considered. For numerical illustrations some numerical data are shown in the, following tables.

Table 4.1: Facility cost, Operating cost and fixed ordering cost for all type of products

Facility Cost (Rs.)			Operating Cost (Rs.)			Fixed ordering Cost (Rs.)		
P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3
25000	50000	70000	3	6	10	250	350	500
22500	45000	65000	2.5	5	9	250	350	500
15000	30000	50000	3	6	10	250	350	500
27500	55000	75000	2.5	5	9	250	350	500
20000	40000	60000	3	6	10	250	350	500

Table 4.2: Inventory holding cost for all type of products and periods

Planning Horizon-1			Planning Horizon-2			Planning Horizon-3			Planning Horizon-4			Planning Horizon-5		
P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3
0.05	0.1	0.15	0.06	0.12	0.17	0.06	0.14	0.19	0.1	0.15	0.25	0.04	0.08	0.12
0.05	0.1	0.15	0.06	0.12	0.17	0.06	0.14	0.19	0.1	0.15	0.25	0.04	0.08	0.12
0.05	0.1	0.15	0.06	0.12	0.17	0.06	0.14	0.19	0.1	0.15	0.25	0.04	0.08	0.12
0.05	0.1	0.15	0.06	0.12	0.17	0.06	0.14	0.19	0.1	0.15	0.25	0.04	0.08	0.12
0.05	0.1	0.15	0.06	0.12	0.17	0.06	0.14	0.19	0.1	0.15	0.25	0.04	0.08	0.12

Table 4.3: Demand of all retailers for all type of products and periods

Planning Horizon-1			Planning Horizon-2			Planning Horizon-3			Planning Horizon-4			Planning Horizon-5		
P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3
1200	1150	1000	1150	1050	950	1125	1000	950	1000	950	900	1400	1250	1200
750	700	650	700	650	600	600	800	700	650	650	600	950	900	850
600	650	600	550	600	550	500	600	700	500	500	500	700	850	800
900	850	750	800	750	700	800	750	600	700	700	700	1100	900	950
1100	1000	950	1000	950	850	1000	900	850	900	800	900	1300	1200	1150
1000	950	900	900	850	800	950	850	800	800	750	750	1100	1100	1100
800	750	650	700	700	600	700	800	700	600	700	625	1000	950	750

Table 4.4: Standard deviations of demand of all retailers for all type of products and periods

Planning Horizon-1			Planning Horizon-2			Planning Horizon-3			Planning Horizon-4			Planning Horizon-5		
P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3	P-1	P-2	P-3
12	11.5	10	11.5	10.5	9.5	11.25	10	9.5	10	9.5	9	14	12.5	12
7.5	7	6.5	7	6.5	6	6	8	7	6.5	6.5	6	9.5	9	8.5
6	6.5	6	5.5	6	5.5	5	6	7	5	5	5	7	8.5	8
9	8.5	7.5	8	7.5	7	8	7.5	6	7	7	7	11	9	9.5
11	10	9.5	10	9.5	8.5	10	9	8.5	9	8	9	13	12	11.5
10	9.5	9	9	8.5	8	9.5	8.5	8	8	7.5	7.5	11	11	11
8	7.5	6.5	7	7	6	7	8	7	6	7	6.25	10	9.5	7.5

4.4 Optimization results: The problem is NLP type and solved through CONPOT solver in AIMMS software on an Intel (R) Core (TM)2 Duo CPU T 6570 with memory 4GB for total cost minimization. The results of all 20 alternatives, for 3 types of products and 5 periods are shown in the following tables

Table 4.5: Variables obtained for Alternative-1, Planning horizon-1, Product-1

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3420.526	453.1264	2.040279	0.999404	160.6264	1870.89	83250	774716.5974
D-2	1897.367	139.4934	0.628666	0.999669	49.49342	998.1767	54000	294945.9484
D-3	3464.102	464.7344	2.092459	0.999396	164.7344	1896.785	185000	1303577.229
D-4	2449.49	232.489	1.047777	0.999572	82.48904	1307.234	83500	600685.5957
D-5	2190.89	185.9912	0.838222	0.999617	65.99123	1161.436	80000	483637.5996
							Total cost	3457562.97

Table 4.6: Variables obtained for Alternative-1, Planning horizon-1, Product-2

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3942.081	429.8905	1.935667	0.999509	152.3905	2123.431	82500	831603.0482
D-2	2336.664	151.1179	0.681055	0.999709	53.61788	1221.95	54000	362445.1572
D-3	3942.081	429.8785	1.935514	0.999509	152.3785	2123.419	176250	1291603.006
D-4	2824.889	220.8646	0.995388	0.999648	78.36459	1490.809	77750	631630.2772
D-5	2509.98	174.3668	0.785833	0.999687	61.86678	1316.857	73750	504863.9889
							Total cost	3622145.478

Table 4.7: Variables obtained for Alternative-1, Planning horizon-1, Product-3

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4449.719	383.4995	1.727468	0.999612	135.9995	2360.859	74750	859160.6583
D-2	2683.282	139.4934	0.628666	0.999766	49.49342	1391.134	49000	392531.7967
D-3	4516.636	395.1054	1.779621	0.999606	140.1054	2398.423	164000	1304447.676
D-4	3286.335	209.292	0.943659	0.999713	74.29205	1717.46	73750	671436.0208
D-5	2792.848	151.2001	0.6821	0.999756	53.70013	1450.124	67250	500184.7619
							Total cost	3727760.913

Table 4.8: Variables obtained for Alternative-1, Planning horizon-2, Product-1

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3331.666	429.8905	1.935667	0.999419	152.3905	1818.224	78000	735538.6356
D-2	1816.59	127.869	0.576277	0.999683	45.36897	953.6641	50500	272624.8904
D-3	3286.335	418.2615	1.883219	0.999427	148.2615	1791.429	167000	1176667.74
D-4	2323.79	209.2401	0.942999	0.999594	74.24014	1236.135	75500	543886.9383
D-5	2049.39	162.7423	0.733444	0.999642	57.74233	1082.437	71500	426173.0824
							Total Cost	8870845.679

Table 4.9: Variables obtained for Alternative-1, Planning horizon-2, Product-2

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3778.889	395.0337	1.778711	0.999529	140.0337	2029.478	76000	770813.2523
D-2	2244.994	139.4934	0.628666	0.99972	49.49342	1171.991	49750	339044.8579
D-3	3778.889	394.9987	1.778266	0.999529	139.9987	2029.443	161000	1194813.105
D-4	2672.078	197.6157	0.890611	0.999667	70.11568	1406.155	71500	572241.8883
D-5	2424.871	162.7423	0.733444	0.999698	57.74233	1270.178	69500	474947.1827
							Total Cost	3351860.287

Table 4.10: Variables obtained for Alternative-1, Planning horizon-2, Product-3

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4312.772	368.3891	1.726065	0.9996	135.8891	2292.275	69500	860831.4108
D-2	2569.047	131.9698	0.628367	0.999755	49.46984	1333.993	45500	393406.3453
D-3	4312.772	372.4884	1.778135	0.999588	139.9884	2296.374	148250	1305932.332
D-4	3098.387	194.1937	0.942409	0.999696	74.19366	1623.387	65750	672361.753
D-5	2683.282	143.5942	0.680755	0.999746	53.59422	1395.235	61000	501126.992
							Total Cost	3733658.833

Table 4.11: Variables obtained for Alternative-1, Planning horizon-3, Product-1

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3217.142	400.8509	1.804969	0.999439	142.1009	1750.672	71625	681254.7396
D-2	1732.051	116.2445	0.523889	0.999698	41.24452	907.2699	46375	249944.6751
D-3	3286.335	418.2615	1.883219	0.999427	148.2615	1791.429	168000	1176560.254
D-4	2387.467	220.8646	0.995388	0.999583	78.36459	1272.098	77750	572407.1846
D-5	2049.39	162.7423	0.733444	0.999642	57.74233	1082.437	71500	426108.1362
							Total Cost	3106274.99

Table 4.12: Variables obtained for Alternative-1, Planning horizon-3, Product-2

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3888.444	418.2615	1.883219	0.999516	148.2615	2092.484	80000	827797.3921
D-2	2244.994	139.4934	0.628666	0.99972	49.49342	1171.991	52000	339865.2514
D-3	3722.902	383.4055	1.726274	0.999536	135.9055	1997.357	158250	1159448.498
D-4	2672.078	197.6157	0.890611	0.999667	70.11568	1406.155	69750	573226.1965
D-5	2592.296	185.9912	0.838222	0.999677	65.99123	1362.139	72000	537570.6311
							Total cost	3437907.969

Table 4.13: Variables obtained for Alternative-1, Planning horizon-3, Product-3

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4449.719	383.4085	1.726312	0.999612	135.9085	2360.768	79750	869409.4262
D-2	2898.275	162.7423	0.733444	0.999747	57.74233	1506.88	52750	448932.9617
D-3	4171.331	336.9359	1.517077	0.999636	119.4359	2205.101	149750	1129939.281
D-4	3098.387	185.9912	0.838222	0.999729	65.99123	1615.185	65750	608759.0214
D-5	2898.275	162.7423	0.733444	0.999747	57.74233	1506.88	65500	536332.9617
							Total cost	3593373.653

Table 4.14: Variables obtained for Alternative-1, Planning horizon-4, Product-1

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3146.427	383.413	1.726369	0.999451	135.913	1709.126	70250	664100.592
D-2	1732.051	116.2445	0.523889	0.999698	41.24452	907.2699	45500	251087.8352
D-3	3098.387	371.7886	1.673981	0.99946	131.7886	1680.982	149000	1051192.139
D-4	2190.89	185.9912	0.838222	0.999617	65.99123	1161.436	67500	488079.7542
D-5	1897.367	139.4934	0.628666	0.999669	49.49342	998.1767	63000	369543.2135
							Total cost	2824003.534

Table 4.15: Variables obtained for Alternative-1, Planning horizon-4, Product-2

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3666.061	371.7925	1.67403	0.999543	131.7925	1964.823	69000	735348.3496
D-2	2049.39	116.2445	0.523889	0.999744	41.24452	1065.94	44750	291620.878
D-3	3549.648	348.5497	1.569331	0.999558	123.5497	1898.374	140500	1055741.286
D-4	2509.98	174.3668	0.785833	0.999687	61.86678	1316.857	61750	513168.4883
D-5	2424.871	162.7423	0.733444	0.999698	57.74233	1270.178	63500	476280.8695
							Total cost	3072159.871

Table 4.16: Variables obtained for Alternative-1, Planning horizon-4, Product-3

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4242.641	348.5563	1.569414	0.99963	123.5563	2244.877	66000	797355.7938
D-2	2449.49	116.2445	0.523889	0.999786	41.24452	1265.989	43000	343327.7351
D-3	4381.78	371.7886	1.673981	0.999618	131.7886	2322.679	146000	1240230.616
D-4	3000	174.3668	0.785833	0.999738	61.86678	1561.867	65250	578665.3212
D-5	2738.613	145.3057	0.654861	0.999761	51.55565	1420.862	64125	488575.2286
							Total cost	3448154.695

Table 4.17: Variables obtained for Alternative-1, Planning horizon-5, Product-1

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3754.997	546.0706	2.458737	0.999345	193.5706	2071.069	99750	932962.7811
D-2	2049.39	162.7423	0.733444	0.999642	57.74233	1082.437	64500	339823.62
D-3	3794.733	557.6803	2.510939	0.999338	197.6803	2095.047	215000	1557811.423
D-4	2569.047	255.7379	1.152555	0.999551	90.73794	1375.261	95000	657434.8367
D-5	2449.49	232.489	1.047777	0.999572	82.48904	1307.234	97000	598793.4256
							Total cost	4086826.086

Table 4.18: Variables obtained for Alternative-1, Planning horizon-5, Product-2

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4249.706	499.5938	2.249449	0.999471	177.0938	2301.947	101000	966370.3038
D-2	2672.078	197.6157	0.890611	0.999667	70.11568	1406.155	66500	458273.2718
D-3	4200	487.9738	2.197116	0.999477	172.9738	2272.974	206500	1467664.327
D-4	3039.737	255.7379	1.152555	0.999621	90.73794	1610.606	91500	721029.5662
D-5	2824.889	220.8646	0.995388	0.999648	78.36459	1490.809	90750	626986.7107
							Total cost	4240324.179

Table 4.19: Variables obtained for Alternative-1, Planning horizon-5, Product-3

	Q	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4959.839	476.3576	2.144832	0.999568	168.8576	2648.777	95750	1049783.315
D-2	3098.387	185.9912	0.838222	0.999729	65.99123	1615.185	63000	497671.19
D-3	5019.96	487.9708	2.197079	0.999562	172.9708	2682.951	204000	1590997.915
D-4	3633.18	255.7379	1.152555	0.999683	90.73794	1907.328	89750	799916.1623
D-5	3000	174.3668	0.785833	0.999738	61.86678	1561.867	79750	564934.0806
							Total cost	4503302.663

The results shown in above tables (Table 4.6 to Table 4.19) represent the result of alternative-1 only. Each of above table shows the ordering quantity, reorder point, expected shortages per cycle, fill rate, safety stocks, average inventory, inventory positions, product miles and the total cost for all 5 distributors for three different types of product and for all 5 planning periods. Similarly results for all three types of products, all five planning periods and for all twenty alternatives can be displayed. Results for rest of nineteen alternatives are not shown here. Now it becomes very difficult that which alternative has to be selected. Here a multi-attribute decision making problem is observed in different alternatives of distribution network and performance criterion. In the next section this problem is solved by one of the most popular MADM approach known as fuzzy TOPSIS approach.

4.5 Selection of preeminent inventory policy through fuzzy TOPSIS approach

Some important inventory variables like ordering quantity, reorder point, expected shortages per cycle, fill rate, safety stocks, average inventory, inventory positions, product miles and total cost are evaluated against different alternatives of distribution networks in the last section. The results of optimizations are quite similar and it is very difficult to decide the best option of inventory policy for all retailers. There are two important criteria's known as "total cost and product-miles". There may be some other criterion, which may play very important role for deciding the best option of inventory policy in a network other than cost and product-miles. There are some other criterion like ordering quantity, reorder point, expected shortages per cycle, fill rate, cycle service level, safety stocks, average inventory and inventory positions beside of cost and product-miles. Collectively all these criterion may play an important role to decide the best inventory policy among all the alternatives of distribution networks. Each criteria cannot be equally important in the given inventory system. These criteria's may have different level of importance in an inventory system in alternative distribution networks. Therefore equal weightage cannot be given on the basis of their importance to each criterion. It is clear that, there are total 20 possible alternatives of distribution networks against some inventory criterion. Each alternative has different level of importance in the network against some criteria. As each network has different level of importance in a supply chain and hence equal weight cannot be given to each alternative according to their level of importance. The objective of this section is to select the best alternative of distribution network for the inventory policy against some performance parameters. Therefore a multi-criteria decision making problem generates here. In this MCDM problem the different criteria may be total cost, product-miles, anticipated shortages per cycle, fill rate, reorder point, cycle service level, safety stock, average inventory and inventory positions and the different alternatives are various options for demand fulfillment in the network.

For solving above MCDM problem a brief literature review has been performed for multi-criteria decision making methods in inventory to find the appropriate methodology. Multiple criteria decision-making (MCDM) methods is considered as a complex decision-making (DM) tool involving both of the quantitative and qualitative factors. In recent years, several MCDM techniques and approaches have been suggested for choosing the optimal probable options (Mardani et al. 2015). First critical analysis of spare parts inventory was carried out using analytical hierarchy process by Gajpal (1994) after that multi-criteria framework for inventory control was developed by Agrell (1995). He considered, ordering quantity, safety inventory and service level as inventory criteria in a continuous review policy. Ramanathan (2006) developed a weighted linear optimization method for ABC inventory classification with multiple criteria. Vencheh (2011) developed a fuzzy analytical hierarchy-data envelopment analysis (Fuzzy AHP-DEA) for the criteria annual dollar usages, lead time, average cost and the limitations of warehouse space.

Kabir and Hasin (2011) compared the AHP model and fuzzy AHP model for inventory classification with criteria unit price, annual demand, criticality, last use date and durability. Balaji and Kumar (2014) developed multi-criteria inventory classification model using analytical hierarchy process using annual usages, unit price, demand, unit weight and shape in automobile industry for rubber components. Duong (2015) proposed multi-metric approach including variance ratio, order rate, fill rate and average inventory. It may be concluded that most of the research work has been carried out for the classification of inventory by using AHP or fuzzy AHP. There is as such no study which gives the best alternative for demand fulfillment against some predefined criteria. So there is a major research gap for such type of problem in which best alternative of distribution network is selected against some criterion of cost and service level measures. So for fulfilling this gap, a study has been carried to find the best alternative of distribution network against the criteria's of cost and service level measures by some suitable MCDM approach. The above problem can be solved by some suitable MCDM approaches for 20 similar solutions for the performance criteria's like, total cost and service level measures. Again service level measures can be divided into three categories via cycle service level, fill rates and product-miles.

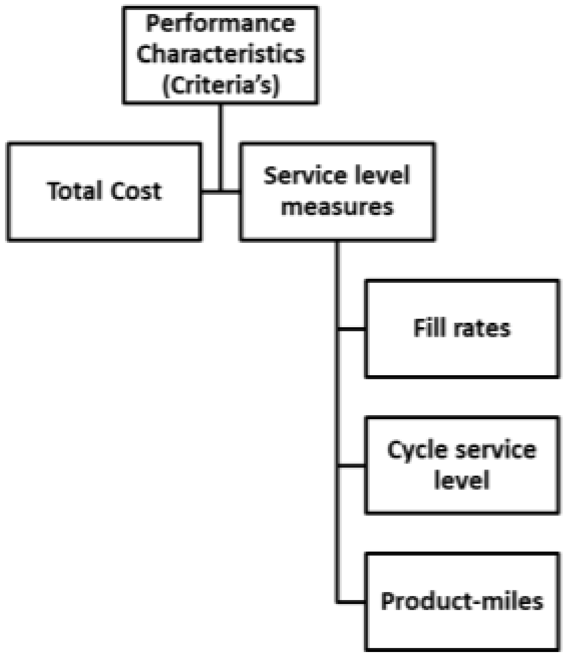


Figure 4.2: Performance characteristics

Therefore this problem may be formulated for 20 alternatives of distribution networks and four performance characteristics of fill rates, cycle service level, product-miles and total cost. Due to similar and identical solutions of twenty alternatives Fuzzy-TOPSIS approach is widely suitable here. Therefore, Fuzzy TOPSIS approach is implemented for finding the best alternative of distribution network against performance criteria's of total cost, fill rates, cycle service level, product-miles due to similarity in results. Before going to fuzzy TOPSIS approach, it is necessary to understand some basic fundamentals of Fuzzy Set theory.

4.6 Fuzzy Set Theory:

For modeling the imprecise and vague information, fuzzy set theory (Zadeh, 1965) is applied by the decision makers in decision making processes.

4.6.1 Fundamentals Definitions:

4.6.1.1 Definitions 1: Fuzzy set

A fuzzy set A in X is defined by (Zadeh, 1965 & Pedrycz et al. 2007):

$A=[X, \mu_A(X)], x \in X$, in which $\mu_A(X){:}X \rightarrow [0,1]$ is the membership function of A and $\mu_A(X)$ is the degree of pertinence of x in A. If $\mu_A(X)$ equals zero, if x does not belong to the fuzzy set A. If $\mu_A(X)$ equals to1, x completely belongs to the fuzzy set A. However, unlike classical set theory, if $\mu_A(X)$ has a value between zero and 1 and 1, x partially belongs to fuzzy set theory A. That is, the pertinence of x is true with degree of membership given by $\mu_A(X)$.

4.6.1.2 Definitions 2: Fuzzy numbers

A fuzzy number is a fuzzy set in which the membership function satisfies the condition of normality Sup $\check{A}$ (x) $_{x\varepsilon X}$ = 1 and of convexity

C_1 C_2 C_j C_m

$$
\begin{array}{cccccc}
 & & C_1 & C_2 & C_3 & C_m \\
 & A1 & X11 & X12 & X1j \ldots\ldots. & X1m \\
D = & A2 & X21 & X22 & X2j \ldots\ldots. & X2m \\
 & An & Xn1 & Xn2 & Xnj \ldots\ldots & Xnm
\end{array}
$$

For all x_1, $x_1 \in X$ and for all, $\lambda \in [0,1]$. The triangular fuzzy number is commonly used in decision making due to intuitive membership function,

$W=[w_1+w_2+.......+w_m]$, given by

$$
\mu_A(x)=\begin{cases} 0 & \text{for } x<l \\ \dfrac{x-l}{m-l} & \text{for} 1 \le x \le m \\ \dfrac{u-x}{u-m} & \text{for } m \le x \le u \\ 0 & \text{for } x > u \end{cases}
$$

In which l, m and u are real numbers with l < m < u. outside the interval [l, u], the pertinence degree is null, and m represents the point in which the degree of pertinence is maximum. Trapezoidal fuzzy numbers are also frequently used in decision making processes.

4.6.1.3 Algebraic operations with fuzzy numbers

For any given real number 'k' and two fuzzy triangular numbers $A=(l_1,m_1,u_1)$ and $B=(l_2,m_2,u_2)$, the main algebraic operations are expressed as follows (Zimmermann, 1991 & Pedrycz et al. 2007):

(1) Addition of two triangular fuzzy numbers

$$A\ (+)\ B=\left(l_1+l_2,\ m_1+m_2,\ u_1+u_2\right) \qquad l_1 \geq 0,\ l_2 \geq 0$$

(2) Multiplication of two triangular fuzzy numbers

$$A\ (X)\ B=\left(l_1 x\, l_2,\ m_1 x m_2,\ u_1 x\, u_2\right) \qquad l_1 \geq 0, l_2 \geq 0$$

(3) Subtraction of two triangular fuzzy numbers

$$A\ (-)\ B=\left(l_1 - l_2,\ m_1 - m_2,\ u_1 - u_2\right) \qquad l_1 \geq 0, l_2 \geq 0$$

(4) Division of two triangular fuzzy numbers

$$A\ (-)\ B=\left(l_1 - l_2,\ m_1 - m_2,\ u_1 - u_2\right) \qquad l_1 \geq 0, l_2 \geq 0$$

(5) Inverse of a triangular fuzzy numbers

$$A^{-1}=\left(\frac{1}{u_1},\frac{1}{m_1},\frac{1}{l_1}\right) \geq 0$$

(6) Multiplication of a triangular fuzzy number by a constant

$$k\ x\ A = \left(k\ x\ l_1,\ k\ x\ m_1,\ k\ x\ u_1\right), \qquad l_1 \geq 0,\ k \geq 0$$

(7) Division of triangular fuzzy number by a constant

$$\frac{A}{k} = \left(\frac{l_1}{k},\ \frac{m_1}{k},\ \frac{u_1}{k}\right), \qquad l_1 \geq 0,\ k \geq 0$$

4.6.1.4 Fuzzy TOPSIS

The fuzzy TOPSIS method was proposed by Chen (2000) to solve the multi-criteria decision making problem under uncertainty. Linguistic variables are used by decision makers $D_r\,(r = 1.....k)$, to assess the weight of the criteria and the ratings of the alternatives. Thus, $W_r^{\,j}$ describes the weight of the j^{th} criterion, $C_j\,(j = 1.......m)$, given by the r^{th} decision maker. Similarly $X_{ij}^{\,r}$ describes the rating of the i^{th} alternative, $A_i\,(i = 1......n)$, with respect to criterion j, given by the r^{th} decision maker. Given that, the method comprises the following steps.

(i) Aggregate the weights of criteria and ratings of alternatives given by k decision makers, as expressed in the following equations.

$$w_j = \frac{1}{k}[w_j^{\,1} + w_j^{\,2} + ...+ w_j^{\,m}]$$

$$x_{ij} = \frac{1}{k}[x_{ij}^{\,1} + x_{ij}^{\,2} + ...+ x_{ij}^{\,k}]$$

(ii) Assemble the fuzzy decision matrix to the alternatives (D) and the criterion(W), according to the following equations.

$$D = \begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & C_j & C_m \end{array} \\ \begin{array}{c} A_1 \\ . \\ A_i \\ . \\ A_n \end{array} & \begin{bmatrix} x_{11} & x_{12} & x_{1j} & x_{1m} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ x_{n1} & x_{n2} & x_{nj} & x_{nm} \end{bmatrix} \end{array}$$

$$W = [w_1 + w_2 + ...+ w_m]$$

(iii) Normalize the fuzzy decision matrix of the alternatives (D) using linear scale transformations. The normalized fuzzy decision matrix R is given by:

$$R = [\tilde{r}_{ij}]_{mxn}$$

$$\tilde{r}_{ij} = \left(\frac{l_{ij}}{u_j^+}, \frac{m_{ij}}{u_j^+}, \frac{u_{ij}}{u_j^+}\right) \text{ and } u_j^+ = \max_i u_{ij}\,(\text{benefit criteria})$$

$$\tilde{r}_{ij}=\left(\frac{l^-_j}{u_{ij}},\frac{l^-_j}{m_{ij}},\frac{u_{ij}}{l_{ij}}\right) \text{ and } l^-_j = \max_i l_{ij}\left(\text{cost criteria}\right)$$

(iv) Compute the weighted normalized decision matrix, V, by multiplying the weights of the evaluation criteria w_j by the elements $\tilde{r}_{ij}$ of the normalized fuzzy decision matrix.

$V=[v_{ij}]_{mxn}$ where v_{ij} is given by the equation

$$v_{ij}=x_{ij} x w_j$$

(v) Define the fuzzy positive ideal solution (FPIS, A^+) and the fuzzy negative ideal solution (FNIS, A^-), according to the following equations

$$A^+=\{v^+_1,..v^+_j,.....,v^+_m\}$$

$$A^-=\{v^-_1,..v^-_j,.....,v^-_m\}$$

Where $v^+_j=(1, 1, 1)$ and $v^-_j=(0, 0, 0)$

(vi) Compute the distances d^+_j and d^-_j of each alternative from respectively v^+_j and v^-_j according to the following equations

$$d_i^+=\sum_{j=1}^{n} d_v\left(v_{ij}, v^+{}_j\right)$$

$$d_i^-=\sum_{j=1}^{n} d_v\left(v_{ij}, v^-{}_j\right)$$

Where d_i^+ and d_i^- represents the distance between two fuzzy numbers according to vertex method. This is expressed as

$$d(\bar{x},\bar{z})=\sqrt{\frac{1}{3}[\left(l_x-l_z\right)^2+\left(m_x-m_z\right)^2+\left(u_x-u_z\right)^2]}$$

(vii) Compute the closeness coefficient, CC_i according to the following equations

$$CC_i=\frac{d^-}{d^-+d^+}$$

(viii) Define the ranking of alternatives according to the closeness coefficient CC_i, in decreasing order. The best alternative is closest to the FPIS and farthest to the FNIS.

4.7 Application of fuzzy TOPSIS for finding best option of distribution network for inventory policy: It has been discussed, that there are total 20 alternatives of distribution networks against four performance criteria's. These four criteria's are total cost (C_1), product-miles (C_2), fill rate (C_3), cycle service level (C_4). Here a MCDM approach has to be implemented to find the best alternative of distribution network, from all twenty alternatives of distribution network. The evaluation of twenty alternatives in each criterion was made on linguistic judgments given by the decision makers, a group of the peoples from academia and industry. Evaluation of weight of the criteria and the ratings of alternatives were made by decision makers according to the linguistic terms as shown in Figures 4.3 and 4.4 respectively.

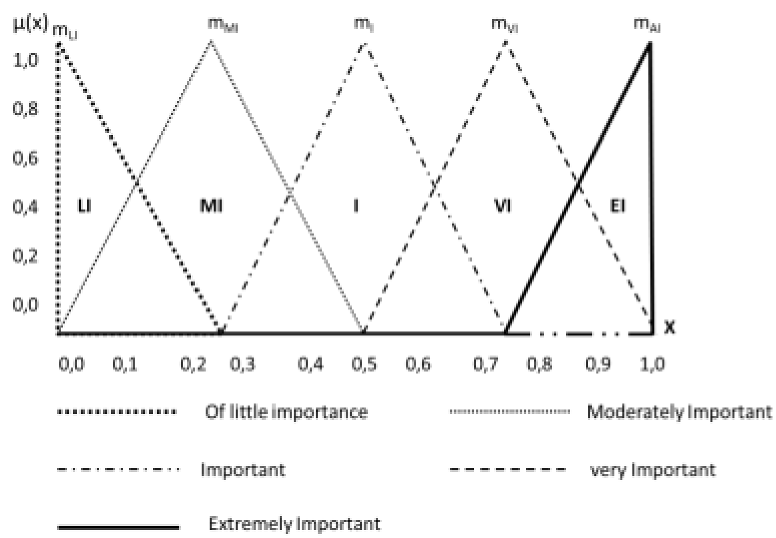


Figure 4.3: Linguistics scale of weights of criteria

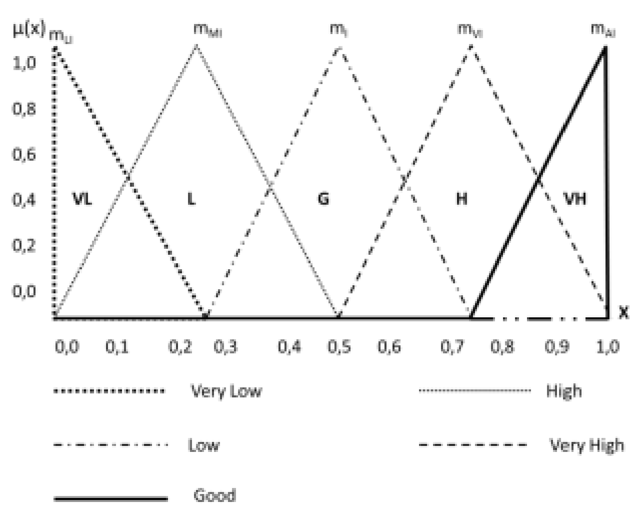


Figure 4.4: Linguistics scale of ratings of alternatives

Triangular fuzzy numbers (TFN) were used to specify the linguistic values of these variables as shown in Tables 4.20 and 4.21 (Lima Junior et al. 2014).

Table 4.20: Linguistic scale to evaluate the weight of criteria

Linguistic Term	Fuzzy triangular number
Little importance (LI)	(0.0, 0.0, 0.25)
Moderately important (MI)	(0.0, 0.25, 0.50)
Important (I)	(0.25, 0.50, 0.75)
Very important (VI)	(0.50, 0.75, 1.0)
Extremely important (EI)	(0.75, 1.0, 1.0)

Table 4.21: Linguistic scale to evaluate the ratings of the alternatives

Linguistic Term	Fuzzy triangular number
Very Low (VL)	(0.0, 0.0, 2.5)
Low (L)	(0.0, 2.5, 5.0)
Good (G)	(2.5, 5.0,7.5)
High (H)	(5.0, 7.5, 10.0)
Very High (VH)	(7.5, 10.0, 10.0)

Table 4.22 shows the linguistic judgments of the weight of criteria and the Table 4.23 represents the fuzzy triangular numbers of the weights of criteria's and Table 4.24 shows the aggregate weights of criteria's in fuzzy triangular numbers. Tables 4.25, 4.26, 4.27, 4.28 and 4.29 show the ratings of alternatives for five decision makers involved in the selection procedure. The linguistic variables shown in Tables 4.25, 4.26, 4.27, 4.28 and 4.29 are converted in to triangular fuzzy number (TFN) in Tables 4.30, 4.31, 4.32, 4.33 and 4.34.

Table 4.22: Linguistic weight of criteria evaluated by DM

	C1	C2	C3	C4
DM1	VI	VI	MI	MI
DM2	EI	I	I	MI
DM3	EI	I	MI	I
DM4	EI	VI	MI	I
DM5	VI	I	I	MI

Table 4.23: Fuzzy triangular numbers of criteria evaluated by DM

	C1	C2	C3	C4
DM1	(0.50. 0.75, 1.0)	(0.50. 0.75, 1.0)	(0.0, 0.25, 0.50)	(0.0, 0.25, 0.50)
DM2	(0.75. 1.0, 1.0)	(0.25, 0.50, 0.75)	(0.25, 0.50, 0.75)	(0.0, 0.25, 0.50)
DM3	(0.75. 1.0, 1.0)	(0.25, 0.50, 0.75)	(0.0, 0.25, 0.50)	(0.25, 0.50, 0.75)
DM4	(0.75. 1.0, 1.0)	(0.50. 0.75, 1.0)	(0.0, 0.25, 0.50)	(0.25, 0.50, 0.75)
DM5	(0.50. 0.75, 1.0)	(0.25, 0.50, 0.75)	(0.25, 0.50, 0.75)	(0.0, 0.25, 0.50)

Table 4.24: Aggregate Fuzzy triangular numbers of weight of criteria

C1	C2	C3	C4
(0.65. 0.90, 1.0)	(0.35, 0.60, 0.85)	(0.10, 0.35, 0.60)	(0.10, 0.25, 0.60)

Table 4.25: Linguistic ratings of alternatives evaluated by first DM

Alternatives	C1	C2	C3	C4
Alternative-1	H	H	G	G
Alternative-2	H	G	G	G
Alternative-3	VH	G	G	G
Alternative-4	VH	G	G	H
Alternative-5	VH	H	G	G
Alternative-6	H	G	H	G

Alternatives	C1	C2	C3	C4
Alternative-7	H	H	H	G
Alternative-8	H	G	H	L
Alternative-9	H	H	G	G
Alternative-10	VH	H	G	L
Alternative-11	VH	H	G	G
Alternative-12	VH	H	H	G
Alternative-13	H	G	H	G
Alternative-14	H	H	G	H
Alternative-15	VH	H	G	L
Alternative-16	VH	G	G	G
Alternative-17	VH	H	H	G
Alternative-18	H	H	H	G
Alternative-19	H	H	G	H
Alternative-20	VH	G	H	G

Table 4.26: Linguistic ratings of alternatives evaluated by second DM

Alternatives	C1	C2	C3	C4
Alternative-1	H	H	G	H
Alternative-2	H	H	G	G
Alternative-3	VH	H	G	G
Alternative-4	VH	H	G	G
Alternative-5	H	G	H	H
Alternative-6	VH	G	G	H
Alternative-7	VH	G	H	H
Alternative-8	VH	G	G	G
Alternative-9	H	G	H	G
Alternative-10	H	H	G	G
Alternative-11	H	H	H	G
Alternative-12	H	G	G	G

Alternatives	C1	C2	C3	C4
Alternative-13	VH	G	G	G
Alternative-14	VH	G	G	H
Alternative-15	VH	G	G	H
Alternative-16	VH	H	G	H
Alternative-17	H	G	H	G
Alternative-18	H	H	H	G
Alternative-19	VH	G	H	G
Alternative-20	H	H	G	H

Table 4.27: Linguistic ratings of alternatives evaluated by third DM

Alternatives	C1	C2	C3	C4
Alternative-1	H	H	G	G
Alternative-2	H	G	G	G
Alternative-3	VH	G	G	G
Alternative-4	VH	G	G	H
Alternative-5	VH	H	G	G
Alternative-6	H	G	H	G
Alternative-7	H	H	H	G
Alternative-8	H	G	H	L
Alternative-9	H	H	G	G
Alternative-10	VH	H	G	L
Alternative-11	VH	H	G	G
Alternative-12	VH	H	H	G
Alternative-13	H	G	H	G
Alternative-14	H	H	G	H
Alternative-15	VH	H	G	L
Alternative-16	VH	G	G	G
Alternative-17	VH	H	H	G
Alternative-18	H	H	H	G

Alternatives	C1	C2	C3	C4
Alternative-19	H	H	G	H
Alternative-20	VH	G	H	G

Table 4.28: Linguistic ratings of alternatives evaluated by fourth DM

Alternatives	C1	C2	C3	C4
Alternative-1	VH	H	G	G
Alternative-2	VH	H	G	H
Alternative-3	VH	H	G	G
Alternative-4	VH	H	G	G
Alternative-5	VH	H	G	H
Alternative-6	VH	H	G	G
Alternative-7	H	G	H	G
Alternative-8	H	G	H	H
Alternative-9	H	G	H	G
Alternative-10	VH	G	G	G
Alternative-11	VH	G	G	G
Alternative-12	VH	G	G	G
Alternative-13	VH	G	G	G
Alternative-14	VH	G	H	G
Alternative-15	H	G	H	H
Alternative-16	H	H	H	H
Alternative-17	H	H	G	G
Alternative-18	VH	H	G	G
Alternative-19	VH	H	G	G
Alternative-20	VH	H	G	H

Table 4.29: Linguistic ratings of alternatives evaluated by fifth DM

Alternatives	C1	C2	C3	C4
Alternative-1	VH	H	H	G

Alternatives	C1	C2	C3	C4
Alternative-2	VH	H	G	G
Alternative-3	VH	H	G	G
Alternative-4	VH	VH	H	G
Alternative-5	VH	VH	H	G
Alternative-6	VH	VH	H	G
Alternative-7	H	VH	G	G
Alternative-8	H	H	G	L
Alternative-9	H	H	G	L
Alternative-10	VH	VH	H	G
Alternative-11	VH	VH	H	G
Alternative-12	VH	VH	H	G
Alternative-13	VH	VH	H	G
Alternative-14	VH	H	G	L
Alternative-15	H	H	G	L
Alternative-16	H	VH	G	L
Alternative-17	H	VH	H	G
Alternative-18	VH	VH	G	G
Alternative-19	VH	VH	G	G
Alternative-20	VH	H	G	L

Table 4.30: FTN of ratings of alternatives evaluated by first DM

Alternatives	C1	C2	C3	C4
Alternative-1	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-2	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-3	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-4	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-5	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-6	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-7	5.0, 7.5, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5

Alternative-8	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	0.0, 2.5, 5.0
Alternative-9	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-10	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0
Alternative-11	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-12	7.5, 10.0, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-13	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-14	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-15	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0
Alternative-16	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-17	7.5, 10.0, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-18	5.0, 7.5, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-19	5.0, 7.5, 10.0	5.0, 7.5, 10.0	(2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-20	7.5, 10.0, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5

Table 4.31: FTN of ratings of alternatives evaluated by second DM

Alternatives	C1	C2	C3	C4
Alternative-1	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-2	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-3	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-4	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-5	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	5.0, 7.5, 10.0
Alternative-6	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-7	7.5, 10.0, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	5.0, 7.5, 10.0
Alternative-8	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-9	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-10	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-11	5.0, 7.5, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-12	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-13	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5

Alternatives	C1	C2	C3	C4
Alternative-14	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-15	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-16	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-17	5.0, 7.5, 10.0	(2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-18	5.0, 7.5, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-19	7.5, 10.0, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-20	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0

Table 4.32: FTN of ratings of alternatives evaluated by third DM

Alternatives	C1	C2	C3	C4
Alternative-1	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-2	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-3	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-4	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-5	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-6	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-7	5.0, 7.5, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-8	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	0.0, 2.5, 5.0
Alternative-9	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-10	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0
Alternative-11	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-12	7.5, 10.0, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-13	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-14	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-15	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0
Alternative-16	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-17	7.5, 10.0, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-18	5.0, 7.5, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5

Alternatives	C1	C2	C3	C4
Alternative-19	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-20	7.5, 10.0, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5

Table 4.33: FTN of ratings of alternatives evaluated by fourth DM

Alternatives	C1	C2	C3	C4
Alternative-1	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-2	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-3	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-4	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-5	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-6	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-7	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-8	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	5.0, 7.5, 10.0
Alternative-9	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-10	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-11	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-12	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-13	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-14	7.5, 10.0, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	2.5, 5.0, 7.5
Alternative-15	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0	5.0, 7.5, 10.0
Alternative-16	5.0, 7.5, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0	5.0, 7.5, 10.0
Alternative-17	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-18	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-19	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-20	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	5.0, 7.5, 10.0

Table 4.34: FTN of ratings of alternatives evaluated by fifth DM

Alternatives	C1	C2	C3	C4
Alternative-1	5.0, 7.5, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-2	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-3	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-4	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-5	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-6	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-7	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	5.0, 7.5, 10.0
Alternative-8	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0	5.0, 7.5, 10.0
Alternative-9	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0	5.0, 7.5, 10.0
Alternative-10	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-11	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-12	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-13	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-14	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0	0.0, 2.5, 5.0
Alternative-15	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0	0.0, 2.5, 5.0
Alternative-16	7.5, 10.0, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0	2.5, 5.0, 7.5
Alternative-17	7.5, 10.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-18	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-19	7.5, 10.0, 10.0	2.5, 5.0, 7.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5
Alternative-20	5.0, 7.5, 10.0	2.5, 5.0, 7.5	0.0, 2.5, 5.0	5.0, 7.5, 10.0

Table 4.35 represents aggregation of all the parameters in TFN resulting from all the decision makers

Table 4.35: Aggregate FTN of ratings of alternatives

Alternatives	C1	C2	C3	C4
Alternative-1	5.5, 8.0, 10.0	5.0, 7.5, 10.0	2.5, 5.0, 7.5	3.0, 5.5, 8.0
Alternative-2	5.5, 8.0, 10.0	3.5, 6.0, 8.5	2.5, 5.0, 7.5	3.0, 5.5, 8.0
Alternative-3	6.9, 9.5, 10.0	3.5, 6.0, 8.5	2.5, 5.0, 7.5	2.5, 5.0, 7.5

Alternative-4	7.5, 10.0,10.0	4.0, 6.5, 9.0	2.5, 5.0, 7.5	3.5, 6.0, 8.5
Alternative-5	7.0, 9.5, 10.0	4.5, 7.0, 9.5	3.0, 5.5, 8.0	3.5, 6.0, 8.5
Alternative-6	6.5, 9.0, 10.0	3.5, 6.0, 8.5	3.5, 6.0, 8.5	3.0, 5.5, 8.0
Alternative-7	6.0, 8.5, 10.0	3.5, 6.0, 8.5	4.5, 7.0, 9.5	3.5, 6.0, 8.5
Alternative-8	5.5, 8.0, 10.0	2.5, 5.0, 7.5	3.5, 6.0, 8.5	2.5, 5.0, 7.5
Alternative-9	5.5, 7.5, 10.0	3.5, 6.0, 8.5	3.0, 5.5, 8.0	3.0, 5.5, 8.0
Alternative-10	7.0, 9.5, 10.0	4.5, 7.0, 9.5	2.5, 5.0, 7.5	1.5, 4.0, 6.5
Alternative-11	7.0, 9.5, 10.0	4.5, 7.0, 9.5	3.0, 5.5, 8.0	2.5, 5.0, 7.5
Alternative-12	7.0, 9.5, 10.0	4.0, 6.5, 9.0	3.5, 6.0, 8.5	2.5, 5.0, 7.5
Alternative-13	6.5, 9.5, 10.0	3.0, 5.5, 8.0	3.5, 6.0, 8.5	2.5, 5.5, 7.5
Alternative-14	6.0, 8.5, 10.0	3.5, 6.0, 8.5	2.5, 5.0, 7.5	3.5, 6.0, 8.5
Alternative-15	6.5, 9.0, 10.0	3.5, 6.0, 8.5	2.5, 5.0, 7.5	2.0, 4.5, 7.0
Alternative-16	7.0, 9,5, 10.0	3.5, 6.0, 8.5	2.5, 5.0, 7.5	3.5, 6.0, 8.5
Alternative-17	6.5, 9.0, 10.0	4.5, 7.0, 9.5	4.0, 6.5, 9.0	2.5, 5.0, 7.5
Alternative-18	6.0, 8.0, 10.0	4.5, 7.0, 9.5	4.0, 6.5, 9.0	2.5, 5.0, 7.5
Alternative-19	6.5, 9.0, 10.0	4.0, 6.5, 9.0	3.0, 5.5, 8.0	3.5, 6.0, 8.5
Alternative-20	6.5, 9.0, 10.0	3.5, 6.0, 8.5	3.0, 5.5, 8.0	4.0, 6.5, 9.5

The normalized fuzzy decision matrix is shown in Table 4.36.

Table 4.36: Normalized fuzzy decision matrix

Alternatives	**C1**	**C2**	**C3**	**C4**
Alternative-1	.55, .80, 1.0	.55, .75, 1.0	.26, .53, .79	.35, .65, .94
Alternative-2	.55, .80, 1.0	.35, .60, .85	.26, .53, .79	.35, .65, .94
Alternative-3	.69, .95, 1.0	.35, .60, .85	.26, .53, .79	.29, .59, .88
Alternative-4	.75, 1.0,1.0	.40, .65, .90	.26, .53, .79	.41, .71, 1.0
Alternative-5	.70, .95, 1.0	.45, .7.0, 9.5	.32,.58, .84	.41, .71, 1.0
Alternative-6	.65, .90, 1.0	.35, .60, .85	.37, .63, .89	.35, .65, .94
Alternative-7	.60, .85, 1.0	.35, .60, .85	.47, .74, 1.0	.41, .71, 1.0
Alternative-8	.55, .80, 1.0	.25, .50, .75	.37, .63, .89	.29, .59, .88

Alternative-9	.55, .75, 1.0	.35, .60, .85	.32, .58, .84	.35, .65, .94
Alternative-10	.70, .95, 1.0	.45, .70, .95	.26, .53, .79	.18, .47, .76
Alternative-11	.70, .95, 1.0	.45, .70, .95	.32, .58, .84	.29, .59, .88
Alternative-12	.70, .95, 1.0	.40, .65, .90	.37, .63, .89	.29, .59, .88
Alternative-13	.65, .95, 1.0	.30, .55, .80	.37, .63, .89	.29, .59, .88
Alternative-14	.60, .85, 1.0	.35, .60, .85	.26, .53, .79	.41, .71, 1.0
Alternative-15	.65, .90, 1.0	.35, .60, .85	.26, .53, .79	.24, .53, .82
Alternative-16	.70, .95, 1.0	.35, .60, .85	.26, .53, .79	.41, .71, 1.0
Alternative-17	.65, .90, 1.0	.45, .70, .95	.42, .68, .95	.29, .59, .88
Alternative-18	.60, .80, 1.0	.45, .70, .95	42, .68, .95	.29, .59, .88
Alternative-19	.65, .90, 1.0	.40, .65, .90	.32, .58, .84	.41, .71, 1.0
Alternative-20	.65, .90, 1.0	.35, .60, .85	.32, .58, .84	.47, .76, 1.0

The weighted normalized fuzzy decision matrix is shown in Table 4.37.

Table 4.37: Weighted normalized fuzzy decision matrix

	C1	**C2**	**C3**	**C4**
Alternative-1	.36, .72, 1.0	.18, .45, .85	.026, .18, .47	.04, .16, .56
Alternative-2	.36, .72, 1.0	.12, .36, .72	.026, .18, .47	.04, .16, .56
Alternative-3	.45, .86, 1.0	.12, .36, .72	.026, .18, .47	.03, .15, .53
Alternative-4	.49, .90, 1.0	.14, .39, .77	.026, .18, .47	.04, .18, .60
Alternative-5	.46, .86, 1.0	.16, .42, .81	.032, .20, .51	.04, .18, .60
Alternative-6	.42, .81, 1.0	.12, .36, .72	.037, .22, .54	.04, .16, .56
Alternative-7	.39, .77, 1.0	.12, .36, .72	.047, .26, .60	.04, .18, .60
Alternative-8	.36, .72, 1.0	.09, .30, .72	.037, .22, .54	.03, .15, .53
Alternative-9	.33, .72, 1.0	.12, .36, .72	.032, .20, .51	.04, .16, .56
Alternative-10	.46, .86, 1.0	.16, .42, .81	.026, .18, .47	.02, .12, .46
Alternative-11	.46, .86, 1.0	.16, .42, .81	.032, .20, .51	.03, .15, .53
Alternative-12	.46, .86, 1.0	.14, .39, .77	.037, .22, .54	.03, .15, .53
Alternative-13	.42, .81, 1.0	.11, .33, .68	.037, .22, .54	.03, .15, .53

	C1	C2	C3	C4
Alternative-14	.39, .77, 1.0	.12, .36, .72	.026, .18, .47	.04, .18, .60
Alternative-15	.42, .81, 1.0	.12, .36, .72	.026, .18, .47	.02, .13, .49
Alternative-16	.46, .86, 1.0	.12, .36, .72	.026, .18, .47	.04, .18, .60
Alternative-17	.42, .81, 1.0	.16, .42, .81	.042, .24, .57	.03, .15, .53
Alternative-18	.39, .72, 1.0	.16, .42, .81	.042, .24, .57	.03, .15, .53
Alternative-19	.42, .81, 1.0	.14, .39, .77	.032, .20, .51	.04, .18, .60
Alternative-20	.42, .81, 1.0	.12, .36, .72	.032, .20, .51	.047, .20, .64

According to Chen (2000), the fuzzy positive ideal solution (FPIS, A^+) and the fuzzy negative ideal solution (FNIS, A^-), are defined as

$$A^+ = \left[(1,1,1),\ (1,1,1),\ (1,1,1),\ (1,1,1),\ (1,1,1),\ (1,1,1),\ (1,1,1),\ (1,1,1),\ (1,1,1)\right]$$
$$A^- = \left[(0,0,0),\ (0,0,0),\ (0,0,0),\ (0,0,0),\ (0,0,0),\ (0,0,0),\ (0,0,0),\ (0,0,0),\ (0,0,0)\right]$$

Table 5.19 shows the distances of the ratings of each alternative from A^+ with respect to each criterion and these distances are calculated by

$$d(\bar{x},\bar{z})=\sqrt{\frac{1}{3}[(l_x-l_z)^2+(m_x-m_z)^2+(u_x-u_z)^2]}$$

In the last column all the distances (d_i^+) are found out by adding all distances of each column. Similarly table 5.20 shows the distances of the ratings of each alternative from A^- with respect to each criterion and it is also calculated by using the formula

$$d(\bar{x},\bar{z})=\sqrt{\frac{1}{3}[(l_x-l_z)^2+(m_x-m_z)^2+(u_x-u_z)^2]}$$

In the last column all the distances (d_i^-) are found out by adding all distances of each column.

Table 4.38: Distances of the ratings of each alternative from A^+

Alternatives	C1	C2	C3	C4	Total Distance (d_i^+)
Alternative-1	0.507545	0.578972	0.793848	0.779482	2.659846531
Alternative-2	0.507545	0.647202	0.793848	0.779482	2.728077053
Alternative-3	0.439612	0.647202	0.793848	0.793936	2.674598342
Alternative-4	0.413383	0.623679	0.793848	0.765407	2.596315948

Alternative-5	0.436902	0.600906	0.778546	0.765407	2.581760169
Alternative-6	0.460437	0.647202	0.763533	0.779482	2.650654084
Alternative-7	0.483985	0.647202	0.734445	0.765407	2.631039145
Alternative-8	0.507545	0.696195	0.763533	0.793936	2.76120912
Alternative-9	0.531115	0.647202	0.778546	0.779482	2.736345257
Alternative-10	0.436902	0.600906	0.793848	0.8239	2.655555549
Alternative-11	0.436902	0.600906	0.778546	0.793936	2.610289323
Alternative-12	0.436902	0.623679	0.763533	0.793936	2.618049526
Alternative-13	0.460437	0.671398	0.763533	0.793936	2.689303586
Alternative-14	0.483985	0.647202	0.793848	0.765407	2.690441957
Alternative-15	0.460437	0.647202	0.793848	0.808749	2.710235345
Alternative-16	0.436902	0.647202	0.793848	0.765407	2.643358736
Alternative-17	0.460437	0.600906	0.748826	0.793936	2.604104892
Alternative-18	0.493997	0.600906	0.748826	0.793936	2.637665353
Alternative-19	0.460437	0.623679	0.778546	0.765407	2.628068073
Alternative-20	0.460437	0.647202	0.778546	0.751732	2.637916813

Table 4.39: Distances of the ratings of each alternative from A^-

	C1	**C2**	**C3**	**C4**	**Total Distance (d_i^-)**
Alternative-1	0.655186	0.564395	0.293827	0.339758	1.853166
Alternative-2	0.655186	0.471385	0.293827	0.339758	1.760156
Alternative-3	0.689632	0.471385	0.293827	0.317683	1.772528
Alternative-4	0.704191	0.502303	0.293827	0.361864	1.862186
Alternative-5	0.69105	0.533311	0.314827	0.361864	1.901052
Alternative-6	0.67848	0.471385	0.335868	0.339758	1.825491
Alternative-7	0.666515	0.471385	0.378045	0.361864	1.877809
Alternative-8	0.655186	0.409903	0.335868	0.317683	1.71864
Alternative-9	0.644528	0.471385	0.314827	0.339758	1.770498
Alternative-10	0.69105	0.533311	0.293827	0.273661	1.791849
Alternative-11	0.69105	0.533311	0.314827	0.317683	1.856871

Alternative-12	0.69105	0.502303	0.335868	0.317683	1.846904
Alternative-13	0.67848	0.440577	0.335868	0.317683	1.772608
Alternative-14	0.666515	0.471385	0.293827	0.361864	1.793591
Alternative-15	0.67848	0.471385	0.293827	0.295648	1.73934
Alternative-16	0.69105	0.471385	0.293827	0.361864	1.818127
Alternative-17	0.67848	0.533311	0.356942	0.317683	1.886417
Alternative-18	0.661337	0.533311	0.356942	0.317683	1.869274
Alternative-19	0.67848	0.502303	0.314827	0.361864	1.857474
Alternative-20	0.67848	0.471385	0.314827	0.383997	1.84869

Finally closeness coefficient (CC_i) is calculated for each alternative and each criterion by using following formula:

$$CC_i = d^- / (d^- + d^+)$$

Table 4.40 represents the Closeness coefficient (CC_i) for each alternative and each criterion.

Table 4.40: Closeness coefficient (CC_i) for each alternative and each criterion

Alternatives	**Closeness Coefficient (CC_i)**
Alternative-1	0.410627
Alternative-2	0.392171
Alternative-3	0.398578
Alternative-4	0.417671
Alternative-5	0.424076
Alternative-6	0.407827
Alternative-7	0.416472
Alternative-8	0.383638
Alternative-9	0.392847
Alternative-10	0.402898
Alternative-11	0.415671
Alternative-12	0.413645
Alternative-13	0.397276

Alternatives	Closeness Coefficient (CC_i)
Alternative-14	0.399995
Alternative-15	0.3909
Alternative-16	0.407516
Alternative-17	0.420089
Alternative-18	0.414755
Alternative-19	0.414103
Alternative-20	0.412046

Now outranking is applied to the above Table 4.41 and the outranking results are shown in Table 4.41. This Table shows that Alternative 5 is the best alternative among all the alternatives, because it has highest closeness coefficient.

Table 4.41: Outranking of alternatives

Alternatives	Closeness Coefficient (CC_i)
Alternative-5	0.424076
Alternative-17	0.420089
Alternative-4	0.417671
Alternative-7	0.416472
Alternative-11	0.415671
Alternative-18	0.414755
Alternative-19	0.414103
Alternative-12	0.413645
Alternative-20	0.412046
Alternative-1	0.410627
Alternative-6	0.407827

Alternatives	Closeness Coefficient (CC_i)
Alternative-16	0.407516
Alternative-10	0.402898
Alternative-14	0.399995
Alternative-3	0.398578
Alternative-13	0.397276
Alternative-9	0.392847
Alternative-2	0.392171
Alternative-15	0.3909
Alternative-8	0.383638

From the fuzzy TOPSIS calculations, it is found that alternative 5 is best among all the alternatives.

4.8 Discussion: It has been found that alternative 5 is best for the given criteria's of total cost, product-miles, cycle service level. In alternative 5 distributors D1 is responsible to fulfill the demands of retailers R1 and R2. Distributor D2 is inactive in the network. Retailers R3, R4 and R5 are pooled for distributor D3 and distributor D3 is responsible to fulfill the demands of retailers, R3, R4 and R5. There is no pooling at retailer R6 and R7. Distributor D4 is responsible to fulfill the demand of retailer R6 and D5 is responsible to fulfill the demand of retailer R7. Alternative 5 can be shown in Figure 4.3 and the inventory policy for alternative 5 can be represented in the following tables:

Now the result analysis is discussed only for alternative 5. In this alternative, the parameters reorder point, safety stocks, expected shortages per cycle, fill rates, average inventory and total expected cost are analyzed for four different uncertainty levels. These four uncertainty levels are 0.2, 0.4, 0.6 and 0.8. The results for alternative 5 are shown in the following tables.

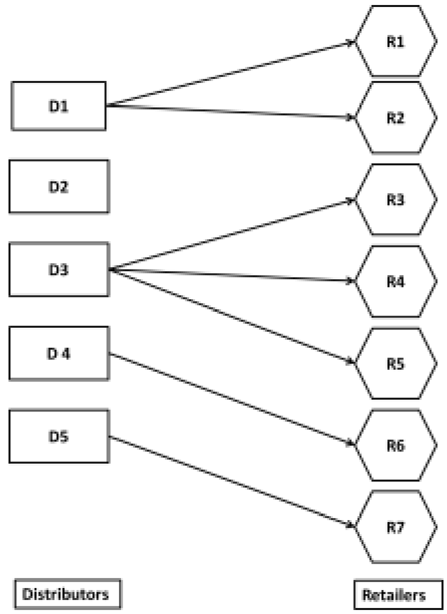


Figure 4.5: Alternative 5

Table 4.42: Alternative-5, Planning period-1, Product-1

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3420.526	301.5149	2.246309	0.999343	9.0149	1719.278	56250	774489.1801
D-3	3949.684	399.8279	2.448905	0.99938	9.8279	1984.67	125000	1613551.846
D-4	2449.49	156.3705	1.587389	0.999352	6.3705	1231.115	45000	600571.4179
D-5	2190.89	125.0964	1.269911	0.99942	5.0964	1100.542	36000	483546.2574
							Total cost	3472158.702

Table 4.43: Alternative-5, Planning period-1, Product-2

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3942.081	286.0765	2.137087	0.999458	8.5765	1979.617	53250	831099.6994
D-3	4582.576	384.3301	2.324861	0.999493	9.3301	2300.618	119250	1651343.451
D-4	2824.889	148.552	1.50802	0.999466	6.052	1418.497	42750	631377.183
D-5	2509.98	117.2779	1.190542	0.999526	4.7779	1259.768	33750	504664.1777
							Total cost	3618484.511

Table 4.44: Alternative-5, Planning period-1, Product-3

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4449.719	255.098	1.893257	0.999575	7.598	2232.458	47750	858293.9479
D-3	5253.57	353.6061	2.144445	0.999592	8.6061	2635.391	110000	1666415.675
D-4	3286.335	140.7334	1.42865	0.999565	5.7334	1648.901	40500	670973.2502
D-5	2792.848	101.6408	1.031803	0.999631	4.1408	1400.565	29250	499850.2366
							Total cost	3695533.11

Table 4.45: Alternative-5, Planning period-2, Product-1

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3331.666	286.0765	2.137087	0.999359	8.5765	1674.41	53250	735279.7705
D-3	3754.997	361.3788	2.212401	0.999411	8.8788	1886.377	113000	1460872.977
D-4	2323.79	140.7334	1.42865	0.999385	5.7334	1167.628	40500	543763.6262
D-5	2049.39	109.4593	1.111172	0.999458	4.4593	1029.154	31500	426077.173
							Total cost	3165993.547

Table 4.46: Alternative-5, Planning period-2, Product-2

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3778.889	262.867	1.960281	0.999481	7.867	1897.311	49000	770258.152
D-3	4395.452	353.6061	2.144445	0.999512	8.6061	2206.332	110000	1527064.321
D-4	2672.078	132.9149	1.349281	0.999495	5.4149	1341.454	38250	571970.145
D-5	2424.871	109.4593	1.111172	0.999542	4.4593	1216.895	31500	474723.3942
							Total cost	3344016.012

Table 4.47: Alternative-5, Planning period-2, Product-3

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4312.772	239.6579	1.783606	0.999586	7.1579	2163.544	44750	811707.496
D-3	5019.96	322.8411	1.953842	0.999611	7.8411	2517.821	100250	1526771.312
D-4	3098.387	125.0964	1.269911	0.99959	5.0964	1554.29	36000	606839.5098
D-5	2683.282	93.82229	0.952433	0.999645	3.8223	1345.463	27000	467634.4334
							Total cost	3412952.752

Table 4.48: Alternative-5, Planning period-3, Product-1

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3217.142	266.8724	2.023921	0.999371	8.1224	1616.693	49125	681021.6169
D-3	3714.835	353.758	2.182299	0.999413	8.758	1866.176	111000	1434904.563
D-4	2387.467	148.552	1.50802	0.999368	6.052	1199.786	42750	572281.3606
D-5	2049.39	109.4593	1.111172	0.999458	4.4593	1029.154	31500	426015.4238
							Total cost	3114222.964

Table 4.49: Alternative-5, Planning period-3, Product-2

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3888.444	278.1582	2.03285	0.999477	8.1582	1952.38	53000	827110.8863
D-3	4347.413	345.8851	2.089393	0.999519	8.3851	2182.092	107250	1491865.956
D-4	2672.078	132.9149	1.349281	0.999495	5.4149	1341.454	38250	572909.1627
D-5	2592.296	125.0964	1.269911	0.99951	5.0964	1301.245	36000	537272.2463
							Total cost	3429158.251

Table 4.50: Alternative-5, Planning period-3, Product-3

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4449.719	255.0174	1.873186	0.999579	7.5174	2232.377	48250	868311.6828
D-3	5079.37	330.4886	1.990576	0.999608	7.9886	2547.674	101750	1553322.294
D-4	3098.387	125.0964	1.269911	0.99959	5.0964	1554.29	36000	608238.3705
D-5	2898.275	109.4593	1.111172	0.999617	4.4593	1453.597	31500	535877.3922
							Total cost	3565749.74

Table 4.51: Alternative-5, Planning period-4, Product-1

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3146.427	255.098	1.893257	0.999398	7.598	1580.811	47750	663715.6471
D-3	3549.648	322.9312	1.976283	0.999443	7.9312	1782.755	101000	1309723.089
D-4	2190.89	125.0964	1.269911	0.99942	5.0964	1100.542	36000	487897.0696
D-5	1897.367	93.82229	0.952433	0.999498	3.8223	952.5056	27000	369406.2001
							Total cost	2830742.006

Table 4.52: Alternative-5, Planning period-4, Product-2

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3666.061	247.333	1.827221	0.999502	7.333	1840.363	46500	734694.9374
D-3	4098.78	307.4836	1.86476	0.999545	7.4836	2056.874	95500	1332847.978
D-4	2509.98	117.2779	1.190542	0.999526	4.7779	1259.768	33750	512868.7716
D-5	2424.871	109.4593	1.111172	0.999542	4.4593	1216.895	31500	476001.1338
							Total cost	3056412.82

Table 4.53: Alternative-5, Planning period-4, Product-3

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4242.641	231.8907	1.717023	0.999595	6.8907	2128.211	43500	796148.305
D-3	5019.96	322.9312	1.976283	0.999606	7.9312	2517.911	101000	1542570.362
D-4	3000	117.2779	1.190542	0.999603	4.7779	1504.778	33750	578074.4509
D-5	2738.613	97.73156	0.992118	0.999638	3.9816	1373.288	28125	488082.8367
							Total cost	3404875.954

Table 4.54: Alternative-5, Planning period-5, Product-1

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	3754.997	363.2782	2.685691	0.999285	10.778	1888.277	68250	932743.4302
D-3	4312.772	476.7293	2.922693	0.999322	11.729	2168.115	149000	1919358.124
D-4	2569.047	172.0075	1.746128	0.99932	7.0075	1291.531	49500	657334.3602
D-5	2449.49	156.3705	1.587389	0.999352	6.3705	1231.115	45000	598702.0833
							Total cost	4108137.998

Table 4.55: Alternative-5, Planning period-5, Product-2

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4249.706	332.3124	2.445043	0.999425	9.8124	2134.665	62750	965901.9159
D-3	4977.951	453.4833	2.736808	0.99945	10.983	2499.959	140500	1937888.861
D-4	3039.737	172.0075	1.746128	0.999426	7.0075	1526.876	49500	720795.1211
D-5	2824.889	148.552	1.50802	0.999466	6.052	1418.497	42750	626784.2353
							Total cost	4251370.133

Table 4.56: Alternative-5, Planning period-5, Product-3

	Q(i)	Ro	ESC	FR	SS	AI	PM	Total Cost
D-1	4959.839	316.8681	2.334325	0.999529	9.3681	2489.287	59750	1048922.072
D-3	5899.152	445.7829	2.686864	0.999545	10.783	2960.359	138000	2072935.516
D-4	3633.18	172.0075	1.746128	0.999519	7.0075	1823.598	49500	799464.0181
D-5	3000	117.2779	1.190542	0.999603	4.7779	1504.778	33750	564625.8005
							Total cost	4485947.406

Tables 4.42 to Tables 4.56 show the results for all three types of products and for all five planning periods which mainly includes ordering quantity, reorder point, expected shortages per cycle, fill rates, safety stocks, average inventory, product-miles and total cost. Now all these parameters are evaluated under different uncertainty levels for demand, supply and combined demand and supply uncertainty. For this we are considering four different uncertainty levels which are 0.2, 0.4, 0.6 and 0.8. Here we are showing the effects of uncertainty levels on all three types of products for planning period 1. Following Tables show the effects of demand uncertainty levels on different types of products.

Table 4.57: ROP vs. demand uncertainty levels in planning period-1

Demand uncertainty = 0.2		Demand uncertainty = 0.4	Demand uncertainty = 0.6	Demand uncertainty = 0.8
PRODUCT 1				
D-1	303.3178	305.1208	306.9238	308.7268
D-3	401.7935	403.7591	405.7247	407.6902
D-4	157.6446	158.9187	160.1928	161.4669
D-5	126.1157	127.1349	128.1542	129.1735
PRODUCT 2				
D-1	292.9378	289.5071	291.2225	292.9378
D-3	391.7942	388.0621	389.9282	391.7942
D-4	153.3935	150.9728	152.1831	153.3935

D-5	121.1002	119.189	120.1446	121.1002
PRODUCT 3				
D-1	256.6176	258.1372	259.6568	261.1764
D-3	355.3273	357.0485	358.7697	360.4909
D-4	141.8801	143.0268	144.1735	145.3202
D-5	102.469	103.2971	104.1253	104.9535

Table 4.58: ESC vs. demand uncertainty levels in planning period-1

Demand uncertainty = 0.2		**Demand uncertainty = 0.4**	**Demand uncertainty = 0.6**	**Demand uncertainty = 0.8**
PRODUCT 1				
D-1	2.695571	3.144833	3.594095	4.043357
D-3	2.938686	3.428466	3.918247	4.408028
D-4	1.904867	2.222345	2.539822	2.8573
D-5	1.523893	1.777876	2.031858	2.28584
Demand uncertainty = 0.2		**Demand uncertainty = 0.4**	**Demand uncertainty = 0.6**	**Demand uncertainty = 0.8**
PRODUCT 2				
D-1	3.846757	2.991922	3.41934	3.846757
D-3	4.18475	3.254806	3.719778	4.18475
D-4	2.714435	2.111227	2.412831	2.714435
D-5	2.142975	1.666758	1.904867	2.142975
PRODUCT 3				
D-1	2.271908	2.650559	3.029211	3.407862
D-3	2.573334	3.002223	3.431112	3.860001
D-4	1.71438	2.00011	2.28584	2.57157
D-5	1.238163	1.444524	1.650885	1.857245

Table 4.59: FR vs. demand uncertainty levels in planning period-1

Demand uncertainty = 0.2		**Demand uncertainty = 0.4**	**Demand uncertainty = 0.6**	**Demand uncertainty = 0.8**
PRODUCT 1				
D-1	0.999212	0.999081	0.998949	0.998818
D-3	0.999256	0.999132	0.999008	0.998884
D-4	0.999222	0.999093	0.998963	0.998834
D-5	0.999304	0.999189	0.999073	0.998957
PRODUCT 2				
D-1	0.999024	0.999241	0.999133	0.999024
D-3	0.999087	0.99929	0.999188	0.999087
D-4	0.999039	0.999253	0.999146	0.999039
D-5	0.999146	0.999336	0.999241	0.999146

PRODUCT 3				
D-1	0.999489	0.999404	0.999319	0.999234
D-3	0.99951	0.999429	0.999347	0.999265
D-4	0.999478	0.999391	0.999304	0.999217
D-5	0.999557	0.999483	0.999409	0.999335

Table 4.60: SS vs. demand uncertainty levels in planning period-1

Demand uncertainty = 0.2		**Demand uncertainty = 0.4**	**Demand uncertainty = 0.6**	**Demand uncertainty = 0.8**
PRODUCT 1				
D-1	10.818	12.621	14.424	16.227
D-3	11.793	13.759	15.725	17.69
D-4	7.6446	8.9187	10.193	11.467
D-5	6.1157	7.1349	8.1542	9.1735
Demand uncertainty = 0.2		**Demand uncertainty = 0.4**	**Demand uncertainty = 0.6**	**Demand uncertainty = 0.8**
PRODUCT 2				
D-1	15.438	12.007	13.722	15.438
D-3	16.794	13.062	14.928	16.794
D-4	10.894	8.4728	9.6831	10.894
D-5	8.6002	6.689	7.6446	8.6002
PRODUCT 3				
D-1	9.1176	10.637	12.157	13.676
D-3	10.327	12.048	13.77	15.491
D-4	6.8801	8.0268	9.1735	10.32
D-5	4.969	5.7971	6.6253	7.4535

Table 4.61: AI vs. demand uncertainty levels in planning period-1

Demand uncertainty = 0.2		**Demand uncertainty = 0.4**	**Demand uncertainty = 0.6**	**Demand uncertainty = 0.8**
PRODUCT 1				
D-1	1721.081	1722.884	1724.687	1726.49
D-3	1986.635	1988.601	1990.566	1992.532
D-4	1232.389	1233.664	1234.938	1236.212
D-5	1101.561	1102.58	1103.599	1104.619
PRODUCT 2				
D-1	1986.478	1983.047	1984.763	1986.478
D-3	2308.082	2304.35	2306.216	2308.082
D-4	1423.338	1420.917	1422.128	1423.338
D-5	1263.59	1261.679	1262.635	1263.59

PRODUCT 3				
D-1	2233.977	2235.497	2237.016	2238.536
D-3	2637.112	2638.834	2640.555	2642.276
D-4	1650.048	1651.194	1652.341	1653.488
D-5	1401.393	1402.221	1403.049	1403.877

Table 4.62: TC vs. demand uncertainty levels in planning period-1

Demand uncertainty = 0.2		Demand uncertainty = 0.4	Demand uncertainty = 0.6	Demand uncertainty = 0.8
PRODUCT 1				
	Total Cost	**Total Cost**	**Total Cost**	**Total Cost**
D-1	774491.9	774494.6	774497.3	774500
D-3	1613555	1613558	1613561	1613564
D-4	600573.3	600575.2	600577.2	600579.1
D-5	483547.8	483549.3	483550.8	483552.4
Demand uncertainty = 0.2		**Demand uncertainty = 0.4**	**Demand uncertainty = 0.6**	**Demand uncertainty = 0.8**
PRODUCT 2				
D-1	831123.7	831111.7	831117.7	831123.7
D-3	1651370	1651357	1651363	1651370
D-4	631394.1	631385.7	631389.9	631394.1
D-5	504677.6	504670.9	504674.2	504677.6
PRODUCT 3				
D-1	858304.2	858314.5	858324.7	858335
D-3	1666427	1666439	1666451	1666462
D-4	670981	670988.7	670996.5	671004.2
D-5	499855.8	499861.4	499867	499872.6

Similarly same variable may be calculated for different supply uncertainty levels. For different supply uncertainty levels, the change in variables can be elaborate in the following tables:

Table 4.63: ROP vs. supply uncertainty levels in planning period-1

Supply uncertainty = 0.2		Supply uncertainty = 0.4	Supply uncertainty = 0.6	Supply uncertainty = 0.8
PRODUCT 1				
D-1	302.3753	304.1846	303.903	304.5947
D-3	400.7659	402.7384	402.4314	403.1855
D-4	156.9785	158.2571	158.0581	158.5469
D-5	125.5828	126.6057	126.4465	126.8375

PRODUCT 2				
D-1	286.8951	288.6165	288.3486	289.0066
D-3	385.2206	387.0932	386.8018	387.5177
D-4	149.1296	150.3442	150.1552	150.6196
D-5	117.7339	118.6928	118.5436	118.9102
PRODUCT 3				
D-1	255.8232	257.3481	257.1108	257.6938
D-3	354.4275	356.1547	355.8859	356.5462
D-4	141.2807	142.4314	142.2523	142.6922
D-5	102.036	102.8671	102.7378	103.0555

Table 4.64: ESC vs. supply uncertainty levels in planning period-1

Supply uncertainty = 0.2		Supply uncertainty = 0.4	Supply uncertainty = 0.6	Supply uncertainty = 0.8
PRODUCT 1				
D-1	2.029246	1.625317	1.683699	1.543128
D-3	2.212264	1.771905	1.835552	1.682304
D-4	1.433998	1.148555	1.189811	1.090475
D-5	1.147198	0.918844	0.951849	0.87238
PRODUCT 2				
D-1	1.930578	1.54629	1.601832	1.468097
D-3	2.100207	1.682153	1.742576	1.597091
D-4	1.362298	1.091127	1.130321	1.035952
D-5	1.075498	0.861416	0.892359	0.817857
PRODUCT 3				
D-1	1.710309	1.369866	1.419071	1.300595
D-3	1.937225	1.551613	1.607347	1.473152
D-4	1.290598	1.0337	1.07083	0.981428
D-5	0.932098	0.746561	0.773377	0.708809

Table 4.65: FR vs. supply uncertainty levels in planning period-1

Supply uncertainty = 0.2		Supply uncertainty = 0.4	Supply uncertainty = 0.6	Supply uncertainty = 0.8
PRODUCT 1				
D-1	0.999407	0.999525	0.999508	0.999549
D-3	0.99944	0.999551	0.999535	0.999574
D-4	0.999415	0.999531	0.999514	0.999555
D-5	0.999476	0.999581	0.999566	0.999602

PRODUCT 2				
D-1	0.99951	0.999608	0.999594	0.999628
D-3	0.999542	0.999633	0.99962	0.999651
D-4	0.999518	0.999614	0.9996	0.999633
D-5	0.999572	0.999657	0.999644	0.999674
PRODUCT 3				
D-1	0.999616	0.999692	0.999681	0.999708
D-3	0.999631	0.999705	0.999694	0.99972
D-4	0.999607	0.999685	0.999674	0.999701
D-5	0.999666	0.999733	0.999723	0.999746

Table 4.66: SS vs. supply uncertainty levels in planning period-1

Supply uncertainty = 0.2		Supply uncertainty = 0.4	Supply uncertainty = 0.6	Supply uncertainty = 0.8
PRODUCT 1				
D-1	9.8753	11.685	11.403	12.095
D-3	10.766	12.738	12.431	13.186
D-4	6.9785	8.2571	8.0581	8.5469
D-5	5.5828	6.6057	6.4465	6.8375
PRODUCT 2				
D-1	9.3951	11.116	10.849	11.507
D-3	10.221	12.093	11.802	12.518
D-4	6.6296	7.8442	7.6552	8.1196
D-5	5.2339	6.1928	6.0436	6.4102
PRODUCT 3				
D-1	8.3232	9.8481	9.6108	10.194
D-3	9.4275	11.155	10.886	11.546
D-4	6.2807	7.4314	7.2523	7.6922
D-5	4.536	5.3671	5.2378	5.5555

Table 4.67: AI vs. supply uncertainty levels in planning period-1

Supply uncertainty = 0.2		Supply uncertainty = 0.4	Supply uncertainty = 0.6	Supply uncertainty = 0.8
PRODUCT 1				
D-1	1720.138	1721.948	1721.666	1722.358
D-3	1985.608	1987.58	1987.273	1988.027
D-4	1231.723	1233.002	1232.803	1233.292
D-5	1101.028	1102.051	1101.892	1102.283

PRODUCT 2				
D-1	1980.435	1982.157	1981.889	1982.547
D-3	2301.508	2303.381	2303.09	2303.805
D-4	1419.074	1420.289	1420.1	1420.564
D-5	1260.224	1261.183	1261.034	1261.4
PRODUCT 3				
D-1	2233.183	2234.708	2234.47	2235.053
D-3	2636.213	2637.94	2637.671	2638.331
D-4	1649.448	1650.599	1650.42	1650.86
D-5	1400.96	1401.791	1401.662	1401.979

Table 4.68: TC vs. supply uncertainty levels in planning period-1

Supply uncertainty = 0.2		Supply uncertainty = 0.4	Supply uncertainty = 0.6	Supply uncertainty = 0.8
PRODUCT 1				
D-1	774490.5	774493.2	774492.8	774493.8
D-3	1613553	1613556	1613556	1613557
D-4	600572.3	600574.2	600573.9	600574.7
D-5	483547	483548.5	483548.3	483548.9
PRODUCT 2				
D-1	831102.6	831108.6	831107.7	831110
D-3	1651347	1651353	1651352	1651355
D-4	631379.2	631383.5	631382.8	631384.4
D-5	504665.8	504669.1	504668.6	504669.9
PRODUCT 3				
D-1	858298.8	858309.1	858307.5	858311.5
D-3	1666421	1666433	1666431	1666436
D-4	670976.9	670984.7	670983.5	670986.5
D-5	499852.9	499858.5	499857.6	499859.8

In the same way, similar variable may be calculated for different combined uncertainty (Demand and supply) levels. For different combined uncertainty levels, the change in variables can be elaborate in the following tables:

Table 4.69: ROP vs. combined uncertainty levels in planning period-1

Combined uncertainty = 0.2		Combined uncertainty = 0.4	Combined uncertainty = 0.6	Combined uncertainty = 0.8
PRODUCT 1				
D-1	310.3059	311.7325	313.0604	316.3891
D-3	410.5551	412.202	413.7349	417.5775
D-4	162.7601	163.7824	164.734	167.1194
D-5	131.4129	132.3274	133.1785	135.3121

PRODUCT 2				
D-1	294.8434	296.233	297.5264	300.7686
D-3	395.1556	396.7706	398.2737	402.0416
D-4	154.937	155.9335	156.861	159.1859
D-5	123.5505	124.4359	125.2601	127.3258
Combined uncertainty = 0.2		**Combined uncertainty = 0.4**	**Combined uncertainty = 0.6**	**Combined uncertainty = 0.8**
PRODUCT 3				
D-1	263.8789	265.1912	266.4127	269.4746
D-3	364.3326	365.8816	367.3234	370.9375
D-4	147.1052	148.0752	148.9779	151.2409
D-5	107.7875	108.6117	109.379	111.3021

Table 4.70: ESC vs. combined uncertainty levels in planning period-1

Combined uncertainty = 0.2		Combined uncertainty = 0.4	Combined uncertainty = 0.6	Combined uncertainty = 0.8
PRODUCT 1				
D-1	2.247084	2.950616	3.60721	5.258144
D-3	3.099782	3.915255	4.675718	6.58609
D-4	1.407957	1.905459	2.370969	3.544911
D-5	1.534028	1.977347	2.392456	3.44016
PRODUCT 2				
D-1	2.290718	2.97567	3.614979	5.22261
D-3	3.211255	4.010806	4.756435	6.62958
D-4	1.44625	1.930763	2.38419	3.527864
D-5	1.552875	1.981604	2.383143	3.396866
PRODUCT 3				
D-1	2.421328	3.067642	3.670985	5.188463
D-3	3.253584	4.020111	4.735012	6.531157
D-4	1.480273	1.951447	2.392465	3.505058
D-5	1.571719	1.969704	2.342653	3.28479

Table 4.71: FR vs. combined uncertainty levels in planning period-1

Combined uncertainty = 0.2		Combined uncertainty = 0.4	Combined uncertainty = 0.6	Combined uncertainty = 0.8
PRODUCT 1				
D-1	0.999343	0.999137	0.998945	0.998463
D-3	0.999215	0.999009	0.998816	0.998333
D-4	0.999425	0.999222	0.999032	0.998553
D-5	0.9993	0.999097	0.998908	0.99843

PRODUCT 2				
D-1	0.999419	0.999245	0.999083	0.998675
D-3	0.999299	0.999125	0.998962	0.998553
D-4	0.999488	0.999317	0.999156	0.998751
D-5	0.999381	0.999211	0.999051	0.998647
Combined uncertainty = 0.2		**Combined uncertainty = 0.4**	**Combined uncertainty = 0.6**	**Combined uncertainty = 0.8**
PRODUCT 3				
D-1	0.999456	0.999311	0.999175	0.998834
D-3	0.999381	0.999235	0.999099	0.998757
D-4	0.99955	0.999406	0.999272	0.998933
D-5	0.999437	0.999295	0.999161	0.998824

Table 4.72: SS vs. combined uncertainty levels in planning period-1

Combined uncertainty = 0.2		Combined uncertainty = 0.4	Combined uncertainty = 0.6	Combined uncertainty = 0.8
PRODUCT 1				
D-1	17.806	19.233	20.56	23.889
D-3	20.555	22.202	23.735	27.578
D-4	12.76	13.782	14.734	17.119
D-5	11.413	12.327	13.179	15.312
PRODUCT 2				
D-1	17.343	18.733	20.026	23.269
D-3	20.156	21.771	23.274	27.042
D-4	12.437	13.433	14.361	16.686
D-5	11.051	11.936	12.76	14.826
PRODUCT 3				
D-1	16.379	17.691	18.913	21.975
D-3	19.333	20.882	22.323	25.937
D-4	12.105	13.075	13.978	16.241
D-5	10.287	11.112	11.879	13.802

Table 4.73: AI vs. combined uncertainty levels in planning period-1

Combined uncertainty = 0.2		Combined uncertainty = 0.4	Combined uncertainty = 0.6	Combined uncertainty = 0.8
PRODUCT 1				
D-1	1728.069	1729.496	1730.824	1734.152
D-3	1995.397	1997.044	1998.577	2002.419
D-4	1237.505	1238.527	1239.479	1241.864
D-5	1106.858	1107.772	1108.624	1110.757

PRODUCT 2				
D-1	1988.384	1989.773	1991.067	1994.309
D-3	2311.443	2313.058	2314.562	2318.329
D-4	1424.882	1425.878	1426.806	1429.131
D-5	1266.041	1266.926	1267.75	1269.816
Combined uncertainty = 0.2		**Combined uncertainty = 0.4**	**Combined uncertainty = 0.6**	**Combined uncertainty = 0.8**
PRODUCT 3				
D-1	2241.238	2242.551	2243.772	2246.834
D-3	2646.118	2647.667	2649.109	2652.723
D-4	1655.273	1656.243	1657.146	1659.409
D-5	1406.711	1407.536	1408.303	1410.226

Table 4.74: TC vs. combined uncertainty levels in planning period-1

Combined uncertainty = 0.2		**Combined uncertainty = 0.4**	**Combined uncertainty = 0.6**	**Combined uncertainty = 0.8**
PRODUCT 1				
D-1	774502.4	774504.5	774506.5	774511.5
D-3	1613568	1613570	1613573	1613578
D-4	600581	600582.5	600584	600587.5
D-5	483555.7	483557.1	483558.4	483561.6
PRODUCT 2				
D-1	831130.4	831135.2	831139.8	831151.1
D-3	1651381	1651387	1651392	1651405
D-4	631399.5	631403	631406.3	631414.4
D-5	504686.1	504689.2	504692.1	504699.3
PRODUCT 3				
D-1	858353.2	858362.1	858370.3	858391
D-3	1666488	1666499	1666508	1666533
D-4	671016.3	671022.8	671028.9	671044.2
D-5	499891.7	499897.3	499902.5	499915.5

4.8.1 Effect of demand uncertainty on performance parameters

It is obvious that there is a very small increment in the reorder point as demand uncertainty level increases. There are rapid increments in the safety stocks under various demand uncertainty levels. Due to this safety stock, inventory holding costs are increased while the other cost of the system is almost remaining same. The ESC is also increasing, as the demand uncertainty is increasing. The behavior of results shows that the fill rate is decreasing, as the demand uncertainty level increases but in all the conditions it is always greater than 99%. It is observed that the average inventory is increasing very slightly as the demand uncertainty levels are increasing. In this case ordering quantity is almost remaining same, only safety stocks are increasing. There is increment in the cost as well as the demand uncertainty is increasing for all types of products and for all planning periods.

4.8.2 Effect of supply uncertainty on performance parameters

For examining the effect of supply uncertainty on performance parameters, same levels of uncertainty (0.2, 0.4, 0.6 and 0.8) and same parameters (reorder point, safety stocks, expected shortages per cycle, fill rates, average inventory and total cost) are considered as discussed in the case of demand uncertainty. It is reported that there is a very small increment in the reorder point as the supply uncertainty level increases from 0.2 to 0.8. Results show that there are fast increments in the safety stocks under various supply uncertainty levels. The expected shortages per cycle are increasing as the supply uncertainty levels are increasing. Results show that the fill rate decreases, as the supply uncertainty level increases while it is always more than 99 %. In this case, the average inventory (Q/2 + SS) is also increasing due to the increments in the safety stocks. It also, has been observed that total costs of the system increases very slowly as the supply uncertainty level are increasing for all types of products and periods.

4.8.3 Combined effect of demand and supply uncertainties on performance parameters

Same levels of uncertainty (0.2, 0.4, 0.6 and 0.8) and the same performance parameters (reorder point, safety stocks, expected shortages per cycle, fill rates, average inventory and total cost) are taken for evaluating the combined effect of both types of uncertainty (demand and supply) on performance parameters. The main observation is that there are large variations in the performance parameters under the combined effect of demand and supply uncertainty as compared to the individual demand and supply uncertainty. In individual demand and supply uncertainty level, the trend of the reorder plot was almost constant, while considering the combined effect of demand and supply uncertainty, the reorder point increases rapidly as compared to the individual type of uncertainty. Under the combined effect of both type of uncertainty the safety stock required is very much high as compared to demand and supply considered individually. The expected shortages per cycle are increasing more rapidly under combined effect of demand and supply uncertainty as compared to the demand and supply uncertainty as individual.

Results are showing that the fill rate is decreasing, as the demand and supply uncertainty is increasing. It is observed that the average inventory is increasing as the demand and supply uncertainty are increasing. The trends of the results show that, the total cost of the system is increasing very fast under the combined effect of uncertainty as compared to the individual uncertainty of demand and supply. The common observations from the results obtained show that various performance parameters are increasing, as the uncertainty levels are increasing. Only the fill rate is showing exceptional behavior. Fill rates are slightly decreasing as uncertainty levels are increasing.

4.9 Summary of the chapter

In this chapter, single-product, single-period risk pooling inventory control model is expanded for multi-product and multi-period scenarios. For solving the above model, 5 distributors, 7 retailers, 3 different types of products and 5 planning periods are considered. Twenty different alternatives of distribution network are considered same as in previous chapter. A series of results are obtained for 20 alternatives, 5 distributors, 3 different types of products and 5 planning periods through optimization. After obtaining the optimization results, here a multi-criteria decision making problem arises between twenty different alternatives and four criterions. Different process variables have been calculated like total cost, product-miles, ordering quantity, reorder point, safety stock, cycle service level, average inventory, inventory positions and expected shortages per cycle for different alternatives of distribution networks. The results of twenty alternatives are so close, that it is very difficult to decide the best alternative against these criteria. For solving the above problem Fuzzy TOPSIS approach is adopted. The fundamentals of fuzzy numbers are disused in this chapter. The detailed procedure for applying Fuzzy TOPSIS approach is also mentioned in this chapter. For applying Fuzzy TOPSIS, the responses of 5 decision makers are collected about this criterion for each of the 20 alternatives. For applying Fuzzy TOPSIS, the weights of criteria and the ratings of the alternatives are aggregated for 5 decision makers. After aggregating the weight of criteria and rating of alternatives a fuzzy decision matrix is formed for 20 alternatives and 4 criteria. The fuzzy normalized decision matrix is formed by dividing the largest number of the corresponding column. For computing the fuzzy normalized weight decision matrix, the matrix of aggregated weights of criteria is multiplied with normalized decision matrix. Now the distances are calculated from FPIS (A+) and FNIS (A-) for each criteria and each alternative. Finally closeness coefficient (CCi) is calculated for each alternative and all these alternatives are outranked based on closeness coefficient (CCi). It is reported that alternative 5 is found the best alternative among all the alternatives based on calculation Fuzzy-TOPSIS approach. The discussion about the result analysis is based on the alternative 5.

CHAPTER 5

CASE STUDIES

5.1 Introduction:

Inventory is an important aspect for any manufacturing firm and it has a great impact on the performance of any business (Routroy and Kodali, 2005). From raw material to final products, the inventory may exist in many forms like raw material inventory, work in progress inventory and finished product inventory etc. (Vrat, 2014) and up to 80% of the total product and other inventory services account in high technology firms. Therefore inventory management is an important activity of any business firm and each firm has its own inventory management (Christopher, 1992). Sometimes, some of the firms have some safety stock for holding inventories to avoid the situations of stock out during lead time and as a result inventory holding cost increases (Gaur and Ravindran, 2006). Due to this, the total cost of a system increase which produces adverse effects on the operational efficiency of the firm (Liao et al., 2011). In this context two inventory control models, have been suggested in a distribution network by implementing the risk pooling approach. First model represents the single-product, single-period risk pooling inventory control model and the second one represents the multi-product, multi-period risk pooling inventory control model. These models mainly include ordering cost, holding cost, facility cost, transportation cost and operating cost. The main decision variables in these models are ordering quantity, reorder point, product-miles and the total cost. Except these variables some other variables are also evaluated, which are expected shortages per cycle (ESC), Fill Rate (FR), safety Stocks (SS), Cycle service level (CSL), Average Inventory (AI) and Inventory Positions (IP) etc.

Two different case studies have been carried out in two different industries for checking the practical applicability and validity of the two proposed models. First case study has been taken for the validation of single-product, single period risk pooling model from a sugar mill distribution network which is situated in Muzaffarnagar, Uttar-Pradesh, India. The second case study has been selected from a milk producing unit situated in Jaipur, Rajasthan, India for the validation of multi-product, multi- period risk pooling model. In the next sections, these case studies are discussed in detail.

5.2 Case study-1:

To get deeper insight of the findings of single product, single period risk pooling inventory control model, a case study has been employed in a sugar manufacturing unit which is situated in Rohana Kalan Distt- Muzaffarnagar, Uttar Pradesh, India. Now this unit is in the control of Indian Potash Limited. Previously this unit was in the under of U P State Sugar corporation limited and it was purchased by Indian Potash Limited in 2010 in the Uttar Pradesh government's ongoing privatization programme. This unit is situated in western Uttar Pradesh in Muzaffarnagar District and in a village called Rohana Kala. It is mainly sugar manufacturing unit which produces two types of sugar. One is sulphitation sugar and other is premier sugar. The main raw material of this unit is sugar cane and the installed capacity of the plant is 2,500 TCD (Tonnes Crushing Daily). The recovery of plant is generally 9 and the production capacity is 2500 bags per day. The major network of distribution of the sulphitation sugar is in Rajasthan and premier sugar is in Gujrat and Punjab. For employing single-product, single period case study, a small network from Rajasthan has been selected which is suggested by the industrial professionals. The distribution network of the sugar distribution of this unit is shown in Figure 5.1.

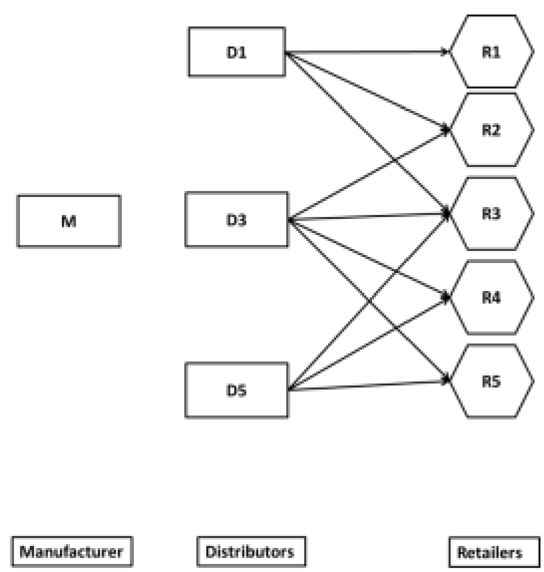


Figure 5.1: Distribution network of a sugar manufacturing unit

For performing this case study, three distributors from Muzaffarnagar and five retailers from Jaipur are selected from this manufacturing unit which is suggested by the industrial experts as shown in Figure 5.1. The distance of each distributor is same for all the retailers, as the distance from Muzaffarnagar to Jaipur is fixed. The name of firms (Distributors and retailers) cannot be opened on the request of experts. According to industrial experts the 3 distributors from Muzaffarnagar are capable to fulfill the demands of five retailers from Jaipur. According to existing contract among retailers and distributors, partial fulfillment is not allowed for retailers. Due to this condition, each distributor of this unit can fulfill the demand of its some specific retailers only. Implementing the above condition of partial fulfillment 9 different alternatives has been generated for fulfilling the demand of all 5 retailers by 3 distributors in the network. The 9 combinations of alternatives are as follows:

Table 5.1: Possible alternatives of distribution network

Alternatives	**Retailers served by Distributor 1**	**Retailers served by Distributor 2**	**Retailers served by Distributor 2**
Alternative-1	R1, R2, R3	R4, R5	-
Alternative-2	R1, R2, R3	R4	R5
Alternative-3	R1	R2, R3	R4, R5
Alternative-4	R1	R2	R3, R4, R5
Alternative-5	R1	R2, R3, R4	R5
Alternative-6	R1, R2	R3, R4	R5
Alternative-7	R1, R2	R3	R4, R5
Alternative-8	R1, R2, R3	-	R5
Alternative-9	R1, R2	-	R3, R4, R5

The main objective of this case study is to find out the best inventory policy in the sugar mill distribution network. For performing the case study, the company has provided some relevant information which is desired for the validation of model. This information is as follows:

Table 5.2: Demand (No. of Bags) and standard deviations (No. of Bags) of demand

Retailers	Demand	Standard Deviation of demand
R1	1500	150
R2	1700	170
R3	3000	300
R4	2000	200
R5	2500	250

Table 5.3: Distance (KM) of Rohana Kalan from Jaipur

	R1	R2	R3	R4	R5
D1	448	448	448		
D2		448	448	448	
D3			448	448	448

Table 5.4: Some other information provided by company

Parameter	Value
Transportation Cost	Rs. 70/ Unit
Operating Cost	Rs. 8/ unit
Inventory holding cost	Rs. 12/ unit
Lead Time	1 day (24 Hours)
Standard Deviations of Lead time	0.25 day (6 hours)

The mathematical representation of model is

The objective function of total cost (Z_1) =

$$\sum_j FOj\left(\frac{\bar{R}}{Q}\right) + \sum_j \left\{\frac{Q_j}{2} + \left(Ro_j - \mu\right)\right\} * C * v + \sum_j \sum_k \left(D_k * T * x_{j-k}\right)$$

$$+ \sum_j \left(OP_j * \bar{R}\right)$$

Product-miles = $\left(\sum_k \sum_j X_{j-k} * D_k\right)$

Subjected to constraints:

$Q \geq 1$

$Ro_j \geq \mu$

$CSL \geq 0.975$,

Where, CSL = NORMSINV (ROP, μ, σ)

The case study has been solved in AIMMS software by using CONPOT solver. Substituting the values of parameters in the programming, this model provides the following results.

The ordering quantity (Q)

Table 5.5: Ordering Quantity

Alternatives	D1	D2	D3
Alternative-1	352.1363	300	1
Alternative-2	352.1363	200	223.6068
Alternative-3	173.2051	306.5942	300
Alternative-4	173.2051	184.3909	387.2983
Alternative-5	173.2051	366.0601	223.6068
Alternative-6	252.9822	316.2278	223.6068
Alternative-7	252.9822	244.949	300
Alternative-8	352.1363	1	300
Alternative-9	252.9822	1	387.2983

Table 5.6: Reorder Point

Alternatives	D1	D2	D3
Alternative-1	232	172	0
Alternative-2	232	80	101
Alternative-3	60	180	172

Alternatives	D1	D2	D3
Alternative-4	60	68	279
Alternative-5	60	251	101
Alternative-6	122	190	101
Alternative-7	122	120	172
Alternative-8	232	0	172
Alternative-9	122	0	279

Table 5.7: Mean Demand during Lead time

Alternatives	D1	D2	D3
Alternative-1	206.6667	150	
Alternative-2	206.6667	66.66667	83.33333
Alternative-3	50	156.6667	150
Alternative-4	50	56.66667	250
Alternative-5	50	223.3333	83.33333
Alternative-6	106.6667	166.6667	83.33333
Alternative-7	106.6667	100	150
Alternative-8	206.6667		150
Alternative-9	106.6667		250

Table 5.8: Safety stocks

Alternatives	D1	D2	D3
Alternative-1	25	22	0
Alternative-2	25	14	18

Alternatives	D1	D2	D3
Alternative-3	10	24	22
Alternative-4	10	12	29
Alternative-5	10	27	18
Alternative-6	16	24	18
Alternative-7	16	20	22
Alternative-8	25	0	22
Alternative-9	16	0	29

Table 5.9: ESC

Alternatives	D1	D2	D3
Alternative-1	0.8522	0.7211	0
Alternative-2	0.8522	0.4589	0.59
Alternative-3	0.3278	0.7866	0.7211
Alternative-4	0.3278	0.3933	0.9833
Alternative-5	0.3278	0.9177	0.59
Alternative-6	0.5244	0.7866	0.59
Alternative-7	0.5244	0.6555	0.7211
Alternative-8	0.8522	0	0.7211
Alternative-9	0.5244	0	0.9833

Table 5.10: Fill Rate

Alternatives	D1	D2	D3
Alternative-1	0.99758001	0.99759646	0
Alternative-2	0.99758001	0.99770571	0.99736162

Alternatives	D1	D2	D3
Alternative-3	0.9981077	0.99743435	0.99759646
Alternative-4	0.9981077	0.99786699	0.99746122
Alternative-5	0.9981077	0.99749299	0.99736162
Alternative-6	0.99792709	0.99751251	0.99736162
Alternative-7	0.99792709	0.99732389	0.99759646
Alternative-8	0.99758001	0	0.99759646
Alternative-9	0.99792709	0	0.99746122

Table 5.11: Total Cost v/s Product-miles

Alternatives	Total Cost	Product-miles
Alternative-1	82346566.6	121633.333
Alternative-2	106821628	158716.667
Alternative-3	106821630	158716.667
Alternative-4	106821612	158716.667
Alternative-5	106821621	158716.667
Alternative-6	106821636	158716.667
Alternative-7	106821639	158716.667
Alternative-8	106821567	158716.667
Alternative-9	106821561	158716.667

Total 9 inventory policies have been received from the optimizations which are tabulated in the above tables. It is clear that except alternative -1, all alternatives from alternatives 2 to alternative 8 have same values of product-miles. On the basis of product-miles any alternative from alternative 2 to alternative 8 may be selected. On the basis of product-miles no alternative can be selected as the best alternative as the weightage of total cost has been ignored. On the other hand, Alternative-1 has lowest value of total cost. On the basis of total cost Alternative-1 may be selected. So it is very difficult to select the best inventory policy.

Now best inventory policy has to be selected from all 9 alternatives. Previously discussed that the main criteria for the selection of best inventory policies are total cost, fill rate, cycle service level and product-miles. Here TOPSIS approach is used for the selection of best inventory policy. The steps involved in TOPSIS approach have already been discussed in Chapter 3. In this context, responses are collected from 5 industrial professionals from sugar mill. The responses of these professionals are shown in the following Tables. Table 5.12 shows the average criteria weights and Table 5.13 shows the average ratings of alternatives of 5 responses for these 4 criterions.

Table 5.12: Average criterion weight

	Total cost (C1)	**Responsive ness (C2)**	**Fill Rate (C3)**	**Service level (C4)**
Criteria weights	0.4	0.2	0.2	0.2

Table 5.13: Average ratings of alternatives

	Total cost (C1)	**Responsive ness (C2)**	**Fill Rate (C3)**	**Service level (C4)**
Alternative-1	9	2	2	1
Alternative-2	8	2	2	1
Alternative-3	7	2	2	1
Alternative-4	8	2	2	1
Alternative-5	5	2	2	1
Alternative-6	9	2	2	1
Alternative-7	8	2	2	1
Alternative-8	7	2	2	1
Alternative-9	9	2	2	1

By applying standard procedure of TOPSIS approach as discussed in Chapter 3, the closeness coefficient of each alternative is obtained by using standard procedure of TOPSIS and is shown in Table 5.14.

Table 5.14: Closeness coefficient

Alternatives								
1	2	3	4	5	6	7	8	9
0.14757	0.43544	0.52228	0.52228	0.61200	0.55830	0.43544	0.52228	0.47557

The highest closeness coefficient is 0.612005 which is of alternative -5. Hence for the provided weightage, the alternative-5 is the best network and the discussion of results is based on the alternative 5.

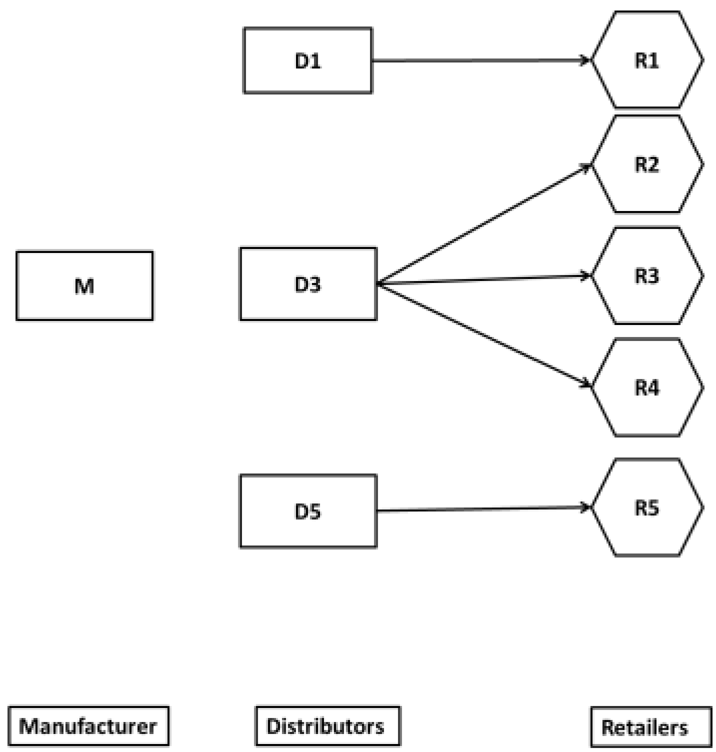


Figure 5.2: Alternative 5

For best inventory policy, the company should follow the alternative 5 which would provide appropriate weightage between the total cost and service level measures. This alternative will provide the cycle service level up to 0.975 with optimum ordering quantity and reorder point with proper weights of cost and responsiveness. Now the effects of uncertainty are evaluated on the performance parameters. The performance parameters taken for study are, reorder point, safety stocks and the total cost. Same uncertainty levels (0.2, 0.4, 0.6 and 0.8) are taken in to consideration as discussed in the previous chapters. The effects of uncertainty levels on the performance parameters under considerations are shown in the following tables and Figures:

Table 5.15: Effect of demand uncertainty on ROP

		0.20	**0.40**	**0.60**	**0.80**
Alternative-5	D-1	62	64	68	71
	D-2	256	262	273	283
	D-3	105	108	115	121

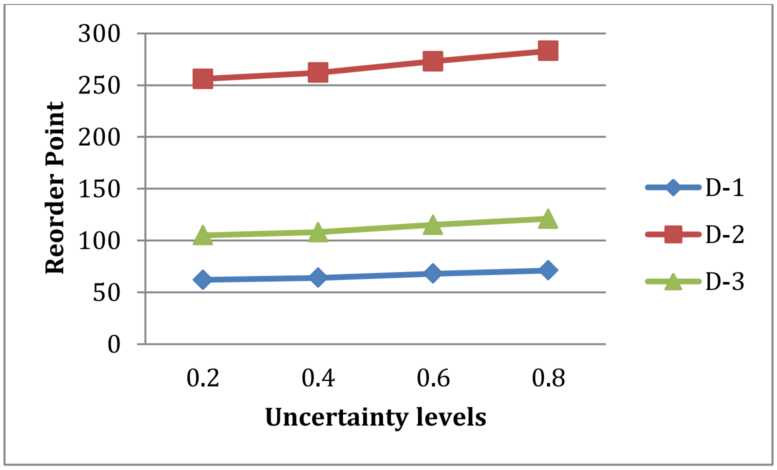


Figure 5.3: Effect of demand uncertainty on ROP

Table 5.16: Effect of demand uncertainty on SS

		0.20	**0.40**	**0.60**	**0.80**
Alternative-5	D-1	12	14	18	21
	D-2	33	38	49	59
	D-3	21	25	32	38

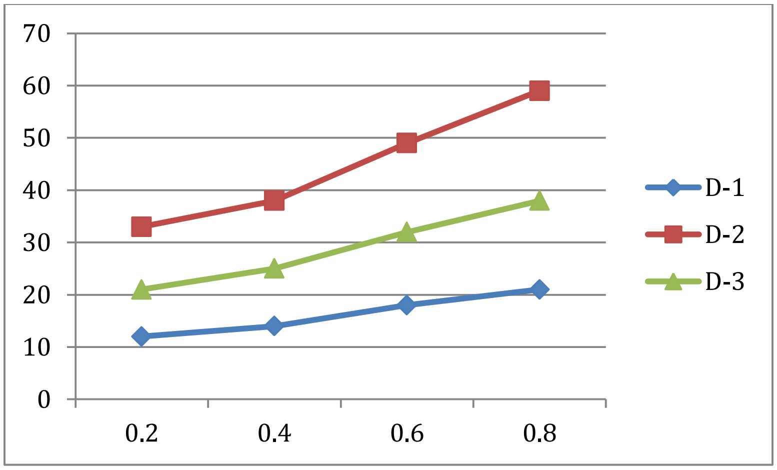


Figure 5.4: Effect of demand uncertainty on SS

Table 5.17: Effect of demand uncertainty on TC

0.20	**0.40**	**0.60**	**0.80**
5019569	5023519	5031422	5038534

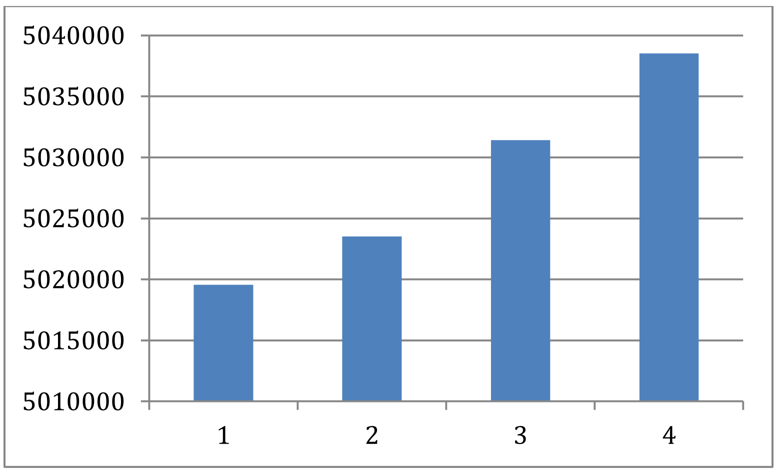


Figure 5.5: Effect of demand uncertainty on TC of system

Table 5.18: Effect of supply uncertainty on ROP

	0.20	**0.40**	**0.60**	**0.80**
D-1	61	62	62	63
D-2	253	256	258	260
D-3	103	104	106	107

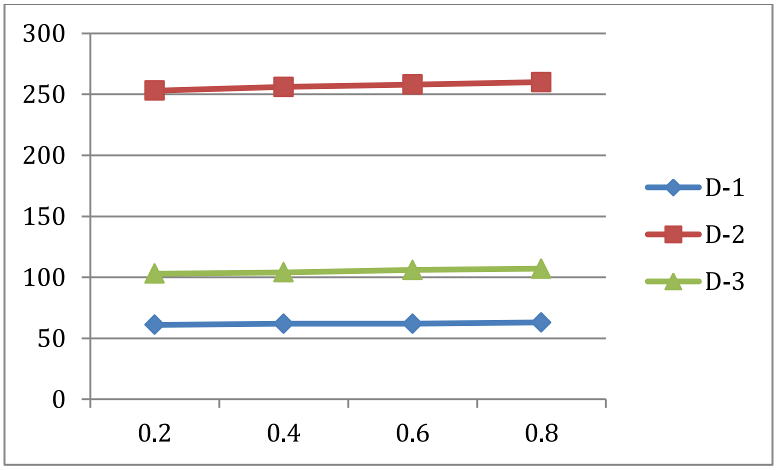


Figure 5.6: Effect of supply uncertainty on ROP

Table 5.19: Effect of supply uncertainty on SS

	0.20	**0.40**	**0.60**	**0.80**
D-1	11	12	12	13
D-2	30	32	35	37
D-3	19	21	22	24

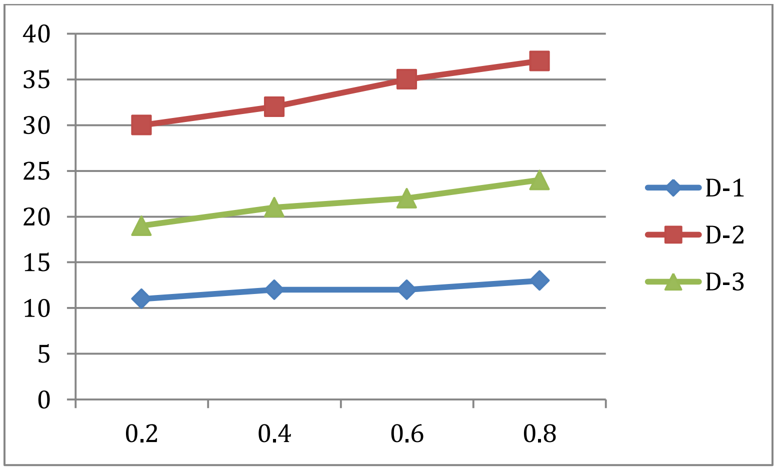


Figure 5.7: Effect of supply uncertainty on SS

Table 5.20: Effect of supply uncertainty on total cost of system

0.20	**0.40**	**0.60**	**0.80**
5017502	5019236	5020850	5022366

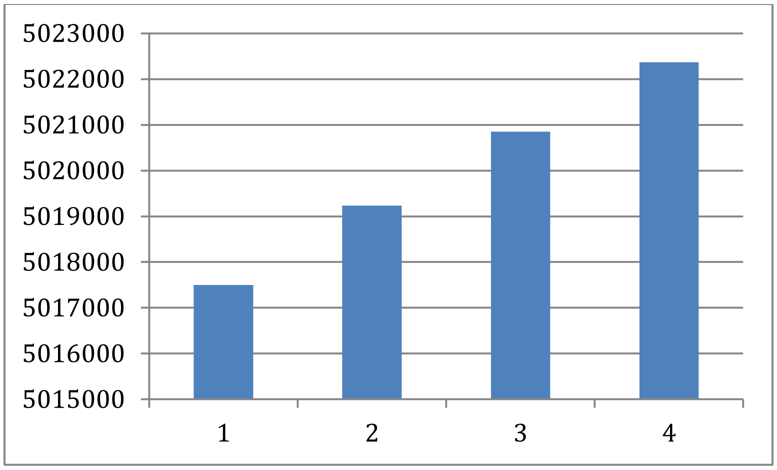


Figure 5.8: Effect of supply uncertainty on TC of system

Table 5.21: Combined effect of both type of uncertainty on ROP

	0.20	**0.40**	**0.60**	**0.80**
D-1	91	99	107	116
D-2	338	341	344	349
D-3	136	142	149	157

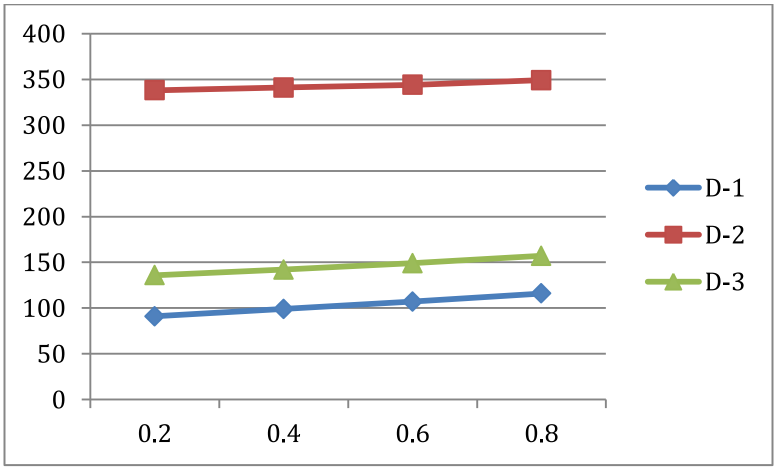


Figure 5.9: Combined effect of both type of uncertainty on ROP

Table 5.22: Combined effect of both type of uncertainty on SS

	0.20	0.40	0.60	0.80
D-1	41	49	57	66
D-2	114	117	121	126
D-3	53	59	66	74

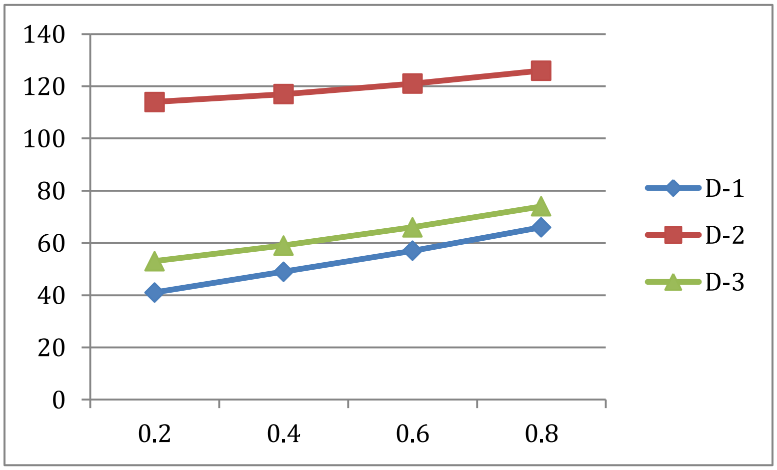


Figure 5.10: Combined effect of both type of uncertainty on SS

Table 5.23: Combined effect of both type of uncertainty on TC

0.20	0.40	0.60	0.80
5071011	5076794	5083642	5091484

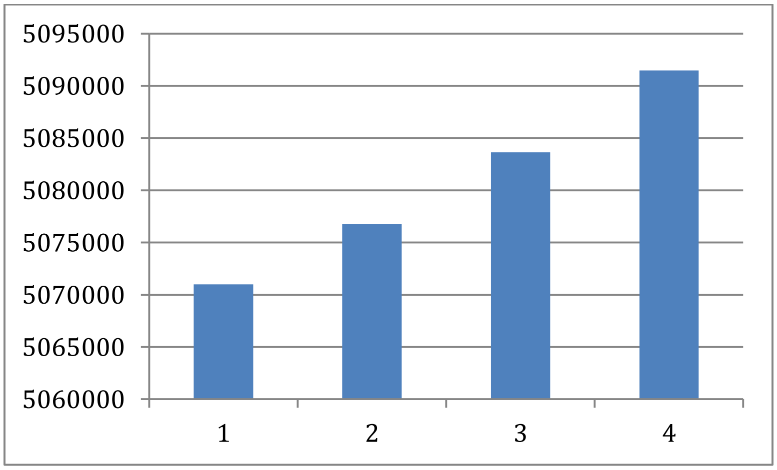


Figure 5.11: Combined effect of both type of uncertainty on TC

The common observations obtained from the above table and graphs is that the reorder point, safety stocks, expected shortages per cycle, average inventory, inventory positions increases as the uncertainty level increases. Only the fill rate decreases after increasing the level of uncertainty. Same types observations are obtained while evaluating the effect of supply uncertainty, demand uncertainty or combined effect of uncertainty on the performance parameters.

5.3 Case Study-2:

For checking the practical applicability and validity for multi-product and multi-period scenarios, one more case study is performed in Jaipur dairy. It is most popular and the largest milk production unit situated in Jaipur Rajasthan. Jaipur Dairy consists of very large network of retail points spread all over Jaipur city and nearby towns through which it sells its milk and milk products. Retail points are the most important market segment for Jaipur Dairy and give all care and attention. They receive regular training in all aspects of customer satisfaction. It is ensured that consumers do not travel/walk more to get milk for his/her daily consumption. Therefore a strong network of retail stores has been established by Jaipur Dairy to fulfill this intention. Supply of liquid milk is two times a day for the benefit of customers, one supply is in morning and another supply is in evening. There are generally four types of liquid milk i.e. Toned, Gold, Double-toned and standard. Liquid milk is transported to rural areas through a fleet of contracted insulated vehicles. Milk and dairy products are sold through their own networks, store agencies, various institutions, and Saras milk shops. Jaipur Dairy is very focused on satisfying and pleasing customers and consumers.

In order to carry out case study on multi-product, multi-period risk pooling model, 4 milk depots and 7 distribution zones, are selected in Jaipur area network which are very crucial in milk distribution network in the context of milk supply. These 4 milk depots in the network are Malaviya nagar, Mansarovar, Vaishali nagar and Durgapura. The 7 distribution zones are Gopalpura, Sodala, Passport office, Birla Mandir, Nirman nagar, Airport and Inox crytal palm. These 4 milk depot have continuous supply of milk from the processing plant therefore all these four depots are capable of to fulfill the demand of all seven distribution zones. These 4 milk depots are denoted by D1, D2, D3 and D4 consecutively and the 7 distribution zones are shown by R1, R2, R3, R4, R5, R6 and R7. Four different types of milk (Toned, Gold, Double-toned and standard) are considered as four different types of products and denoted by P1, P2, P3 and P4. The supply time of milk i.e. morning and evening are considered as the periods of demands. Morning period is treated as period-1 and evening period is treated as period 2. The objective of second case study is to find out the optimum inventory policy for all 20 scenarios along with minimized cost. The network of the milk distribution from four milk depot to seven distribution zones is depicted in the following Figure 5.12:

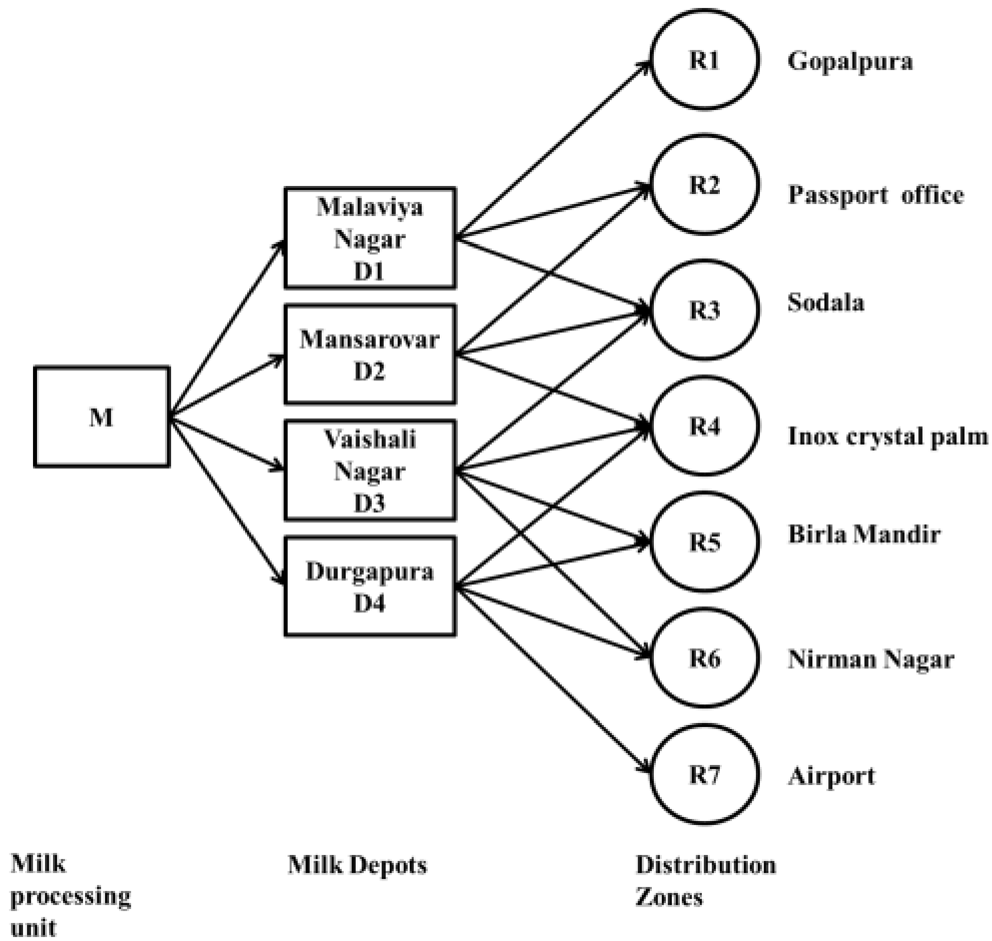


Figure 5.12: Milk distribution network for Jaipur dairy

In the above Figure M denotes, milk processing unit, D1, D2, D3 and D4 denote the four milk depot and R1, R2, R3, R4, R5, R6 and R7 denote the 7 milk distribution zones. The various arrows show the flow of milk in the network. For example, the three distribution zones R1, R2 and R3 can fulfill their demand by milk depot D1. Similarly demands of zones R2, R3 and R4 can be fulfilled by zone D2. In the same way, all the possibility of milk flow in the network can be explained. The different possible options of distribution network can be shown in the following table.

Table 5.24: Possible distribution networks

Alternatives	Retail points served by milk depot D1	Retail points served by milk depot D2	Retail points served by milk depot D3	Retail points served by milk depot D4
Alternative-1	R1, R2	R3	R4, R5	R6, R7
Alternative-2	R1, R2	R3	R4, R5, R6	R7
Alternative-3	R1, R2	R3	R4,	R5, R6, R7
Alternative-4	R1, R2	-	R3, R4, R5, R6	R7
Alternative-5	R1, R2	-	R3, R4, R5	R6, R7
Alternative-6	R1, R2	-	R3, R4,	R5, R6, R7
Alternative-7	R1, R2	-	R3	R4, R5, R6, R7
Alternative-8	R1, R3	R2	R4, R5, R6	R7
Alternative-9	R1, R3	R2	R4, R5	R6, R7
Alternative-10	R1, R3	R2	R4	R5, R6, R7
Alternative-11	R1, R3	R2	-	R4, R5, R6, R7
Alternative-12	R1, R2, R3	-	R4, R5, R6	R7
Alternative-13	R1, R2, R3	-	R4, R5	R6, R7
Alternative-14	R1, R2, R3	-	R4	R5, R6, R7
Alternative-15	R1, R2, R3	R4	-	R5, R6, R7
Alternative-16	R1, R2, R3	-	-	R4, R5, R6, R7
Alternative-17	R1	R2, R3	R4, R5, R6	R7
Alternative-18	R1	R2	R3, R4, R5, R6	R7
Alternative-19	R1	R2	R3, R4	R5, R6, R7
Alternative-20	R1	R2	R3, R4, R5	R6, R7

For performing the case study for multi-product, multi-period scenarios some important information have been collected from Jaipur dairy. This information's may be shown in the following tables:

Table 5.25: Morning Demand

Retailers	Morning Demand (Litre)			
	Tonned	Gold	Double Tonned	Standard
R1	1744	1308	872	436
R2	1628	1221	814	407
R3	2036	1527	1018	509
R4	1388	1041	694	347
R5	1625	1105	785	425
R6	1850	1175	805	435
R7	9250	1325	1020	505

Table 5.26: Evening Demand

Retailers	Evening Demand (Litre)			
	Tonned	Gold	Double Tonned	Standard
R1	1080	810	540	270
R2	936	702	468	234
R3	1464	1098	732	366
R4	1488	1116	744	372
R5	1480	1185	735	295
R6	1235	1180	725	310
R7	1180	1105	695	305

Table 5.27: Standard deviations of morning demand

Retailers	Std. dev. of morning Demand (Litre)			
	Tonned	Gold	Double Tonned	Standard
R1	349	262	174	87
R2	326	244	163	81
R3	407	305	204	102
R4	278	208	139	69
R5	325	221	157	85
R6	370	235	161	87
R7	322	265	204	101

Table 5.28: Standard deviations of evening demand

Retailers	Std. Dev. of evening Demand (Litre)			
	Tonned	Gold	Double Tonned	Standard
R1	216	162	108	54
R2	187	140	94	47

Retailers	Std. Dev. of evening Demand (Litre)			
	Tonned	Gold	Double Tonned	Standard
R3	293	220	146	73
R4	298	223	149	74
R5	296	237	147	59
R6	247	236	145	62
R7	236	221	139	61

Table 5.29: Facility cost (Rs.) for every product

Facility Cost	Tonned	Gold	Double Tonned	Standard
D-1	200000	2250000	245000	235000
D-2	200000	2250000	245000	235000
D-3	200000	2250000	245000	235000
D-4	200000	2250000	245000	235000

Table 5.30: Distance matrix (Distance from milk depot to retailer in km)

Distance Matrix (Actual)	R1	R2	R3	R4	R5	R6	R7
D-1	3	4.7	6.7				
D-2		6.7	7	3.8			
D-3			9.4	4.4	5.5	8.1	
D-4				6.7	3.6	5.3	3.9

Table 5.31: Other informations provided by industry experts

Parameters	Value
Lead time	3 Hrs
Standard Dev. of LT	30 minutes
Transportation cost	0.55 paisa/liter
Ordering Cost	1 Rs/order
Inventory holding cost	1.00 Rs/liter

Optimization results: This case study is also solved in AIMMS software by using above information as input parameters. For solving above case study, the non-linear programming is done in AIMMS software and solved through CONPOT solver on an Intel (R) Core (TM)2 Duo CPU T 6570 with memory 4GB for the total cost minimization. The objective function and constraints of multi-product, multi-period inventory control model are shown below.

$$Z1=\text{MINMIZATION}[\sum_{l}\sum_{k}\sum_{i}F^{l}_{i,k}+\sum_{l}\sum_{k}\sum_{i}OP^{l}_{i,k}*R^{l}_{i,k}+\sum_{l}\sum_{k}\sum_{j}\sum_{i}T_{i,j}*x_{i,j}*D_{j}$$

$$+\sum_{l}\sum_{k}\sum_{i}FO_{i}(R^{l}_{i,k}/Q^{l}_{i,k})+\sum_{l}\sum_{k}\sum_{i}\left\{Q^{l}_{i,k}/2+\left(Ro^{l}_{i,k}-\mu^{l}_{i,k}\right)\right\}*C^{l}_{i,k}*v\}]$$

Subjected to constraints:

$Q^{l}_{i,k} \geq 0$; for all i and k

$Ro^{l}_{i,k} \geq \mu^{l}_{i,k}$ for all i and k

$CSL \geq 0.95$

$CSL = \text{NORMDIST}\left(Ro_{i,k}{}^{l}, \mu_{i,k}{}^{l}, \sigma_{Li,k}{}^{l}, 1\right)$

The results of for 4 types of products (Toned, Gold, Double Toned and Standard) are depicted as P-1, P-2, P-3 and P-4 in the following tables. The 2 periods (Morning and Evening) are shown by period-1 and period-2 in the following tables. In this case study only four variables are calculated for all alternatives. These variables are ordering quantity, reorder point, product miles and the total cost.

In this case no effect of uncertainty is evaluated on the system variables. The results for 4 types of products (Toned, Gold, Double Toned and Standard), 2 periods (Morning and Evening) and for all alternatives of distribution network are shown in the following tables:

Table 5.32: Ordering quantity for all Alternatives

	Ordering Quantity (Q)							
	Alternative-1							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	101160	75870	50580	25290	60480	45360	30240	15120
D-2	61080	45810	30540	15270	43920	32940	21960	10980
D-3	90390	64380	44370	23160	89040	69030	44370	20010
D-4	333000	75000	54750	28200	72450	68550	42600	18450
	Alternative-2							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	101160	75870	50580	25290	60480	45360	30240	15120
D-2	61080	45810	30540	15270	43920	32940	21960	10980
D-3	145890	99630	68520	36210	126090	104430	66120	29310
D-4	277500	39750	30600	15150	35400	33150	20850	9150

	Alternative-3							
	Period-1				Period-2			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	101160	75870	50580	25290	60480	45360	30240	15120
D-2	61080	45810	30540	15270	43920	32940	21960	10980
D-3	41640	31230	20820	10410	44640	33480	22320	11160
D-4	381750	108150	78300	40950	116850	104100	64650	27300
	Alternative-4							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	101160	75870	50580	25290	60480	45360	30240	15120
D-3	206970	145440	99060	51480	170010	137370	88080	40290
D-4	277500	39750	30600	15150	35400	33150	20850	9150
	Alternative-5							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	101160	75870	50580	25290	60480	45360	30240	15120
D-3	151470	110190	74910	38430	132960	101970	66330	30990
D-4	333000	75000	54750	28200	72450	68550	42600	18450
	Alternative-6							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	101160	75870	50580	25290	60480	45360	30240	15120
D-3	102720	77040	51360	25680	88560	66420	44280	22140
D-4	381750	108150	78300	40950	116850	104100	64650	27300
	Alternative-7							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	101160	75870	50580	25290	60480	45360	30240	15120
D-3	61080	45810	30540	15270	43920	32940	21960	10980
D-4	423390	139380	99120	51360	161490	137580	86970	38460
	Alternative-8							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	113400	85050	56700	28350	76320	57240	38160	19080
D-2	48840	36630	24420	12210	28080	21060	14040	7020
D-3	145890	99630	68520	36210	126090	104430	66120	29310
D-4	277500	39750	30600	15150	35400	33150	20850	9150

	Alternative-9							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	113400	85050	56700	28350	76320	57240	38160	19080
D-2	48840	36630	24420	12210	28080	21060	14040	7020
D-3	90390	64380	44370	23160	89040	69030	44370	20010
D-4	333000	75000	54750	28200	72450	68550	42600	18450
	Alternative-10							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	113400	85050	56700	28350	76320	57240	38160	19080
D-2	48840	36630	24420	12210	28080	21060	14040	7020
D-3	41640	31230	20820	10410	44640	33480	22320	11160
D-4	381750	108150	78300	40950	116850	104100	64650	27300
	Alternative-11							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	113400	85050	56700	28350	76320	57240	38160	19080
D-2	48840	36630	24420	12210	28080	21060	14040	7020
D-4	423390	139380	99120	51360	161490	137580	86970	38460
	Alternative-12							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	162240	121680	81120	40560	104400	78300	52200	26100
D-3	145890	99630	68520	36210	126090	104430	66120	29310
D-4	277500	39750	30600	15150	35400	33150	20850	9150
	Alternative-13							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	162240	121680	81120	40560	104400	78300	52200	26100
D-3	90390	64380	44370	23160	89040	69030	44370	20010
D-4	333000	75000	54750	28200	72450	68550	42600	18450
	Alternative-14							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	162240	121680	81120	40560	104400	78300	52200	26100
D-4	423390	139380	99120	51360	161490	137580	86970	38460

	Alternative-15							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	162240	121680	81120	40560	104400	78300	52200	26100
D-2	41640	31230	20820	10410	44640	33480	22320	11160
D-4	381750	108150	78300	40950	116850	104100	64650	27300
	Alternative-16							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	52320	39240	26160	13080	32400	24300	16200	8100
D-2	109920	82440	54960	27480	72000	54000	36000	18000
D-3	145890	99630	68520	36210	126090	104430	66120	29310
D-4	277500	39750	30600	15150	35400	33150	20850	9150
	Alternative-17							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	52320	39240	26160	13080	32400	24300	16200	8100
D-2	48840	36630	24420	12210	28080	21060	14040	7020
D-3	206970	145440	99060	51480	170010	137370	88080	40290
D-4	277500	39750	30600	15150	35400	33150	20850	9150
	Alternative-18							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	52320	39240	26160	13080	32400	24300	16200	8100
D-2	48840	36630	24420	12210	28080	21060	14040	7020
D-3	151470	110190	74910	38430	132960	101970	66330	30990
D-4	333000	75000	54750	28200	72450	68550	42600	18450
	Alternative-19							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	52320	39240	26160	13080	32400	24300	16200	8100
D-2	48840	36630	24420	12210	28080	21060	14040	7020
D-3	102720	77040	51360	25680	88560	66420	44280	22140
D-4	381750	108150	78300	40950	116850	104100	64650	27300
	Alternative-20							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	52320	39240	26160	13080	32400	24300	16200	8100
D-2	48840	36630	24420	12210	28080	21060	14040	7020
D-3	151470	110190	74910	38430	132960	101970	66330	30990
D-4	333000	75000	54750	28200	72450	68550	42600	18450

Table 5.33: Reorder point for all alternatives

	Reorder Point (Ro)							
	Alternative-1							
	Period-1				Period-2			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	284	246	201	142	220	190	156	110
D-2	221	191	156	111	187	162	133	94
D-3	269	227	188	136	267	235	188	127
D-4	516	245	209	150	241	234	185	121
	Alternative-2							
	Period-1				Period-2			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	284	246	201	142	220	190	156	110
D-2	221	191	156	111	187	162	133	94
D-3	342	282	234	170	318	289	230	153
D-4	471	178	156	110	168	163	129	86
	Alternative-3							
	Period-1				Period-2			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	284	246	201	142	220	190	156	110
D-2	221	191	156	111	187	162	133	94
D-3	183	158	129	91	189	164	134	94
D-4	553	294	250	181	306	289	227	148
	Alternative-4							
	Period-1				Period-2			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	284	246	201	142	220	190	156	110
D-3	407	341	282	203	369	332	265	180
D-4	471	178	156	110	168	163	129	86
	Alternative-5							
	Period-1				Period-2			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	284	246	201	142	220	190	156	110
D-3	348	297	245	175	326	286	230	157
D-4	516	245	209	150	241	234	185	121

	Alternative-6							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	284	246	201	142	220	190	156	110
D-3	287	248	203	143	266	231	188	133
D-4	553	294	250	181	306	289	227	148
	Alternative-7							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	284	246	201	142	220	190	156	110
D-3	221	191	156	111	187	162	133	94
D-4	582	334	282	203	359	332	264	175
	Alternative-8							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	301	261	213	151	247	214	175	124
D-2	198	171	140	99	150	130	106	75
D-3	342	282	234	170	318	289	230	153
D-4	471	178	156	110	168	163	129	86
	Alternative-9							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	301	261	213	151	247	214	175	124
D-2	198	171	140	99	150	130	106	75
D-3	269	227	188	136	267	235	188	127
D-4	516	245	209	150	241	234	185	121
	Alternative-10							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	301	261	213	151	247	214	175	124
D-2	198	171	140	99	150	130	106	75
D-3	183	158	129	91	189	164	134	94
D-4	553	294	250	181	306	289	227	148
	Alternative-11							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	301	261	213	151	247	214	175	124
D-2	198	171	140	99	150	130	106	75
D-4	582	334	282	203	359	332	264	175

	Alternative-12							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	360	312	255	180	289	250	204	144
D-3	342	282	234	170	318	289	230	153
D-4	471	178	156	110	168	163	129	86
	Alternative-13							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	360	312	255	180	289	250	204	144
D-3	269	227	188	136	267	235	188	127
D-4	516	245	209	150	241	234	185	121
	Alternative-14							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	360	312	255	180	289	250	204	144
D-4	582	334	282	203	359	332	264	175
	Alternative-15							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	360	312	255	180	289	250	204	144
D-2	183	158	129	91	189	164	134	94
D-4	553	294	250	181	306	289	227	148
	Alternative-16							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	205	177	145	102	161	139	114	80
D-2	297	257	210	148	240	208	170	120
D-3	342	282	234	170	318	289	230	153
D-4	471	178	156	110	168	163	129	86
	Alternative-17							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	205	177	145	102	161	139	114	80
D-2	198	171	140	99	150	130	106	75
D-3	407	341	282	203	369	332	265	180
D-4	471	178	156	110	168	163	129	86

	Alternative-18							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	205	177	145	102	161	139	114	80
D-2	198	171	140	99	150	130	106	75
D-3	348	297	245	175	326	286	230	157
D-4	516	245	209	150	241	234	185	121
	Alternative-19							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	205	177	145	102	161	139	114	80
D-2	198	171	140	99	150	130	106	75
D-3	287	248	203	143	266	231	188	133
D-4	553	294	250	181	306	289	227	148
	Alternative-20							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	205	177	145	102	161	139	114	80
D-2	198	171	140	99	150	130	106	75
D-3	348	297	245	175	326	286	230	157
D-4	516	245	209	150	241	234	185	121

Table 5.34 Product-miles for all alternatives

	Product-miles							
	Alternative-1							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	12884	9663	6442	3221	7639	5729	3820	1910
D-2	14252	10689	7126	3563	10248	7686	5124	2562
D-3	15045	10658	7371	3864	14687	11428	7316	3259
D-4	45880	11395	8245	4275	11148	10564	6553	2833
	Alternative-2							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	12884	9663	6442	3221	7639	5729	3820	1910
D-2	14252	10689	7126	3563	10248	7686	5124	2562
D-3	30030	20175	13892	7388	24691	20986	13189	5770
D-4	36075	5168	3978	1970	4602	4310	2711	1190

	Alternative-3							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	12884	9663	6442	3221	7639	5729	3820	1910
D-2	14252	10689	7126	3563	10248	7686	5124	2562
D-3	6107	4580	3054	1527	6547	4910	3274	1637
D-4	51730	15373	11071	5805	16476	14830	9199	3895
	Alternative-4							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	12884	9663	6442	3221	7639	5729	3820	1910
D-3	49168	34529	23461	12172	38452	31307	20069	9211
D-4	36075	5168	3978	1970	4602	4310	2711	1190
	Alternative-5							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	12884	9663	6442	3221	7639	5729	3820	1910
D-3	34183	25012	16940	8649	28449	21749	14197	6700
D-4	45880	11395	8245	4275	11148	10564	6553	2833
	Alternative-6							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	12884	9663	6442	3221	7639	5729	3820	1910
D-3	25246	18934	12623	6311	20309	15232	10154	5077
D-4	51730	15373	11071	5805	16476	14830	9199	3895
	Alternative-7							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	12884	9663	6442	3221	7639	5729	3820	1910
D-3	19138	14354	9569	4785	13762	10321	6881	3440
D-4	61030	22348	15720	8130	26445	22307	14184	6387
	Alternative-8							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	18873	14155	9437	4718	13049	9787	6524	3262
D-2	10908	8181	5454	2727	6271	4703	3136	1568
D-3	30030	20175	13892	7388	24691	20986	13189	5770
D-4	36075	5168	3978	1970	4602	4310	2711	1190

	Alternative-9							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	18873	14155	9437	4718	13049	9787	6524	3262
D-2	10908	8181	5454	2727	6271	4703	3136	1568
D-3	15045	10658	7371	3864	14687	11428	7316	3259
D-4	45880	11395	8245	4275	11148	10564	6553	2833
	Alternative-10							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	18873	14155	9437	4718	13049	9787	6524	3262
D-2	10908	8181	5454	2727	6271	4703	3136	1568
D-3	6107	4580	3054	1527	6547	4910	3274	1637
D-4	51730	15373	11071	5805	16476	14830	9199	3895
	Alternative-11							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	18873	14155	9437	4718	13049	9787	6524	3262
D-2	10908	8181	5454	2727	6271	4703	3136	1568
D-4	61030	22348	15720	8130	26445	22307	14184	6387
	Alternative-12							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	26525	19894	13262	6631	17448	13086	8724	4362
D-3	30030	20175	13892	7388	24691	20986	13189	5770
D-4	36075	5168	3978	1970	4602	4310	2711	1190
	Alternative-13							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	26525	19894	13262	6631	17448	13086	8724	4362
D-3	15045	10658	7371	3864	14687	11428	7316	3259
D-4	45880	11395	8245	4275	11148	10564	6553	2833
	Alternative-14							
	Period-1				**Period-2**			
	P-1	**P-2**	**P-3**	**P-4**	**P-1**	**P-2**	**P-3**	**P-4**
D-1	26525	19894	13262	6631	17448	13086	8724	4362
D-4	61030	22348	15720	8130	26445	22307	14184	6387

	Alternative-15							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	26525	19894	13262	6631	17448	13086	8724	4362
D-2	5274	3956	2637	1319	5654	4241	2827	1414
D-4	51730	15373	11071	5805	16476	14830	9199	3895
	Alternative-16							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	5232	3924	2616	1308	3240	2430	1620	810
D-2	25160	18870	12580	6290	16519	12389	8260	4130
D-3	30030	20175	13892	7388	24691	20986	13189	5770
D-4	36075	5168	3978	1970	4602	4310	2711	1190
	Alternative-17							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	5232	3924	2616	1308	3240	2430	1620	810
D-2	10908	8181	5454	2727	6271	4703	3136	1568
D-3	49168	34529	23461	12172	38452	31307	20069	9211
D-4	36075	5168	3978	1970	4602	4310	2711	1190
	Alternative-18							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	5232	3924	2616	1308	3240	2430	1620	810
D-2	10908	8181	5454	2727	6271	4703	3136	1568
D-3	34183	25012	16940	8649	28449	21749	14197	6700
D-4	45880	11395	8245	4275	11148	10564	6553	2833
	Alternative-19							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	5232	3924	2616	1308	3240	2430	1620	810
D-2	10908	8181	5454	2727	6271	4703	3136	1568
D-3	25246	18934	12623	6311	20309	15232	10154	5077
D-4	51730	15373	11071	5805	16476	14830	9199	3895
	Alternative-20							
	Period-1				Period-2			
	P-1	P-2	P-3	P-4	P-1	P-2	P-3	P-4
D-1	5232	3924	2616	1308	3240	2430	1620	810
D-2	10908	8181	5454	2727	6271	4703	3136	1568
D-3	34183	25012	16940	8649	28449	21749	14197	6700
D-4	45880	11395	8245	4275	11148	10564	6553	2833

Table 5.35: Total cost for all alternatives

Alternatives	Total cost
Alternative-1	26812172
Alternative-2	26890528
Alternative-3	26759364
Alternative-4	26965867
Alternative-5	26887578
Alternative-6	26834900
Alternative-7	26894769
Alternative-8	26927484
Alternative-9	26849127
Alternative-10	26796319
Alternative-11	26855494
Alternative-12	26880193
Alternative-13	26801837
Alternative-14	26808204
Alternative-15	26733369
Alternative-16	26937032
Alternative-17	27013041
Alternative-18	26934752
Alternative-19	26882074
Alternative-20	26934752

From Table 5.35, alternative 15 has the minimum cost. Therefore this network should be followed by the company for the optimum inventory policy and the minimum cost. The alternative 15 is shown in the following Figure 5.13:

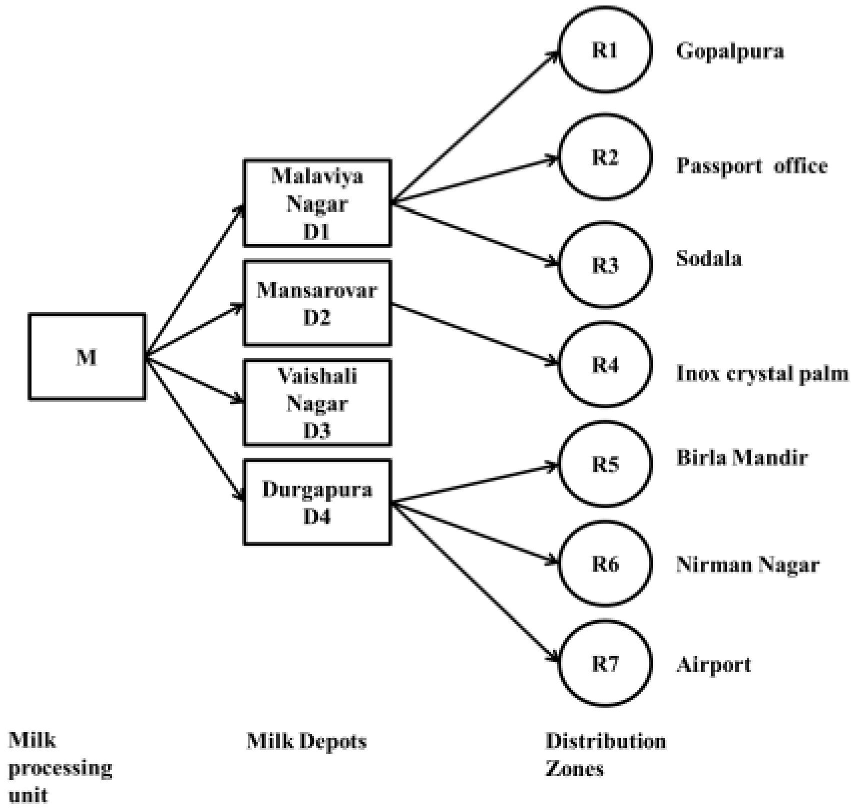


Figure 5.13: Suggested optimum network for Jaipur dairy

The above Figure shows that, depot, D1 is fulfilling the demands of three distribution zones, R1, R2 and R3 (Distribution zones R1, R2 and R3 are pooled for depot D1). Depot D2 is responsible to fulfill the demand of only distribution zone R4 and distributor D3 is inactive. This facility may be closed. The depot, D4 is responsible to fulfill the demand of distribution zone R5, R6 and R7 (Distribution zones R5, R6 and R7 are pooled for depot D4). This is the optimum network which provides the minimum cost and optimum inventory policy.

5.4 Summary of the Chapter

Two different inventory control models have been suggested using risk pooling approach in distribution network in Chapter 3 and Chapter 4. Chapter 3 deals with single-product, single period inventory control model while Chapter 4 deals with multi-product, multi-period inventory control model. Two different case studies have been performed in two different industries for evaluating the practical applicability of these two models. Chapter 5 deals with these two case studies. The purpose of two case studies is to check the acceptability of these two models in various types of industries. For the case study purpose and the validation of these two models two different industries have been chosen. For the validation of single-product, single-period inventory control model, a case study is taken from a sugar mill distribution network. Sugar is a highly demanded product and its shelf life is longer. For this case study some relevant data has been collected and validate these data through the proposed model. Another case study has been taken from Jaipur dairy for the validation of multi-product, multi-period inventory control model. Four different types of milk are considered as four different types of products. Morning and evening supply of milk are treated as the periods of demands. For the validation purpose some relevant data has been collected from Jaipur dairy and this data is validated through the proposed model. As milk is a perishable product and has lower shelf life. Therefore it may be concluded that this model is widely applicable for various types of products and industries.

CHAPTER 6

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

6.1 Introduction

The final chapter of this thesis represents the detailed discussion on concluding remarks, findings of this research and some suggestions for future research directions. The emphasis of this research is on how risk pooling approach contributed to the body of research in controlling inventory under different types of uncertainties. This chapter also shows the limitations of this research and some potential research areas. Section 6.2 represents the detailed discussion on concluding remarks. The limitations of this study are discussed in Section 6.3 and section 6.4 suggests some future research directions.

6.1.1 Managerial Implications

In this dissertation, inventory aggregation approach is used for controlling the inventory in a distribution network. This approach is very much useful for reducing the safety inventory under different types of uncertainties. For this, a mathematical model is developed for multiproduct, multi planning periods in a distribution network by using inventory aggregation approach with a fixed cycle service level under different types of uncertainties. For checking the practical applicability of model, a case study is performed in a sugar mill distribution network. This model clearly suggests the optimum inventory policy under different type of uncertainties. This model also suggests to the inventory managers about to keep the safety inventory level under various uncertainty levels. This model has a number of managerial implications for controlling inventory under different types and levels of uncertainties.

- This model suggests to the inventory manager about the level of reorder point and ordering quantities under different types and levels of uncertainties.
- This model suggests inventory manager the level of safety inventory under different types and levels of uncertainties.

- This model suggests the best inventory policy to the inventory manager with the appropriate weight of cost and responsiveness.
- This model suggests the best inventory policy with the optimum facility in a distribution network for a minimum cost.
- The practical applicability of this model can be checked for different types of products in a distribution network.

6.2 Concluding remarks and discussion

Inventory management is a crucial part of supply chain management philosophy under uncertainty and inventory represents from twenty to sixty percent of most manufacturing firms' total assets. In this context, a detailed literature review has been done and it is found that there are numerous inventory control models under uncertainty suggested by various researchers. On the basis of this literature review, some research gaps are identified and some future research directions are also suggested. After going in to deeper insight, it is found that, inventory aggregation is a suitable approach for reducing the level of safety inventory under uncertainty. It is decided here, that a mathematical model will be developed for controlling inventory in a distribution network by incorporating the inventory aggregation approach for getting desired service level under different types of uncertainties. For executing this research, a detailed investigation has been done for inventory aggregation approach and a mathematical model is developed for controlling inventory incorporating inventory aggregation approach. The main contributions of this study may be summarized as follows:

1. For executing the above study, a small network is considered between distributors and retailers for simplicity of calculations. For numerical illustration 5 distributors and 7 retailers are considered in this network. An important condition is applied that one distributor can fulfill the demand of more than one retailer and one retailer cannot get supply from more than one distributor due to some business conditions. After imposing the above condition, different scenarios of demand fulfillment are generated. For numerical illustration, total 20 scenarios of demand fulfillment are generated by imposing the above condition.
2. In this study, inventory aggregation is incorporated in the modeling for reducing safety stock under uncertain demand and supply. As a result, it reduces the inventory holding cost at

different facilities in the network. For finding best inventory policy at distributor, the retailer's demands are aggregated in different scenarios of demand fulfillment. This is inventory aggregation approach, as a result safety inventory is reduced and overall network cost is also reduced.

3. For developing a mathematical model mainly ordering cost, inventory holding cost, transportation cost, facility cost and operating cost for facility are considered in the network and inventory aggregation approach is incorporated in the modeling. As, the safety inventory is considered for maintaining desired service level against the uncertainty, so there is no consideration of shortages, backorders and lost sales due to shortages.

4. Initially, the model is developed for single product and single planning period. The model is tested with some numerical examples and results are found out. The model provides best inventory policy for different scenarios of demand fulfillment with minimum cost.

5. After solving, the single product, single period problem, the modeling is extended to the multiple products and the multiple periods. For solving the above problem, three different types of products and five planning periods are considered. Rest of the conditions and scenarios are same as in the previous models. This model also provides the inventory policies for different scenarios, for different products and in different planning periods along with total cost.

6. After solving the above problem, some other performance parameters are also calculated in excel sheet for evaluating the performance of the system. These performance parameters are Expected shortages per cycle (ESC), Fill Rate (FR), Average inventory (AI) and Inventory position (IP).

7. There are numbers of different scenarios of demand fulfillment (according to the present study, 20 possible scenarios) and some performance parameters like Total cost (TC), Responsiveness (Res), Reorder point (Ro), Safety stock (SS), Expected shortages per cycle (ESC), Fill Rate (FR), Average inventory (AI) and Inventory position (IP). Out of these, best inventory policy has to be selected. For this Fuzzy-TOPSIS approach is implemented in between performance parameters and different scenarios of demand fulfillment. Finally a best inventory policy is selected based on some decision maker's perception.

8. As a conclusion, this study helps in the selection of best alternative of demand fulfillment along with the appropriate weightage to performance parameters of an inventory system. This study suggests to an inventory manager about to keep the level of safety inventory, reorder point

and ordering quantity under different types and levels of uncertainties. This study also gives an idea, what may be the effect of different types of uncertainties on the total cost of the system.

6.3 Limitations of present study

In this research, a mathematical model is developed using risk pooling approach for optimizing inventory policy. A number of tests and numerical illustrations are carried to solve problem. For checking the practical applicability, two case studies from sugar mill and milk processing plant are also performed. After all this research has some limitations as this research is not exception as well. While presenting this thesis all issues are considered seriously still, few points are noticed which could be identified as the limitations of this research work. The limitations of this research work can be deliberated as follows:

“This study is carried out for a small network while a large network can be considered for solving. In this study, the problem is solved through the solver, while some more advanced computational algorithms may be used as solution approaches. For solving the research problem, the input data is taken from the previously published research papers, other sources for the input data are not considered in this study. In this study, some performance parameters are evaluated against some inventory policies. For finding best inventory policy Fuzzy TOPSIS approach is implemented between performance parameters and inventory policies on the basis of Decision makers’ perception. There may be some inconsistencies in the decision of decision makers which may affect the result of study. The network in this study is tested for local network, it may be tested globally”.

6.4 Future research directions

The potential future scope of this research may be suggested as follows:

- In this study, a small and simple network is taken for the easiness of study. A large network and complex network can be selected for the study.
- This problem is applied in a sugar mill distribution network and tested for local network. This problem can be tested for global network for optimizing inventory policy.
- Manufacturer is out of scope in the present study. In future studies, manufacturer can be incorporated in the network.

- Some evolutionary algorithms can be implemented as solution approaches for getting improved results.
- In the present study, only one case study from sugar mill is performed for single product. In the same way, more case studies can be performed for single product as well as multiproduct.
- In the present case study, a fixed weightage of cost and responsive are taken. Some different case studies can be performed for different type of product and for different weightage between cost and responsiveness.
- In this research, demand, supply and combined effects of demand and supply uncertainty are investigated. Risk factors are not considered in this study and risk factors can be embedded into this study.
- Risk pooling approach is a suitable approach for reducing safety inventory and it can be integrated with the concept of product modularity.
- An integrated approach of risk pooling, risk pooling and the concept of mass customization may be developed. This may be the potential research area.

6.5 Summary of the chapter

The current chapter deals with mainly conclusion of the present study and suggests some future research directions. In this chapter managerial implications and the limitations of the present study has been discussed in a detailed manner.

REFERENCES

1. Arnold J.R.T. (1998), “Introduction to Materials Management”, Prentice Hall. (https://perso.crans.org/fortineau/ressources_maxnce_nouchet/Introduction%20to%20 Materials%20Management.pdf)
2. Christopher M. (1992), Logistic and Supply Chain Management, London, Pitman Pub. (http://www.icesi.edu.co/blogs/supplychain0714/files/2014/07/Martin_Christopher_Logistics_and_Supply_Chain_Management_4th_Edition____2011-1.pdf)
3. Routroy, S and Kodali, R. (2005),"Differential evolution algorithm for supply chain inventory planning", Journal of Manufacturing Technology Management, Vol. 16, No. 1 pp. 7 – 17 (https://www.emeraldinsight.com/doi/full/10.1108/17410380510574059)
4. Agrawal, P. (2014), “Effect of uncertain and turbulent environment on organizational design”, Economics and business Journal: Enquires and perspective, Vol. 5, No. 1, pp. 11-24. (https://pdfs.semanticscholar.org/3e10/89cd5c81e88c945ac4e3996ff86de7d3a8e5.pdf)
5. Govindan, K., Fattahi, Md., and Keyvanshokooh, E. (2017), “Supply chain network design under uncertainty: A comprehensive review and future research directions”, European Journal of Operational Research, Vol. 263, No. 1, pp. 108-141. (https://www.sciencedirect.com/science/article/pii/S0377221717303429)
6. Gaur, S. and Ravindran, A.R. (2006), “A bi-criteria model for the inventory aggregation problem under risk pooling”, Computers and Industrial Engineering, Vol. 51, No. 1, pp. 482-501. (https://www.sciencedirect.com/science/article/pii/S0360835206001082)
7. Lee, H.L. and Billington, C. (1992), “Managing supply chain inventories: opportunities and pitfalls”, MIT Sloan Management Review; spring 1992; Vol. 33, No.3, pp. 65-74. (https://sloanreview.mit.edu/article/managing-supply-chain-inventory-pitfalls-and-opportunities/)
8. Angkiriwang, R., Pujawan, I.N. & Santosa, B. (2014), “Managing uncertainty through supply chain flexibility: reactive vs. proactive approaches”, Production and Manufacturing Research, Vol. 2, No. 1, pp. 50-70. (https://www.tandfonline.com/doi/full/10.1080/21693277.2014.882804)

9. Ganeshan, R. (1999), “Managing supply chain inventories: A multiple retailer, one warehouse, multiple supplier model”, International Journal of production Economics, Vol. 59, No. 1-3, pp. 341-354.
(https://www.sciencedirect.com/science/article/pii/S0925527398001157)
10. Etienne, E.C. (2005), “Supply chain responsiveness and the inventory illusion”, Supply Chain Forum, Vol. 6, No. 1, pp. 48-65.
(https://www.tandfonline.com/doi/abs/10.1080/16258312.2005.11517138)
11. Constantin, A. (2016), “Inventory management, service level and safety stock”, Journal of Public Administration, Finance and Law, Vol. X, No. 6, pp. 145-153. (http://www.jopafl.com/uploads/issue9/INVENTORY_MANAGEMENT_SERVICE_LEVEL_AND_SAFETY_STOCK.pdf)
12. Jones, Thomas C and Riley, Daniel W. (1985), “Using Inventory for Competitive Advantage through Supply Chain”, International Journal of Physical Distribution & Logistics Management, Vol. 15, No. 5, pp. 16 – 26.
(https://www.emeraldinsight.com/doi/abs/10.1108/eb014615)
13. Scott, Charles and Westbrook, Roy (1991),"New Strategic Tools for Supply Chain Management", International Journal of Physical Distribution & Logistics Management, Vol. 21, No. 1, pp. 23 – 33.
(https://www.emeraldinsight.com/doi/abs/10.1108/09600039110002225)
14. New, Stephen J. and Payne, Philip (1995),"Research frameworks in logistics", International Journal of Physical Distribution & Logistics Management, Vol. 25, No. 10, pp. 60 – 77.
(https://www.emeraldinsight.com/doi/full/10.1108/09600039510147663)
15. Farley, G.A. (1997), “Discovering supply chain management: a round table discussion”, APICS-The performance advantage, Vol. 7, No.1, pp. 38-39. (http://www.sciepub.com/reference/44383)
16. Ganeshan, R and Harisson, T.P (2002), “An introduction to supply chain management”, Department of Management Science and Information Systems, Penn State University, Vol. xx, No. yy, pp. 2-7.
(http://lcm.csa.iisc.ernet.in/scm/supply_chain_intro)

17. Chopra, S. (2003), "Designing the distribution network in a supply chain", Transportation Research Part E, Vol. 39, No. 2, pp. 123-140. (https://www.kellogg.northwestern.edu/faculty/chopra/htm/research/deliverynetwork.pdf)
18. Ravert, D. (2013), "Delivering sustainability through supply chain distribution network redesign", Central European Business Review, Vol. 2, No. 3, pp. 22-29. (https://cebr.vse.cz/pdfs/cbr/2013/03/04.pdf)
19. Keskinocak, P. and Tayur, S. (2001), "Quantitative analysis for internet-based supply chain", Interfaces (Informs), Vol. 31, No. 2, pp. 70-89. (https://dl.acm.org/citation.cfm?id=511789)
20. Tagaras, G. (1989), "Effects of Pooling on the Optimization and Service Levels of Two-Location Inventory Systems", IIE Transactions, Vol. 21, No.3, pp. 250-257. (https://www.tandfonline.com/doi/abs/10.1080/07408178908966229)
21. Tagaras, G. (1999), "Pooling in multi-location periodic distribution system", Omega, International Journal of Management Science, Vol. 27, No. 1, pp. 39-59. (https://www.sciencedirect.com/science/article/pii/S0305048398000309)
22. Weng, Z. Kevin, (1999), "Risk pooling over demand uncertainty in the presence of product modularity", International Journal of production Economics, Vol. 62, No.1-2, pp. 75-85. (https://www.sciencedirect.com/science/article/pii/S0925527398002266)
23. Xu, Kefeng. and Evers, Philip T. (2003), "Managing single echelon inventories through demand aggregation and the feasibility of a correlation matrix" Computers and Operations Research, Vol. 30, No. 2, pp. 297-308. (https://www.sciencedirect.com/science/article/pii/S0305054801000971)
24. Eppen, G. D. (1979), "Note—Effects of Centralization on Expected Costs in a Multi-Location Newsboy Problem", Management Science, Vol. 25, No.5, pp. 498-501. (https://pubsonline.informs.org/doi/10.1287/mnsc.25.5.498)
25. Caplin, Andrew S. (1985), "The Variability of Aggregate Demand with (S, s) Inventory Policies", Econometrica, Vol. 53, No. 6, pp. 1395-1409. (https://www.jstor.org/stable/1913214?seq=1#metadata_info_tab_contents)

26. Chang, Pao-Long and Lin Chin-Tsai (1991), “Centralized effect on expected costs in a multi-location news boy problem”, Journal of the Operations Research, The Operations Research Society of Japan, Vol. 34, No. 1, pp. 87-92. (https://www.jstor.org/stable/2630280?seq=1#metadata_info_tab_contents)
27. C.A. Weber, J.R. Current & W.C. Benton (1991), “Vendor Selection Criteria and Methods”, European Journal of Operational Research, Vol. 50, No. 1, p.p. 2-18. (https://www.sciencedirect.com/science/article/abs/pii/037722179190033R)
28. Verwijmeren, M., Vander Vlist, P. and Van Donselaar, K. (1996), ``Networked inventory management information systems: materializing supply chain management'', International Journal of Physical Distribution & Logistics Management, Vol. 26 No. 6, pp. 16-31. (https://www.emeraldinsight.com/doi/full/10.1108/09600039610125170)
29. Karwowski W., Evans G.W. (1986), “Fuzzy concepts in production management research: a review”, International Journal of Production Research, Vol. 24, No. 1, pp. 129-147. (https://www.tandfonline.com/doi/abs/10.1080/00207548608919718)
30. Park K.S. (1987), “Fuzzy-Set Theoretic Interpretation of Economic Order Quantity”, IEEE Transactions on Systems, Man, and Cybernetics, Vol. 17, No. ,pp. 1082-1084. (https://ieeexplore.ieee.org/iel7/21/6324864/06499320.pdf)
31. Geary, S., Childerhouse, P., and Towill, D. R. (2002). “Uncertainty and the seamless supply chain” Supply Chain Management Review, Vol. 6, No. 4, pp. 52-61. (https://trid.trb.org/view/623370)
32. Minner, S. (2003), “Multiple-supplier inventory models in supply chain management: a review”, International Journal of Production Economics, Vol. 81, No. 82, pp. 265–279. (https://www.sciencedirect.com/science/article/pii/S0925527302002888)
33. Simchi-Levi, D., Kaminsky, P., and Simchi-Levi, E. (2000), “Designing and managing the supply chain”, Irwin McGraw-Hill, Boston, MA. (https://trove.nla.gov.au/work/11547870?q&l-edition=McGraw+Hill%2FIrwin+Series+Operations+And+Decision+Sciences)

34. Giannoccaro, I., Pontrandolfo, P., and Scozzi, B. (2003), "A fuzzy echelon approach for inventory management in supply chains". European. Journal of Operational Research, Vol. 149, pp. 185–196. (https://www.sciencedirect.com/science/article/pii/S0377221702004411)
35. Ganeshan, R. (1999), "Managing supply chain inventories: a multiple retailer, one warehouse, multiple supplier models", International journal of Production Economics., Vol. 59, pp. 341–354. (https://www.sciencedirect.com/science/article/pii/S0925527398001157)
36. Towill, D. R. (1996), "Industrial dynamics modeling of supply chains", Logistics Information Management, Vol. 9, pp. 43–56. (https://www.emeraldinsight.com/doi/10.1108/09600039610113182)
37. Rau, H., Wu, M.-Y., and Wee, H.-M. (2003), "Integrated inventory model for deteriorating items under a multi-echelon supply chain environment". International Journal of Production Economics, Vol. 86, pp. 155–168. (https://www.sciencedirect.com/science/article/pii/S0925527303000483)
38. Diks, E. B. and de Kok, A. G. (1998), "Optimal control of a divergent multi-echelon inventory system". European Journal of Operational Research, Vol. 111, pp. 75–97. (https://www.sciencedirect.com/science/article/pii/S0377221797003275)
39. Chiang, W. K. and Monahan, G. E. (2005), "Managing inventories in a two-echelon dual-channel supply chain". European Journal of Operational Research, Vol. 162, pp. 325–341. (https://www.sciencedirect.com/science/article/pii/S0377221703006830)
40. Moinzadeh, K. and Aggarwal, P. K. (1997), "An information based multi-echelon inventory system with emergency orders". Operations Research, Vol. 45, No. 5, PP. 694. (https://pubsonline.informs.org/doi/10.1287/opre.45.5.694)
41. Kalchschmidt, M., Zotteri, G., and Verganti, R. (2003), "Inventory management in a multi-echelon spare parts supply chain". International Journal of Production Economics, Vol. 81, No. 82, pp. 397–413. (https://www.sciencedirect.com/science/article/pii/S0925527302002840)

42. Clark, A. and Scarf, H. (1960), "Optimal policies for a multi-echelon inventory problem". Management Science, Vol. 6, pp. 475–490. (https://pubsonline.informs.org/doi/10.1287/mnsc.6.4.475)

43. Bollapragada, S., Akella, R., and Srinivasan, R. (1998), "Centralized ordering and allocation policies in a two-echelon system with non-identical warehouses". European Journal of Operational research, Vol. 106, pp. 74–81. (https://www.sciencedirect.com/science/article/pii/S0377221797001483)

44. Tee, Y. S. and Rossetti, M. D. (2002), "A robustness study of a multi-echelon inventory model via simulation". International Journal of Production Economics, Vol. 80, pp. 265–277. (https://www.sciencedirect.com/science/article/pii/S0925527302002591)

45. Dong, L. and Lee, H. L. (2003), "Optimal policies and approximations for a serial multi-echelon inventory system with time-correlated demand". Operations research, Vol.51, No. 6, pp.969. (apps.olin.wustl.edu/workingpapers/pdf/2002-03-006.pdf)

46. Van der Vorst, J. G. A. J., Beulens, A. J. M., and Van Beek, P. (2000), "Modelling and simulating multi-echelon food systems". European Journal of Operational research, Vol. 122, pp. 354–366. (https://www.sciencedirect.com/science/article/pii/S0377221799002386)

47. Bessler, S. A. and Veinott, A. F. (1965), "Optimal policy for a dynamic multi-echelon inventory model". *Naval Research. Logistics Q.*, Vol. 13, No. 4, pp. 355–389. (https://onlinelibrary.wiley.com/doi/abs/10.1002/nav.3800130402)

48. Eppen, G. and Schrage, L. (1981), "Centralized ordering policies in a multi-warehouse system with lead times and random demand". Multi-level production-inventory control systems: theory and practice (Ed. L. B. Schwarz), North-Holland, Amsterdam. (https://ocw.mit.edu/courses/...management/...management.../lec2_schoenmeyr.pdf)

49. Van der Heijden, M. C. (1999), "Multi-echelon inventory control in divergent systems with shipping frequencies". European Journal of Operational research, Vol. 116, pp. 331–351.

(https://www.sciencedirect.com/science/article/pii/S0377221798000484)

50. Rosenbaum, B. A. (1981), “Service level relationships in a multi- echelon inventory system”. Management Science, Vol. 27, No. 8, pp. 926–945. (https://dl.acm.org/citation.cfm?id=2892626)
51. Federgruen, A. and Zipkin, P. (1984), “Computational issues in an infinite-horizon, multi-echelon inventory model”. Operations Research, Vol. 32, pp. 818–836. (https://dl.acm.org/citation.cfm?id=2777278)
52. Schwarz, L. B., Deuermeyer, B. L., and Badinelli, R. D. (1985), “Fill rate optimization in a one-warehouse N-identical retailer distribution system”. Management Science, Vol. 31, No. 4, pp. 488–498. (https://pubsonline.informs.org/doi/abs/10.1287/mnsc.31.4.488?journalCode=mnsc)
53. Jackson, P. L. (1988), “Stock allocation in a two-echelon distribution system or “what to do until your ship comes in”. Management Science, Vol. 34, pp. 880–895. (https://pubsonline.informs.org/doi/pdf/10.1287/mnsc.34.7.880)
54. Jackson, P. L. and Muckstadt, J. A. (1989) “Risk pooling in a two-period, two-echelon inventory stocking and allocation problem”. Naval Research. Logistic Q., Vol. 36, No. 1, pp. l–26. (https://onlinelibrary.wiley.com/doi/abs/10.1002/1520-6750%28198902%2936%3A1%3C1%3A%3AAID-NAV3220360102%3E3.0.CO%3B2-S)
55. Erkip, N., Hausman, W. H., and Nahmias, S. (1990), “Optimal centralized ordering policies in multi-echelon inventory systems with correlated demands”. Management Science, Vol. 36, No. 3, pp. 38l–392. (https://pubsonline.informs.org/doi/10.1287/mnsc.36.3.381)
56. Axsater, S. (1990), “Simple solution procedures for a class of two-echelon inventory problems”. Operations Research, Vol. 38, No. 1, pp. 64. (https://www.jstor.org/stable/171298)
57. Verrijdt, J. H. C. M. and de Kok, A. G. (1996), “Distribution planning for a divergent depot less two-echelon network under service constraints”. European Journal of Operational Research, Vol. 89, pp. 341–354.

(https://pdfs.semanticscholar.org/1476/1a9fb750c81ff1430fb404124007fb857f6e.pdf)

58. Thomas, D. J. and Griffin, P. M. (1996), "Coordinated supply chain management". European Journal of Operational Research, Vol. 94, pp. 1–15. (https://www.sciencedirect.com/science/article/pii/0377221796000987)
59. Parker, R. P. and Kapuscinski, R. (2004), "Optimal policies for a capacitated two-echelon inventory system". Operations Research, Vol. 52, No. 5, pp. 739. (https://pubsonline.informs.org/doi/10.1287/opre.1040.0131)
60. Hadley, G. and Whitin, T. M. (1963), "Analysis of inventory systems", Prentice-Hall, Englewood Cliffs, NJ. (www.worldcat.org/title/analysis-of-inventory-systems/oclc/234586)
61. Sherbrooke, C. C. (1968), "Metric: a multi-echelon technique for recoverable item control". Operations Research, Vol. 16, pp. 122–141. (https://pubsonline.informs.org/doi/10.1287/opre.16.1.122)
62. Sherbrooke, C. C. (1992) "Optimal inventory modeling of systems multi-echelon techniques", John Wiley & Sons, New York. (https://www.researchgate.net/.../247067108_Optimal_Inventory_Modeling_of_Systems...)
63. Moinzadeh, K. and Lee, H. L. (1986), "Batch size and stocking levels in multi-echelon repairable systems". Management Science, Vol. 32, pp. 1567–1591. (https://pubsonline.informs.org/doi/10.1287/mnsc.32.12.1567)
64. Svoronos, A. and Zipkin, P. (1988), "Estimating the performance of multi-level inventory systems". Operations Research. Vol. 36, pp. 57–72. (https://pubsonline.informs.org/doi/10.1287/opre.36.1.57)
65. Nahmias, S. and Smith, S. A. (1994), "Optimizing inventory levels in a two-echelon retailer system with partial lost sales". Management Science, Vol. 40, pp. 582–596. (https://pubsonline.informs.org/doi/10.1287/mnsc.40.5.582)
66. Aggarwal, P. K. and Moinzadeh, K. (1994), "Order expedition in multi-echelon production/distribution systems". IIE Transactions, Vol. 26, pp. 86–96. (https://www.tandfonline.com/doi/abs/10.1080/07408179408966599)
67. Pyke, D. F. (1990), "Priority repair and dispatch policies for repairable-

item logistics systems". Naval Research. Logistics, Vol. 37, pp. 1–30. (https://onlinelibrary.wiley.com/.../1520-6750%28199002%2937%3A1%3C1%3A%3AA...)

68. Dada, M. (1992), "A two-echelon inventory system with priority shipments". Management Science, Vol. 38, pp. 1140–1153. (https://pubsonline.informs.org/doi/10.1287/mnsc.38.8.1140)
69. Alfredsson, P. and Verrijdt, J. (1999), "Modeling emergency supply flexibility in a two-echelon inventory system". Management Science, Vol. 45, pp. 1416–1431. (https://pure.tue.nl/ws/files/4442947/459319.pdf)
70. Seferlis, P. and Giannelos, G. F. (2004), "A two-layered optimization-based control strategy for multi-echelon supply chain networks". Computers and Chemical Engineering, Vol. 28, pp. 799–809. (https://www.sciencedirect.com/science/article/pii/S0098135404000481)
71. Andersson, J. and Melchiors, P. (2001), "A two-echelon inventory model with lost sales". International Journal of Production Economics, Vol. 69, pp. 307–315. (https://www.sciencedirect.com/science/article/pii/S0377221706005674)
72. Axsater, S. (2003), "Approximate optimization of a two-level distribution inventory system". International Journal of Production Economics, Vol. 81–82, pp. 545–553. (https://www.sciencedirect.com/science/article/pii/S0925527302002700)
73. Silver, E. A., Pyke, D. F., and Peterson, R. (1998), "Inventory management and production planning and scheduling", Third edition, John Wiley, New York. (https://www.researchgate.net/publication/239386432_Inventory_Management_and_Production_Planning_and_Scheduling_Third_Edition)
74. Mitra, S. and Chatterjee, A. K. (2004a), "Echelon stock based continuous review (R; Q) policy for fast moving items". Omega, Vol. 32, No. 161–166. (https://www.sciencedirect.com/science/article/pii/S0305048303001348)

75. Hariga, M. (1998), “A single-period, multi-echelon stochastic model under a mix of assemble to order and assemble in advance policies”. Naval Research Logistics, Vol. 45, pp. 599–614. (https://onlinelibrary.wiley.com/.../%28SICI%291520-6750%28199809%2945%3A6%3...)
76. Chen, F. (1999), “Worst-case analysis of (R; Q) policies in a two- stage serial inventory system with deterministic demand and backlogging”. Operations Research Letters, Vol. 25, pp. 51–58. (https://www.sciencedirect.com/science/article/pii/S0167637799000358)
77. Axsater, S. and Zhang, W.-F. (1999), “A joint replenishment policy for multi-echelon inventory control”. International Journal of Production Economics, Vol. 59, pp. 243–250. (https://www.sciencedirect.com/science/article/pii/S0925527398000188)
78. Nozick, L. K. and Turnquist, M. A. (2001), “A two echelon inventory allocation and distribution center location analysis”. Transportations Research Part E, Vol. 37, pp. 425–441. (https://www.sciencedirect.com/science/article/pii/S1366554501000072)
79. So, K. C. and Zheng, X. (2003), “Impact of supplier’s lead-time and forecast demand updating on retailer’s order quantity variability in a two-level supply chain”. International Journal of Production Economics, Vol. 86, pp. 169–179. (https://www.sciencedirect.com/science/article/pii/S0925527303000501)
80. Forsberg, R. (1996), “Exact evaluation of (R, Q)-policies for two- level inventory systems with Poisson demand”. European Journal of Operational Research, Vol. 96, pp. 130–138. (https://www.sciencedirect.com/science/article/pii/S0377221796001373)
81. Graves, S. C. (1996), “A Multi-echelon inventory model with fixed replenishment intervals”. Management Science, Vol. 32, No. 1, pp. 1. (https://pubsonline.informs.org/doi/10.1287/mnsc.42.1.1)
82. Mohebbi, E. and Posner, M. J. M. (1998), “Sole versus dual sourcing in a continuous-review inventory system with lost sales”. Computers and

Industrial Engineering, Vol. 34, No. 2, 321–336. (https://www.sciencedirect.com/science/article/pii/S0360835297002799)

83. Dekker, R., Kleijn, M. J., and de Kok, A. G. (1998), "The break quantity rule's effect on inventory costs in a 1-warehouse, N-retailers distribution system". International Journal of production Economics, Vol. 56–57, pp. 61–68. (https://www.sciencedirect.com/science/article/pii/S0925527397000674)
84. Korugan, A. and Gupta, S. M. (1998), "A multi-echelon inventory system with returns". Computers and Industrial Engineering, Vol. 35(1–2), pp. 145–148. (https://www.sciencedirect.com/science/article/pii/S0360835298000412)
85. Van der Heijden, M. C., Diks, E., and de Kok, T. (1999), "Inventory control in multi-echelon divergent systems with random lead times". OR Spectrum, Vol. 21, pp. 331–359. (https://link.springer.com/article/10.1007/s002910050093)
86. Andersson, J. and Marklund, J. (2000), "Decentralized inventory control in a two-level distribution system". European Journal of Operational Research, Vol. 127, No. 483–506. (https://www.sciencedirect.com/science/article/pii/S037722179900332X)
87. Cachon, G. P. and Fisher, M. (2000), "Supply chain inventory management and the value of shared information". Management Science, Vol. 46, No. 8, pp. 1032–1048. (pulsar.wharton.upenn.edu/~cachon/pdf/cachon_fisher_ms.pdf)
88. Axsater, S. (2000), "Exact analysis of continuous review (R, Q) policies in two-echelon inventory systems with compound Poisson demand". Operations Research, Vol. 48, No. 5, pp. 686–696. (https://pubsonline.informs.org/doi/abs/10.1287/opre.48.5.686.12403?journalCode=opre)
89. Axsater, S. (2001), "Scaling down multi-echelon inventory problems". International Journal of production Economics, Vol. 71, pp. 255–261. (https://www.sciencedirect.com/science/article/pii/S0925527300001237)
90. Tsiakis, P., Shah, N., and Pantelides, C. C. (2001), "Design of multi-echelon supply chain networks under demand uncertainty". Industrial & Engineering

Chemistry Research (ACS Publications), Vol. 40, pp. 3585–3604. (https://pubs.acs.org/doi/abs/10.1021/ie0100030)

91. Moinzadeh, K. (2002), "A multi-echelon inventory system with information exchange". Management Science, Vol. 48, No. 3, pp. 414. (https://pubsonline.informs.org/doi/10.1287/mnsc.48.3.414.7730)
92. Tang, O. and Grubbström, R. W. (2003), "The detailed coordination problem in a two-level assembly system with stochastic lead times". International Journal of production Economics, Vol. 81–82, pp. 415–429. (https://www.sciencedirect.com/science/article/pii/S0925527302002967)
93. Chiu, H. N. and Huang, H. L. (2003), "A multi-echelon integrated JIT inventory model using the time buffer and emergency borrowing policies to deal with random delivery lead times". International Journal of production Economics, Vol. 41, No. 13, pp. 2911–2931. (https://www.tandfonline.com/doi/abs/10.1080/0020754031000101303)
94. Mitra, S. and Chatterjee, A. K. (2004), "Leveraging information in multi-echelon inventory systems". European Journal of Operational Research, Vol. 152, pp. 263–280. (https://www.sciencedirect.com/science/article/pii/S0377221702006562)
95. Chen, C.-L. and Lee, W.-C. (2004), "Optimization of multi echelon supply chain networks with uncertain sales prices". Journal of Chemical Engineering Japan, Vol. 37, No. 7, pp.822–834. (https://www.jstage.jst.go.jp/article/jcej/37/7/37_7_822/_pdf)
96. Jalbar, B. A., Gutierrez, J. M., and Sicilia, J. (2005), "Single cycle policies for the one-warehouse N-retailer inventory/ distribution system". Omega, Vol. 34, No. 2, pp. 196-208. (https://www.sciencedirect.com/science/article/pii/S0305048304001549)
97. Seifbarghy, M. and Jokar, M. R. A. (2005), "Cost evaluation of a two-echelon inventory system with lost sales and approximately Poisson demand". International Journal of Production Economics, Vol. 102, No. 2, pp. 244-254. (https://www.sciencedirect.com/science/article/pii/S0925527305000757)
98. Han, C. and Damrongwongsiri, M. (2005), "Stochastic modeling of a two-

echelon multiple sourcing supply chain system with genetic algorithm". Journal of Manufacturing Technology and Management, Vol. 16, No. 1, pp. 87–108. (https://www.emeraldinsight.com/doi/10.1108/17410380510574103)

99. Axsater, S., Forsberg, R., and Zhang, W.-F. (1994), "Approximating general multi-echelon inventory systems by Poisson models". International Journal of Production Economics, Vol. 35, pp.201–206. (https://www.sciencedirect.com/science/article/pii/0925527394900825)

100. Jönsson, H. and Silver, E. A. (1987), "Analysis of a two-echelon inventory control system with complete redistribution". Management Science, Vol. 33, pp. 215–227. (https://dl.acm.org/citation.cfm?id=2872209)

101. Dekker, R., Frenk, J. B. G., Kleijn, M. J., de Kok, A. G., and Piersma, N. (1995) "On the use of break quantities in multi-echelon distribution systems". Inventory Modelling (Ed. L. Bogataj), Lecture Notes of the International Postgraduate Summer School, vol. 1, Budapest, ISIR. (https://www.researchgate.net/...use_of_break_quantities_in_multi--echelon_distributio...)

102. Wang, Y., Cohen, M. A., and Zheng, Y.-S. (2000), "A two-echelon repairable inventory system with stocking center dependent depot replenishment lead times". Management Science, Vol. 46, No. 11, pp. 1441–1453. (https://pubsonline.informs.org/doi/10.1287/mnsc.46.11.1441.12081)

103. Iida, T. (2001), "The infinite horizon non-stationary stochastic multi-echelon inventory problem and near myopic policies". European Journal of Operational Research, Vol. 134, No. 525–539. (https://www.sciencedirect.com/science/article/abs/pii/S0377221700002757)

104. Chen, F. and Song, J. S. (2001), "Optimal policies for multi- echelon inventory problems with Markov-modulated demand". Operations Research, Vol. 49, No. 2, pp. 226. (https://pubsonline.informs.org/doi/10.1287/opre.49.2.226.13528)

105. Chen, F. Y., Feng, Y., and Simchi-Levi, D. (2002), "Uniform distribution of inventory positions in two-echelon periodic review systems with batch-ordering policies and interdependent demands". European Journal of

Operational Research, Vol. 140, pp. 648–654.
(https://www.sciencedirect.com/science/article/abs/pii/S037722170100203X)

106. Minner, S., Diks, E. B., and de Kok, A. G. (2003), "A two-echelon inventory system with supply lead time flexibility". IIE Transactions, Vol. 35, pp. 117–129.
(https://www.tandfonline.com/doi/abs/10.1080/07408170304383)

107. Johansen, S. G. (2005), "Base-stock policies for the lost sales inventory system with Poisson demand and Erlangian lead times". International Journal of Production Economics, Vol. 93–94, pp. 429–437.
(https://www.sciencedirect.com/science/article/pii/S0925527304002610)

108. Ng, W. K., Piplani, R., and Viswanathan, S. (2003), "Simulation workbench for analyzing multi-echelon supplies chains". Integrated manufacturing System, Vol. 14, No. 5, pp. 449.
(https://www.emeraldinsight.com/doi/10.1108/09576060310477852)

109. Martel, A. (2003), "Policies for multi-echelon supply; DRP systems with probabilistic time-varying demands". ABI/INFORM Global, INFOR, Vol. 41, No. 1, pp. 71.
(https://www.tandfonline.com/doi/abs/10.1080/03155986.2003.11732669)

110. Kiesmüller, G. P., de Kok, T. G., Smits, S. R., and van Laarhoven, P. J. M. (2004), "Evaluation of divergent N-echelon (s, nQ)-policies under compound renewal demand". OR Spectrum, Vol. 26, pp. 547–577.
(https://link.springer.com/article/10.1007/s00291-004-0172-8)

111. Liberopoulos, G. and Koukoumialos, S. (2005), "Tradeoffs between base stock levels, numbers of Kanbans, and planned supply lead times in production/inventory systems with advance demand information". International Journal of Production Economics, Vol. 96, No. 2, pp. 213-232.
(https://www.sciencedirect.com/science/article/pii/S0925527304001495)

112. Axsater, S. (2001), "A Framework for decentralized multi-echelon inventory control". IIE Transactions, Vol. 33, pp. 91–97.
(https://link.springer.com/article/10.1023/A:1007694700789)

113. Lau, A. H. L. and Lau, H.-S. (2003) “Effects of a demand-curve’s shape on the optimal solutions of a multi-echelon inventory/pricing model”. European Journal of Operational Research, Vol. 147, pp. 530–548. (https://www.sciencedirect.com/science/article/pii/S0377221702002916)

114. Yoo, Y.-J., Kim, W.-S., and Rhee, J. T. (1997), “Efficient inventory management in multi-echelon distribution systems”. Computers and Industrial. Engineering, Vol. 33, No. (3–4), pp. 729–732. (https://www.sciencedirect.com/science/article/pii/S0360835297002337)

115. Jalbar, B. A., Gutierrez, J. M., and Sicilia, J. (2005) “Integer-ratio policies for distribution/inventory systems”. International Journal of Production Economics, Vol. 93–94, pp. 407–415. (Integer-ratio policies for distribution/inventory systems)

116. Liang, W.-Y. and Huang, C. C. (2006), “Agent-based demand forecast in multi-echelon supply chain”. Decision Support Systems, Vol.42, No. 1, pp. 390-407. (https://www.sciencedirect.com/science/article/pii/S0167923605000102)

117. Köchel, P. and Nielander, U. (2005), “Simulation-based optimization of multi-echelon inventory systems”. International Journal of Production Economics, Vol. 93–94, pp. 505–513. (https://www.sciencedirect.com/science/article/pii/S0925527304002683)

118. Sleptchenko, A., van der Heijden, M. C., and van Harten, A. (2002), “Effects of finite repair capacity in multi-echelon, multi-indenture service part supply systems”. International Journal of Production Economics, Vol. 79, pp. 209–230. (https://www.sciencedirect.com/science/article/pii/S092552730200155X)

119. Braun, M. W., Rivera, D. E., Flores, M. E., Carlyle, W. M., and Kempf, K. G. (2003) “A model predictive control framework for robust management of multi-product, multi-echelon demand networks”. Annual. Review. In Control, Vol. 27, pp. 229–245. (https://www.sciencedirect.com/science/article/pii/S1367578803000312)

120. Chen, C.-L. and Lee, W.-C. (2004), "Multi-objective optimization of multi-echelon supply chain networks with uncertain product demands and prices". Computers and Chemical Engineering, Vol. 28, pp. 1131–1144. (https://www.sciencedirect.com/science/article/pii/S0098135403002473)

121. Chandra, C. and Grabis, J. (2005), "Application of multi- steps forecasting for restraining the bullwhip effect improving inventory performance under autoregressive demand". European Journal of Operational Research, Vol. 166, pp. 337–350.
(https://www.sciencedirect.com/science/article/pii/S0377221704001225)

122. Alfaro, J.A. and Corbett, C.J. (2003), "The value of SKU rationalization in practice (the pooling effect under suboptimal inventory policies and nonnormal demand)", Production and Operations Management, Vol. 12, No. 1, pp. 12-29.
(https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1937-5956.2003.tb00195.x)

123. Anupindi, R. Bassok, Y. (1999), "Centralization of stocks: Retailers vs. manufacturer", Management Science, Vol. 45, No. 2, pp. 178-191. (https://pubsonline.informs.org/doi/pdf/10.1287/mnsc.45.2.178)

124. Anupindi, R., Chopra, S., Deshmukh, S. D., Van Mieghem, J. A. and Zemel, E. (2006), "Managing business process flows: Principles of operations management, Pearson/Prentice Hall.
(https://www.scholars.northwestern.edu/en/publications/managing-business-process-flows-principles-of-operations-manageme)

125. Ballou, R. H. and Burnetas, A. (2000), "Planning virtual inventories", Technical Memorandum 742, Department of Operations, Weatherhead School of Management, Case Western Reserve University, Cleveland, OH.

126. Ballou, R. H. (2004), "Business Logistics/Supply Chain Management, Pearson/Prentice Hall.
(https://www.pearson.com/...Business-Logistics-Supply-Chain-Management.../PGM170...)

127. Bendoly, E. (2004), "Integrated inventory pooling for firms servicing both on-line and store demand", Computers and Operations Research, Vol. 31, No.

9, pp. 1465-1480.
(https://www.sciencedirect.com/science/article/pii/S0305054803001023)

128. Benjaafar, S. and Kim J. S. (2001), “When does higher demand variability lead to lower safety stock?” Technical report, Department of Mechanical Engineering, University of Minnesota, Minneapolis, MN.
(https://www.researchgate.net/publication/248428116_When_Does_Higher_Demand_Variability_Lead_to_Lower_Safety_Stocks)

129. Benjaafar, S., Elhafsi, M. and De Vericourt, F. (2004a), “Demand allocation in multiple-product, multiple-facility, make-to-stock systems” Management Science, Vol. 50, No. 10, pp. 1431-1448.
(https://pubsonline.informs.org/doi/10.1287/mnsc.1040.0250)

130. Benjaafar, S., Kim J. S. and Vishwanadham, N. (2004b), “On the effect of product variety in production-inventory system” Annals of Operations Research, Vol. 126, No. 1-4, pp. 71-101.
(https://link.springer.com/article/10.1023/B:ANOR.0000012276.87248.c7)

131. Bowersox, D. J., Closs, D.J. and Helferich, O. K. (1986), “Logistical Management: A system integration of physical distribution, Manufacturing support, and Materials Procurement, MacMillan.
(https://trid.trb.org/view/261521)

132. Cachon, G. and Terwiesch, C. (2009), “Matching supply with demand: An Introduction to Operations Management” McGraw-Hill.
(https://www.researchgate.net/.../246774245_Matching_Supply_With_Demand_An_Intr...)

133. Cachon, G. P., Randall, T. and Schmidt, G. M. (2007), “In search of the bullwhip effect”, Manufacturing and Service Operations Management, Vol. 9, No. 4, pp. 457-479.
(https://pubsonline.informs.org/doi/10.1287/msom.1060.0149)

134. Campbell, J. Y., Lettau, M., Malkiel, B. G. and Xu, Y. (2001), “Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk” Journal of Finance, Vol. 56, No. 1, pp. 1-43. (https://rady.ucsd.edu/faculty/directory/valkanov/pub/.../Campbell_etal_JoF_2

000.pdf)

135. Carter, P. L. and Monczka, R. M. (1978), "MRO inventory pooling" Journal of Purchasing and Materials Management, Vol. 14, No. 3, pp. 27-33. (https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1745-493X.1978.tb00431.x)

136. Caves, D. W., Christiansen, L. R. and Tretheway, M. W. (1984), "Economies of density versus economies of scale: Why trunk and local service airlines costs differ" Rand Journal of Economics, Vol. 15, No. 4, pp. 471-488. (https://www.jstor.org/stable/2555519)

137. Chandler, A. D. (1990), "Scale and Scope- The dynamics of industrial capitalism" Harvard University Press. (https://www.researchgate.net/.../39109624_Alfred_D_Chandlerjr_Scale_and_Scope_The...)

138. Chen, M. and Lin, C. (1989), "Effects of centralization on expected costs in a multi-location newsboy problem" Journal of Operational Research Society, Vol. 40, No. 6, pp. 597-602. (https://www.jstor.org/stable/2630280)

139. Chopra, S. and Meindl, P. (2007), "Supply Chain Management: Strategy, Planning and Operations" Pearson.

140. Das, C. and Tyagi, R. (1997), "Role of inventory and transportation costs in determining the optimal degree of centralization" Transportation Research Part E: Logistics and Transportation Review", Vol. 33, No. 3, pp. 171-179. (https://www.sciencedirect.com/science/article/pii/S1366554597000197)

141. Eppen, G. D. (1979), "Effects of centralization on expected costs in multi-location newsboy problem" Management science, Vol. 25, No. 5, pp. 498-501. (https://www.jstor.org/stable/2630280)

142. Evers, P. T. and Beier, F. J. (1993), "The portfolio effect and multiple consolidation points: A critical assessment of square root law" Journal of Business Logistics, Vol. 14, No. 2, pp. 109-125. (https://www.jstor.org/stable/2630280)

143. Evers, P. T. (1995), "Expanding the square root law: An analysis of both safety and cycle stocks" Logistics and Transportation Review, Vol. 31, No. 1, pp. 1-20. (https://trid.trb.org/view.aspx?id=424905)

144. Evers, P. T. (1996), "The impacts of transshipments on safety stock requirements" Journal of Business Logistics, Vol. 17, No. 1, pp. 109-133. (https://trid.trb.org/view.aspx?id=560390)

145. Evers, P. T. (1997), "Hidden benefits of emergency transshipments" Journal of Business Logistics, Vol. 20, No. 1, pp. 55-76. (https://trid.trb.org/view.aspx?id=570472)

146. Eynan, A. and Fouque, T. (2003), "Capturing the risk pooling effect through demand reshape" Management Science, Vol. 49, No. 6, pp. 704-717. (https://pubsonline.informs.org/doi/pdf/10.1287/mnsc.49.6.704.16020)

147. Eynan, A. and Fouque, T. (2005), "Benefiting from risk pooling effect: Internal (component commonality) vs. External (demand reshape) efforts" International Journal of Services and Operations Management, Vol. 1, No. 1, pp. 90-99. (https://www.inderscienceonline.com/doi/abs/10.1504/IJSOM.2005.006320)

148. Fawcett, P. McLeish, R. E. and Ogden, I. D. (1992), "Logistics Management" Pitman. (https://www.sciencedirect.com/science/article/pii/S1877705815019098)

149. Gerchak, Y. and Gupta, D. (1991), "On apportioning costs to customer in centralized continuous review inventory system" Journal of Operations Management, Vol. 10, No.4, pp. 546-551. (https://www.sciencedirect.com/science/article/pii/027269639190010U)

150. Gerchak, Y. and He, Q.-M. (2003), "On the relation the benefits of risk pooling and variability of demand", IIE Transactions, Vol. 35, No. 11, pp. 1027-1032. (https://www.tandfonline.com/doi/abs/10.1080/07408170304399)

151. Gerchak, Y. and Mossman, D. (1992), "On the effect of demand randomness on inventories and costs", Operations Research, Vol. 40, No. 4, pp. 804-808. (https://pubsonline.informs.org/doi/pdf/10.1287/opre.40.4.804)

152. Graman, G. A. and Magazine, M. J. (2006), "Implementation issues influencing the decision to adopt postponement", International Journal of Operations and Production Management, Vol. 26, No. 10, pp. 1068-1083. (https://www.emeraldinsight.com/doi/10.1108/01443570610691076)

153. Hamies, Y. Y. (2009), "Risk Modeling, Assessment, and Management", Johns Wiley and Sons. (https://onlinelibrary.wiley.com/doi/book/10.1002/9780470422489)

154. Hartman, B. C. and Dror, M. (2003), “Optimizing centralized inventory operations in a cooperative game theory setting”, IIE Transactions, Vol. 35, No. 3, pp. 243-258. (https://www.tandfonline.com/doi/abs/10.1080/07408170304363)

155. Hartman, B. C. and Dror, M. and Shaked, M. (2000), “Cores of inventory centralization games”, Games and Economic Behavior, Vol. 31, No. 1, pp. 26-49. (https://www.sciencedirect.com/science/article/pii/S0899825699907327)

156. Hearn, G. (2007), “Shred or Dread” Airfinanace Journal, April1, 2007(299), 9. (https://airfinancejournal.com/articles/2049289/shred-or-dread)

157. Heskett, J. L. Glaskowsky, N. A. J. and Ivie, R. M. (1974), “Business Logistics” Ronald Press. (https://www.econbiz.de/.../business-logistics...ronald-press.../10004019404)

158. Hubbard, D. (2009), “The failure of Risk Management: Why It’s Broken and How to Fix it” John Wiley and Sons. (https://www.wiley.com/.../The+Failure+of+Risk+Management%3A+Why+It%27s+Bro...)

159. Jackson, P. L. and Muckstadt, J.A. (1989), “Risk pooling in a two-period, two-echelon inventory stocking and allocation problem” Naval Research Logistics, Vol. 36, pp. 1-26. (https://onlinelibrary.wiley.com/.../1520-6750%28198902%2936%3A1%3C1%3A%3AA...)

160. Jönsson, H. and Silver, E. A. (1987), “Analysis of two-echelon inventory control system with complete redistribution”, Management Science, Vol. 33, No. 2, pp. 215-227. (https://pubsonline.informs.org/doi/10.1287/mnsc.33.2.215)

161. Knight, F. H. (2005), “Risk, Uncertainty and Profit” Cosimo, originally published by the University of Chicago Press. (https://mises.org/sites/default/files/Risk,%20Uncertainty,%20and%20Profit_4.pdf)

162. Lin, C. T., Chen, C. B. and Hsieh, H. J. (2001), “Effects of centralization on expected profits in multi-location newsboy problem” Journal of Operational Research Society, Vol. 52, No. 7, pp. 839-844. (https://link.springer.com/article/10.1057/palgrave.jors.2601077)

163. Maister, D. H. (1976), “Centralization of inventories and the square root law”,

International Journal of Physical Distribution", Vol. 6, No. 3, pp. 124-134. (https://www.emeraldinsight.com/doi/10.1108/eb014366)

164. McKinnon, A. C. (1989), "Physical Distribution System" Rouledge. (https://doi.org/10.1002/pts.2770020407)

165. Milner, J. M. and Kouvelis, P. (2007), "Inventory, speculation, and sourcing strategies in the presence of online exchanges" Manufacturing and Service Operations Management, Vol. 9, No. 3, pp. 312-331. (https://pubsonline.informs.org/doi/10.1287/msom.1060.0137)

166. Monezka, R. M. and Carter, P. L. (1976), "The economies of vendor pooling in Academy of Management Proceedings" pp. 207-211.

167. Netessine, S. and Rudi, N. (2006), "Supply Chain Choice on the Internet", Management Science, Vol. 52, No. 6, pp. 844-864. (https://pubsonline.informs.org/doi/10.1287/mnsc.1060.0512)

168. Pishchulov, G. (2008), "Inventory Rationing and Sharing in Pre-sell Distribution with Mobile Communication Technologies" Shaker. (https://www.sciencedirect.com/journal/international-journal-of-production.../vol/.../2)

169. Scherer, F. M. and Ross, D. (1990), "Industrial Market Structure and Economic Performance" Houghton Miffilin Company. (https://papers.ssrn.com/abstract=1496716)

170. Schward, L.B. (1981), "Physical Distribution: The analysis of inventory and location" AIIE Transactions, Vol. 13, No. 2, pp. 138-150. (https://www.tandfonline.com/doi/pdf/10.1080/05695558108974546)

171. Simchi-Levi, D., Kaminsky, P. and Simchi-Levi, E. (2008), "Designing and Managing the supply chains: Concepts, Strategies and Case studies", McGraw-Hill/Irwin. (https://www.emeraldinsight.com/doi/10.1108/ijqrm.2000.17.7.812.1)

172. Soanes, C. and Hawker, S. (2008), "Compact Oxford English Dictionary of Current English, Oxford University Press. (https://global.oup.com/.../compact-oxford-english-dictionary-of-current-english-9780..)

173. Srinivasan, R. Jayaraman, R. Roundy, R. and Tayur, S. (1992), "Procurement of

common components in stochastic environment” Research Report RC-18580.
(https://pdfs.semanticscholar.org/e291/63be88fcd8fc195d7de66f9ed960e45fd1af.pdf)

174. Sussams, J. (1986), “Buffer stocks and the square root law”, Focus on Physical Distribution and Logistics Management, Vol. 5, No. 5, pp. 8-10.
(https://www.isye.gatech.edu/~spyros/courses/IE6201/Spring-14/square-root-law.pdf)

175. Tagaras, G. (1989), “Effects of pooling on optimization and service levels of two-location inventory systems” IIE Transactions, Vol. 21, No. 3, pp. 250-257.
(https://www.tandfonline.com/doi/abs/10.1080/07408178908966229)

176. Teo, C. P., Ou, J. and Goh, M. (2001), “Impact on inventory costs with consolidation of distribution centers” Vol. 33, No. 2, pp. 99-110.
(https://link.springer.com/article/10.1023/A:1007646817627)

177. Thomas, D. J. and Tyworth, J. E. (2006), “Pooling lead-time risk by order splitting: A critical review”, Transportation Research: Part E, Vol. 42, No. 4, pp. 245-257.
(https://www.sciencedirect.com/science/article/pii/S1366554505000037)

178. Tomlin, B. and Wang, Y. (2005), “On the value of mix flexibility and dual sourcing in unreliable newsvendor network” Manufacturing and Service Operations Management, Vol. 7, No.1, pp. 37-57.
(mba.tuck.dartmouth.edu/pages/faculty/brian.tomlin/pub/MixFlexibilityMSOM.pdf)

179. Tyagi, R. and Das, C. (1998), “Extension of the square root law for safety stock to demands with unequal variances” Journal of Business Logistics”, Vol. 19, No. 2, pp. 197-203.
(https://www.tandfonline.com/doi/full/10.1080/00207543.2015.1071895?src=recsys)

180. Tyagi, R. and Das, C. (1999), “Grouping customers for better allocation of resources to serve correlated demands” Computers and Operations Research, Vol. 26, No. 10-11, pp. 1041-1058.
(https://www.sciencedirect.com/science/article/pii/S0305054899000234)

181. Wanke, P. F. and Saliby, E. (2009), “Consolidation effects: Whether and how inventories should be pooled” Transportation Research: Part E, Vol. 45, No. 5.
(https://www.sciencedirect.com/science/article/pii/S1366554509000192)

182. Xu, K. and Evers, P. T. (2003), “Managing single echelon inventories through

demand aggregation and the feasibility of a correlation matrix", Computers and Operations Research, Vol. 30, No. 2, pp. 297-309.
(https://www.sciencedirect.com/science/article/pii/S0305054801000971)

183. Zhang, D. and Copper, W. L. (2005), "Revenue management for parallel flights with customer-choice behavior" Operations research, Vol. 53, No. 3, pp. 415-431.
(citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.448.5698&rep=rep1...)

184. Zinn, W. and Bowersox, D. J. (1988), "Planning and physical distribution with the principle of postponement", Journal of Business Logistics, Vol. 9, No. 2, pp. 117-136.
(www.scirp.org/(S(lz5mqp453edsnp55rrgjct55))/reference/ReferencesPapers.aspx?...)

185. Zinn, W. (1990), "Developing heuristics to estimate the impact of postponement on safety stock" International Journal of Logistics Management, Vol. 1, No. 2, pp. 11-16.
(https://www.emeraldinsight.com/doi/pdfplus/10.1108/09574099010804554)

186. Zinn, W., Levy, M. and Bowersox, D. J. (1989), "Measuring the effect of inventory centralization/decentralization on aggregate safety stocks: The "square root law" revisited", Journal of Business Logistics,, Vol. 10, No. 1, pp. 1-14.
(https://trid.trb.org/view/321247)

187. Brandimarte, P. (2006), "Multi-item capacitated lot-sizing with demand uncertainty", International Journal of Production Research, Vol. 44, No. 15, pp. 2997-3022.
(https://www.tandfonline.com/doi/abs/10.1080/00207540500435116)

188. Boute, Robert N., Disney, S. M., Lambrecht, M. R. and Houdt, B. V. (2007), "An integrated production and inventory model to dampen upstream demand variability in the supply chain" European Journal of Operational Research, Vol. 178, pp. 121-142.
(https://www.sciencedirect.com/science/article/pii/S0377221706000646)

189. Feng, Y. and Viswanathan, S. (2007), "Impact of Demand Uncertainty on Coordinating Supply Chain Inventories through Common Replenishment Epochs", Journal of Operations Research Society, Vol. 58, No. 7, pp. 964-971.
(https://www.tandfonline.com/doi/abs/10.1057/palgrave.jors.2602219)

190. Lin, Yu-Jen (2008), "A periodic review inventory model involving fuzzy

expected demand short fuzzy back order", Computers and Industrial Engineering, Vol. 54, pp. 666-676. (https://www.sciencedirect.com/science/article/pii/S0360835207002173)

191. Lau, H. C. and Song, H. (2008), "Multi-echelon repairable item inventory system with limited repair capacity under non stationary demands" International Journal of Inventory Research, Vol. 1, No. 1, pp. 67-92. (https://www.inderscienceonline.com/doi/abs/10.1504/IJIR.2008.019209)

192. Seth, D. and Pandey, M K. (2009), "A multiple-item inventory model for a non-stationary demand", Production Planning & Control, Vol. 20, No. 3, pp. 242-253. (https://www.tandfonline.com/doi/abs/10.1080/09537280902843607)

193. Routroy, S. and Maddala, K. C. (2009), "Multi-echelon supply chain inventory planning with demand and lead time uncertainty " International Journal of Operational Research, Vol. 5, No. 3, pp. 251. (https://www.inderscienceonline.com/doi/abs/10.1504/IJOR.2009.025195)

194. Mahnman, M., Yadollahpour, M. R., Famil-Dardashti, V. and Hejazi, S. R. (2009), "Supply chain modeling in uncertain environment with bi-objective approach", Computers and Industrial Engineering, Vol. 56, pp. 1535-1544. (https://www.sciencedirect.com/science/article/pii/S0360835208002453)

195. Huang, S.-H. and Lin, P.-C. (2010), "A modified ant colony optimization algorithm for multi-item inventory routing problems with demand uncertainty", Transportation Research Part E, Vol. 46, pp. 598-611. (https://www.sciencedirect.com/science/article/pii/S1366554510000177)

196. Yang, M-F and Lo, M-C, (2010), "Considering single-vendor and multiple buyers integrated supply chain inventory model with lead time reduction", Part B: Journal of Engineering Manufacture, Vol. 225, pp. 747-759. (https://journals.sagepub.com/doi/10.1243/09544054JEM1710)

197. Schmitt, A. J., Snyder, L. V. and Shen, Z- J. (2010), "Inventory systems with stochastic demand and supply: Properties and approximations" European Journal of Operational Research, Vol. 206, pp. 313-328. (https://www.sciencedirect.com/science/article/pii/S0377221710001608)

198. Lam, C. Y. and Ip, W. H. (2011), "A customer satisfaction inventory model for

supply chain integration" Expert Systems with Applications, Vol. 38, pp. 875-883. (https://www.sciencedirect.com/science/article/pii/S0957417410006913)

199. Sajjadi, S. R. and Cheraghi, S. H. (2011), "Multi-products location-routing problem integrated with inventory under stochastic demand" International Journal of Industrial and System Engineering" Vol. 7, No. 4, pp. 454-475. (https://www.inderscienceonline.com/doi/abs/10.1504/IJISE.2011.039670)

200. Yeo, W. M. and Yuan, X-M. (2011), "Optimal inventory policy with supply uncertainty and demand cancellation" European Journal of Operational Research" Vol. 211, pp. 26-34. (https://www.sciencedirect.com/science/article/pii/S0377221710007332)

201. Fernandes, R., Gouveia, J. B. and Pinho, C. (2011), "Target setting under uncertainty", International Journal of Business Performance and Supply Chain Modelling, Vol. 3, No. 4, pp. 297-315. (https://www.inderscience.com/info/inarticle.php?artid=43816)

202. Dey, O. and Chakraborty, D. (2012), "A fuzzy random periodic review system: A technique for real life application" International Journal of Operational Research, Vol. 13, No. 4, pp. 395-405. (https://www.researchgate.net/.../264834817_A_fuzzy_random_periodic_review_system...)

203. He, Y. and Zhao, X. (2012), "Coordination in multi-echelon supply chain under supply and demand uncertainty" International Journal of Production Economics, Vol. 139, pp. 106-115. (https://www.sciencedirect.com/science/article/pii/S0925527311001964)

204. Rahim, M. K. I. A. and Aghezzaf, EI-H. (2012), "Implementing a vendor managed inventory policy in a two stage supply chain with stochastic demands" Proceedings of the 14th IFAC symposium on Information Control Problems in Manufacturing Bucharest, Romania, May 23-25. (https://www.sciencedirect.com/science/article/pii/S1474667016332141)

205. Schmitt, A. J. and Synder, L. V. (2012), "Infinite-horizon models for inventory control under yield uncertainty and disruptions", Computers and Operations Research, Vol. 39, pp. 850-862.

(https://www.sciencedirect.com/science/article/pii/S0305054810001681)

206. Özen, U., Dogru, M. K. and Tarim, S. A. (2012), “Static-dynamic uncertainty strategy for a single-item stochastic inventory control problem” Omega, Vol. 40., pp. 348-357. (https://www.sciencedirect.com/science/article/pii/S0305048311001228)

207. Taleizadeh, N. A., Niaki, S. T. A., Aryanezhad, M.-B. and Shafii, N. (2013), “A hybrid method of fuzzy simulation and genetic algorithm to optimize constrained inventory control systems with stochastic replenishments and fuzzy demand” Information Sciences, Vol. 220, pp. 425-441. (https://www.sciencedirect.com/science/article/pii/S0020025512004999)

208. Guo, C. and Li, X. (2014), “A multi-echelon inventory system with supplier selection and order allocation under stochastic demand” International Journal of Production Economics, Vol. 151, pp. 37-47. (https://www.sciencedirect.com/science/article/pii/S092552731400019X)

209. Firouzi, F., Baglieri, E. and Jaber, M. Y. (2014), “Two-product inventory management with fixed costs and supply uncertainty” Applied Mathematical Modelling, Vol. 38, pp. 5635-5650. (https://www.sciencedirect.com/science/article/pii/S0307904X14001218)

210. Wang, D., Qin, Z. and Kar, S. (2015), “A novel single period inventory problem with uncertain random demand and its application” Applied Mathematics and Computation, Vol. 269, pp. 133-145. (https://www.sciencedirect.com/science/article/pii/S0096300315008838)

211. Pasandideh, S. H. R., Niaki, S. T. A. and Asadi, K. (2015), “Bi-objective optimization of a multi-product multi-period, three echelon supply chain problem under uncertain environments: NSGA-II and NRGA, Information Sciences, Vol. 292, PP. 55-74. (https://dl.acm.org/citation.cfm?id=2941895)

212. Lin, L. and Song, S. (2015) “Hybrid NSGA-II algorithm on robust multi-objective inventory management problem” Transaction of Institute of Measurement and control, Vol. 37, No. 7, pp. 909-918. (https://journals.sagepub.com/doi/abs/10.1177/0142331214536200)

213. Kumar, P. and Dutta, D. (2015), “Multi-objective linear fraction inventory model of multi-products with price dependent demand rate in fuzzy environment”

International Journal of Mathematics in Operational Research, Vol. 7, No. 5, pp. 547-565. (https://www.inderscienceonline.com/doi/abs/10.1504/IJMOR.2015.071280)

214. Movahed, K. K. & Zhang, Z.-H. (2015), "Robust design of (s, S) inventory policy parameters in supply chains with demand and lead time uncertainties, International Journal of Systems Science" Vol. 46, No.12, pp. 2258-2268. (https://www.tandfonline.com/doi/abs/10.1080/00207721.2013.860637)

215. Thorsen, A. and Yao, T. (2017), "Robust inventory control under demand and lead time" Annals of Operations Research, Vol. 257, No. 1-2, pp. 207-236. (https://link.springer.com/article/10.1007/s10479-015-2084-1)

216. Sadeghi, J. and Niaki, S. T. A. (2015), "Two parameters tuned multi-objective evolutionary algorithms for a bi-objective vendor managed inventory model with trapezoidal fuzzy demand" Applied Soft Computing, Vol. 30, pp. 567-576. (https://www.sciencedirect.com/science/article/pii/S1568494615001040)

217. Chu, Y., You, F., Wassick, J. M. and Agarwal, A. (2015), "Simulation-based optimization framework for multi-echelon inventory system under uncertainty" Computers and Chemical Engineering, Vol. 73, pp. 1-16. (https://www.sciencedirect.com/science/article/pii/S0098135414003032)

218. Moghaddam, R. T. and Raziei, Z. (2016), "A new bi-objective location-routing-inventory problem with fuzzy demand" IFAC-Papers Online, Vol. 49, No. 12, pp. 1116-1121. (https://www.sciencedirect.com/science/article/pii/S2405896316309193)

219. Attar, A., Raissi, S. and Khalili-Damghani, K. (2016), "Simulation-optimization approach for continuous- review, base stock inventory model with general compound demands, random lead times, and lost sales", Vol. 92, No. 6, pp. 1-18. (https://journals.sagepub.com/doi/10.1177/0037549716644055)

220. Puga, M. S. and Tancrez, J. S. (2017), "A heuristics algorithm for solving large location-inventory problems with demand uncertainty" European Journal of Operational Research, Vol. 259, pp. 413-423. (https://www.sciencedirect.com/science/article/pii/S0377221716308712)

221. Qiu, R., Sun, M. and Lim, Y. F. (2017), "Optimizing (s, S) Policies for Multi-period Inventory Models with Demand Distribution Uncertainty: Robust Dynamic

Programming Approaches" European Journal of Operational Research, Vol. 261, No. 3, pp. 880-892. (https://www.sciencedirect.com/science/article/pii/S0377221717301492)

222. Zadeh, L. A. (1965), "Fuzzy Sets", INFOR. Control, Vol.8, pp. 338-353. (https://www.sciencedirect.com/science/article/pii/S001999586590241X)

223. Zimmermann, H. J. (1991), "Fuzzy Set Theory and its Application, Second edition, Kluwer Academic, Boston. (https://kashanu.ac.ir/.../H_-J_%20Zimmermann%20%20(auth_)%20Fuzzy%20Set%20T...)

224. Pedrycz, W. and Gomide, F. (2001), "Fuzzy System Engineering-Towards Human-centric Computing", Wiley, New Jersy. (https://www.wiley.com/.../Fuzzy+Systems+Engineering%3A+Toward+Human+Centric...)

225. Lima Junior, F. R., Osiro, L. and Carpinetti, L. C. R. (2014), "A comparison between Fuzzy AHP and Fuzzy TOPSIS methods to supplier selection", Applied Soft Computing, Vol. 21, pp. 194-209. (https://www.sciencedirect.com/science/article/pii/S1568494614001203)

226. Moghassem, A. R. (2010), "Application of TOPSIS approach on parameters selection problem for rotor spinning machine", Fibers and Polymers, Vo. 11, No.4, pp. 669–675. (https://link.springer.com/article/10.1007/s12221-010-0669-7)

227. Hwang C. L. and Yoon K. (1981), "Methods for Multiple Attribute Decision Making. In: Multiple Attribute Decision Making. Lecture Notes in Economics and Mathematical Systems", Vol. 186, Springer, Berlin, Heidelberg. (https://link.springer.com/chapter/10.1007/978-3-642-48318-9_3)

228. Gajpal, P. P., Ganesh, L.S. and Rajendran, C. (1994), "Critically analysis of spare parts using analytical hierarchy process" International Journal of Production Economics, Vol. 35, pp. 293-297. (https://www.sciencedirect.com/science/article/pii/0925527394900957)

229. Agrell, P. J., (1994), "A multi-criteria frame work for inventory control", International Journal of Production Economics", Vol. 41, pp. 59-70. (https://www.sciencedirect.com/science/article/pii/0925527394000549)

230. Ramanathan, R. (2006), "ABC inventory classification with multiple-criteria using weight linear optimization" Computers and Operations Research", Vol. 33, pp. 695-700. (https://www.sciencedirect.com/science/article/pii/S0305054804001790)

231. Vencheh, A. H. and Mohamadghasemi, A. (2011), "A fuzzy AHP-DEA approach for multiple criteria ABC inventory classification", Expert Systems with Applications Vol. 38, pp.3346-3352. (https://www.sciencedirect.com/science/article/pii/S0957417410009279)

232. Kabir, G. and Hasin, M. A. A. (2011), "Comparative Analysis of AHP and Fuzzy AHP models for Multi-criteria inventory classification" International Journal of Fuzzy Logic Systems (IJFLS), Vol. 1, No.1, pp. 1-17. (https://pdfs.semanticscholar.org/727c/8a414856185a6358e4cd93088e8b77c3e794.pdf)

233. Balaji, K. and Kumar, V. S. S. (2014), "Multi-criteria Inventory ABC classification in an automobile rubber manufacturing industry", Procedia CIRP, Science Direct, Vol. 17, pp. 463-468. (https://www.sciencedirect.com/science/article/pii/S2212827114003849).

234. Duong, L N. K., Wood, L. C. and Wang, W. Y. C. (2015), "A multi-criteria inventory management system for perishable and sustainable products" Procedia-Manufacturing, Vol. 2, pp. 66-76 (https://www.sciencedirect.com/science/article/pii/S235197891500013X)

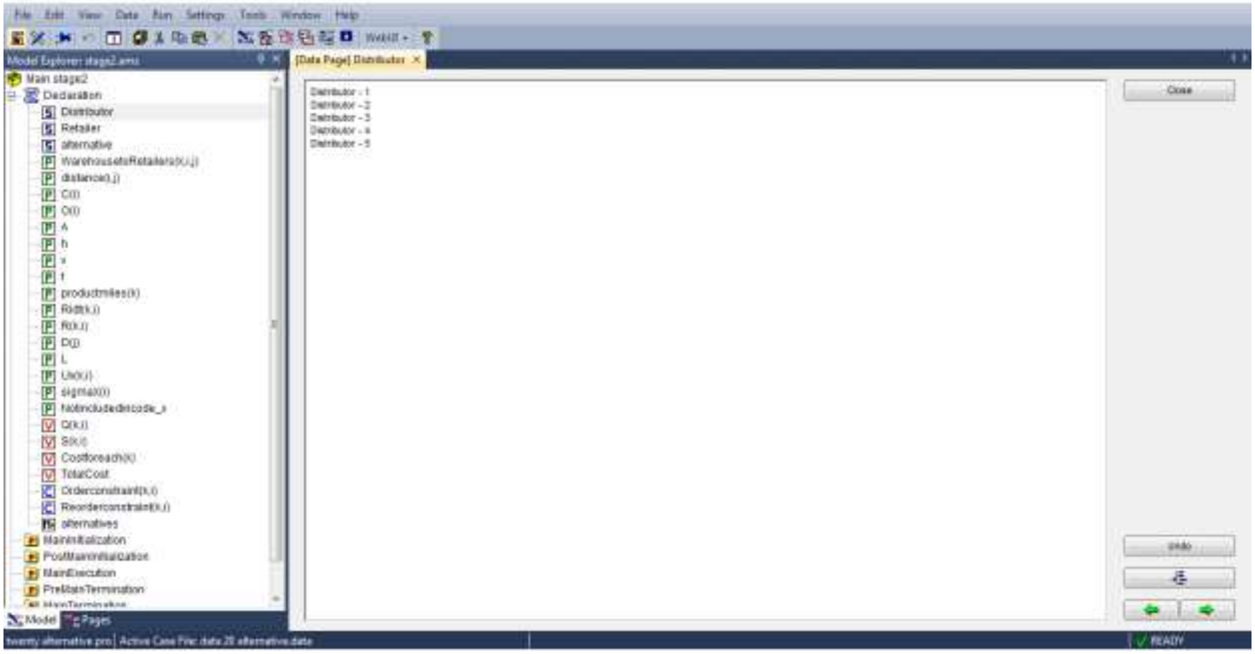
235. Mardani, A., Jusoh, A., Nor, MD, Khalifah, Z., Zakwan, N. Z. and Valipour, N. (2015), "Multiple criteria decision-making techniques and their applications – a review of the literature from 2000 to 2014" Economic Research, Vol. 28, No. 1, pp. 516-571. (https://www.tandfonline.com/doi/full/10.1080/1331677X.2015.1075139)

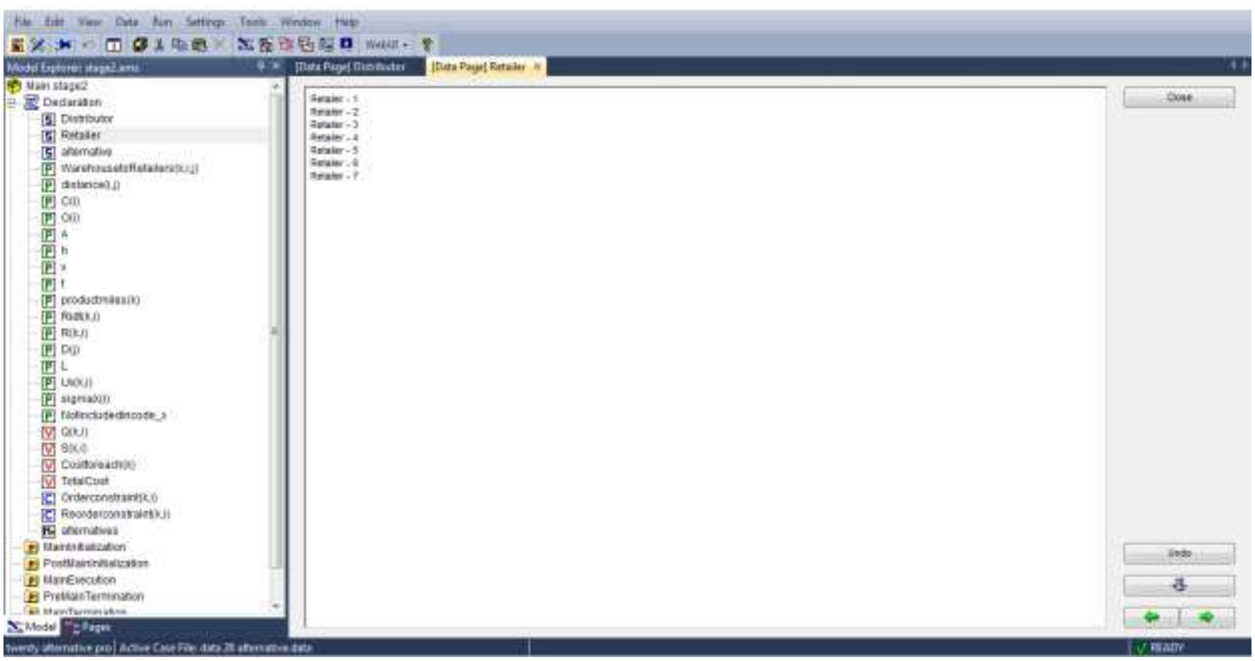
Appendix-I: AIMMS Codes and screen shots for single product, single period risk pooling inventory control model

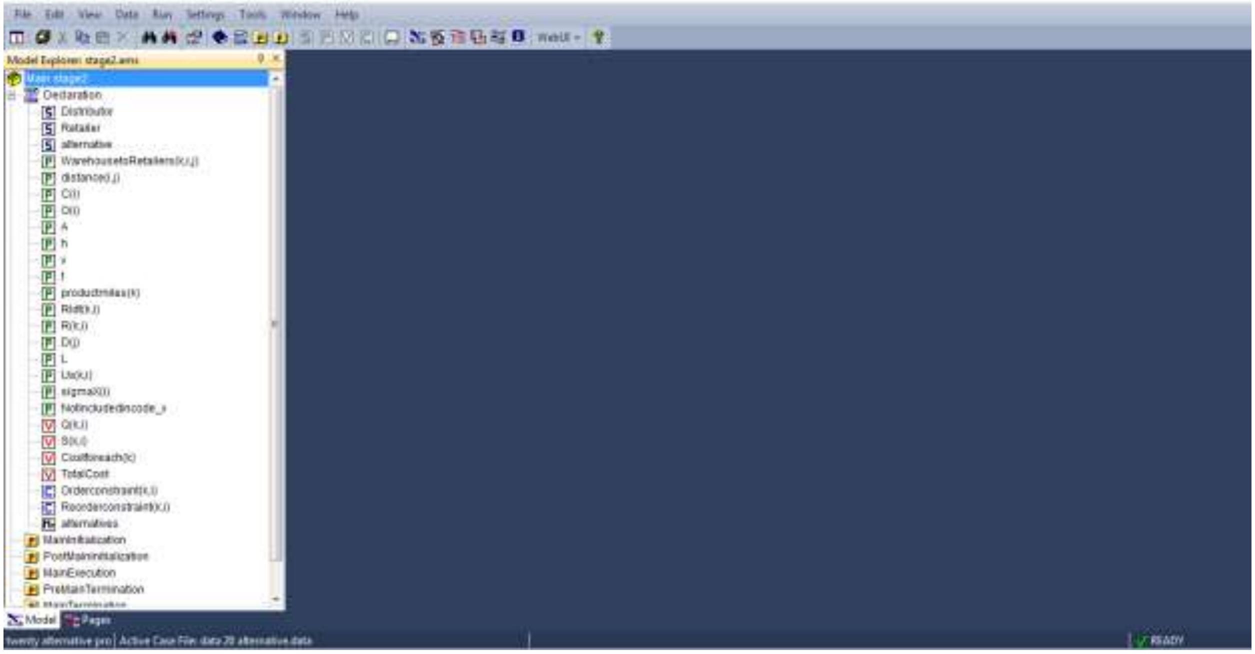
```
Set Distributor {
        Index: i;
        Definition: ElementRange(from:1,to:5,prefix:'Distributor - ');
        Comment: {
            "Dist-1 is \'2\' from stage 1
            Dist-2 is \'3\' from stage 1
            Dist-3 is \'8\' from stage 1
            Dist-4 is \'10\' from stage 1"
        }
    }
    Set Retailer {
        Index: j;
        Definition: ElementRange(from:1,to:7,prefix:'Retailer - ');
    }
    Set alternative {
        Index: k;
        Definition: ElementRange(from:1,to:20,prefix:'Aternative - ');
    }
    Parameter WarehousetoRetailers {
        IndexDomain: (k,i,j);
    }
    Parameter distance {
        IndexDomain: (i,j);
        Comment: "Data is given in paper";
    }
    Parameter C {
        IndexDomain: i;
        Comment: "ata is given in paper";
    }
    Parameter O {
        IndexDomain: i;
        Comment: "ata is given in paper";
    }
    Parameter A {
        Comment: "ata is given in paper";
    }
    Parameter h {
        Comment: "data is given in paper";
    }
    Parameter v {
        Comment: "data is given in paper";
    }
    Parameter t {
        Comment: "data is given in paper";
    }
    Parameter productmiles {
        IndexDomain: k;
        Definition: {
            sum(i,(sum(j,(D(j)*distance(i,j)*WarehousetoRetailers(k,i,j)))));
        }
        Comment: "Data is not required here..Can be deleted from here. Have been
calculated in excel sheet attactehd";
    }
    Parameter Ridt {
        IndexDomain: (k,i);
        Definition: {
            sum(j,distance(i,j)*WarehousetoRetailers(k,i,j)*D(j));
        }
```

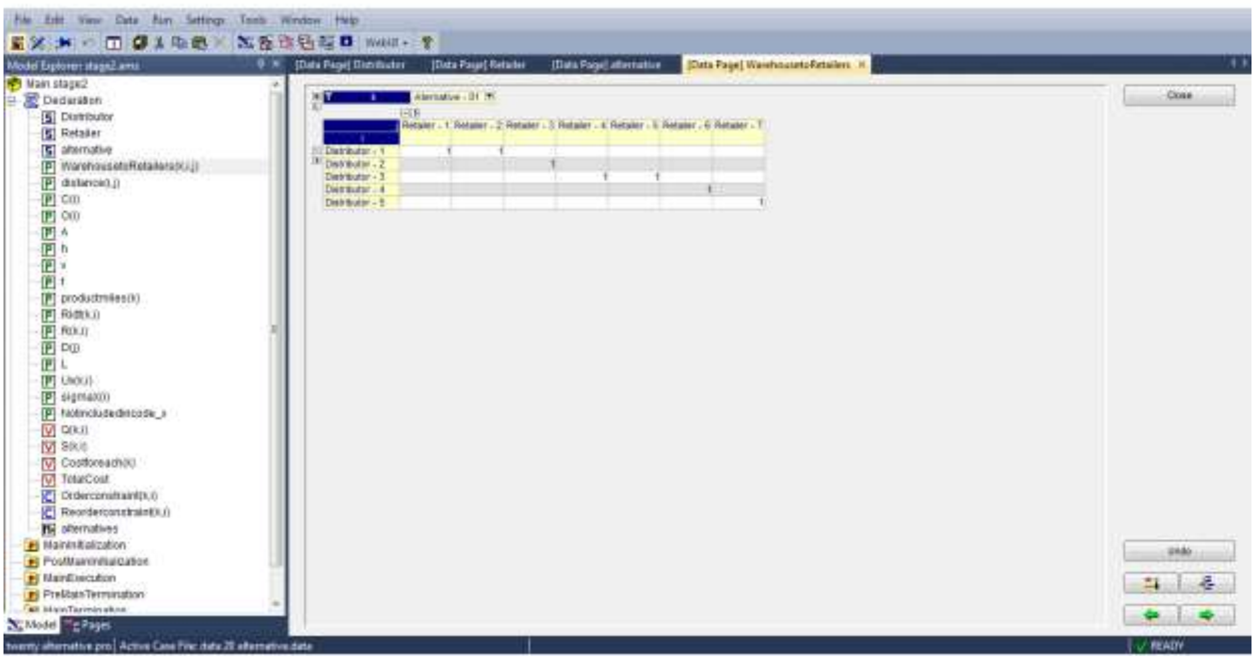
```
        Comment: "data is calculated from Excelsheet. Formula is given in paper page
496/line no. 11";
    }
    Parameter R {
        IndexDomain: (k,i);
        Definition: {
            sum(j,WarehousetoRetailers(k,i,j)*D(j));
        }
    }
    Parameter D {
        IndexDomain: j;
    }
    Parameter L {
        Comment: "Taken from paper = .13 (Why .13 not found )";
    }
    Parameter Ux {
        IndexDomain: (k,i);
        Definition: {
            R(k,i)*L;
        }
        Comment: "It is muX-- Mean demand during lead time (Excel sheet)";
    }
    Parameter sigmaX {
        IndexDomain: i;
        Comment: "Not required here. No input";
    }
    Parameter Notincludedincode_x {
        Comment: {
            "!!In this programe the one constaint ESC<TSC (Page 497 line no. 9) is not
captured.
            !!This is inporporated in solevr. The output of this progamme is S(k,i)
i.e. reorder point and Q(k,i)(EOQ)"
        }
    }
    Variable Q {
        IndexDomain: (k,i);
        Range: free;
    }
    Variable S {
        IndexDomain: (k,i);
        Range: nonnegative;
    }
    Variable Costforeach {
        IndexDomain: k;
        Range: free;
        Definition: {
            sum(i,A*(12*(R(k,i)))+(Q(k,i)/2+(S(k,i)-
Ux(k,i))*h*v+12*Ridt(k,i)*t+C(i)+12*O(i)*R(k,i)));
        }
    }
    Variable TotalCost {
        Range: free;
        Definition: {
            sum(k,sum(i,A*(12*(R(k,i)/Q(k,i)))+(Q(k,i)/2+(S(k,i)-
Ux(k,i))*h*v+12*Ridt(k,i)*t+C(i)+12*O(i)*R(k,i))));
        }
    }
    Constraint Orderconstraint {
        IndexDomain: (k,i);
        Definition: {
            Q(k,i)>=1;
        }
    }
```

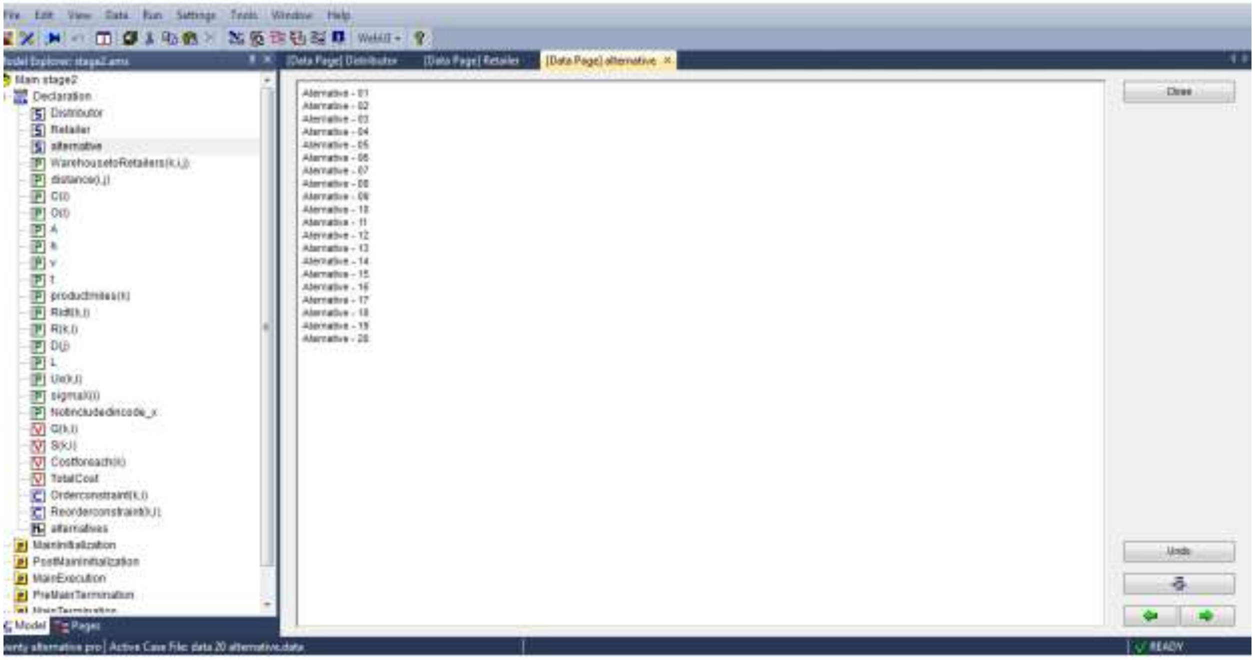
```
    Constraint Reorderconstraint {
        IndexDomain: (k,i);
        Definition: {
            S(k,i)>=Ux(k,i);
        }
    }
    MathematicalProgram alternatives {
        Objective: TotalCost;
        Direction: minimize;
        Constraints: AllConstraints;
        Variables: AllVariables;
        Type: Automatic;
    }
    Procedure MainInitialization {
        Comment: "Add initialization statements here that do NOT require any library
being initialized already.";
    }
    Procedure PostMainInitialization {
        Comment: {
            "Add initialization statements here that require that the libraries are
already initialized properly,
            or add statements that require the Data Management module to be
initialized."
        }
    }
    Procedure MainExecution {
        Body: {
            solve alternatives;
        }
    }
    Procedure PreMainTermination {
        Body: {
            return DataManagementExit();
        }
        Comment: {
            "Add termination statements here that require all libraries to be still
alive.
            Return 1 if you allow the termination sequence to continue.
            Return 0 if you want to cancel the termination sequence."
        }
    }
    Procedure MainTermination {
        Body: {
            return 1;
        }
        Comment: {
            "Add termination statements here that do not require all libraries to be
still alive.
            Return 1 to allow the termination sequence to continue.
            Return 0 if you want to cancel the termination sequence.
            It is recommended to only use the procedure PreMainTermination to cancel
the termination sequence and let this procedure always return 1."
        }
    }
}
```

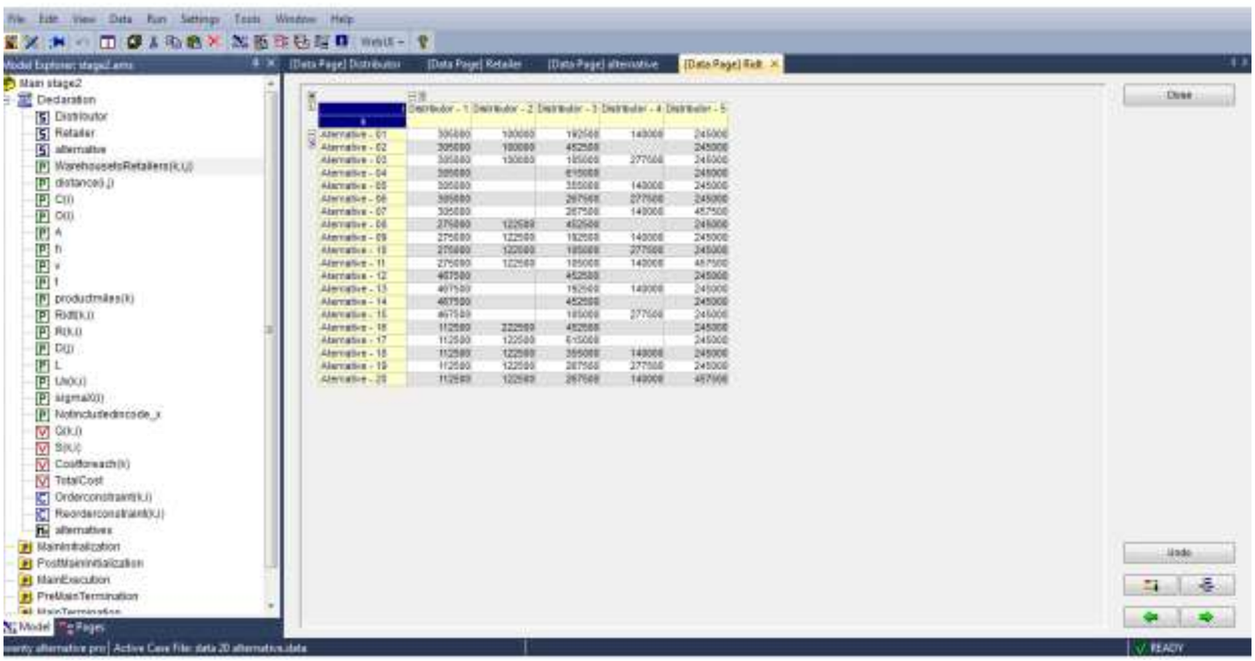


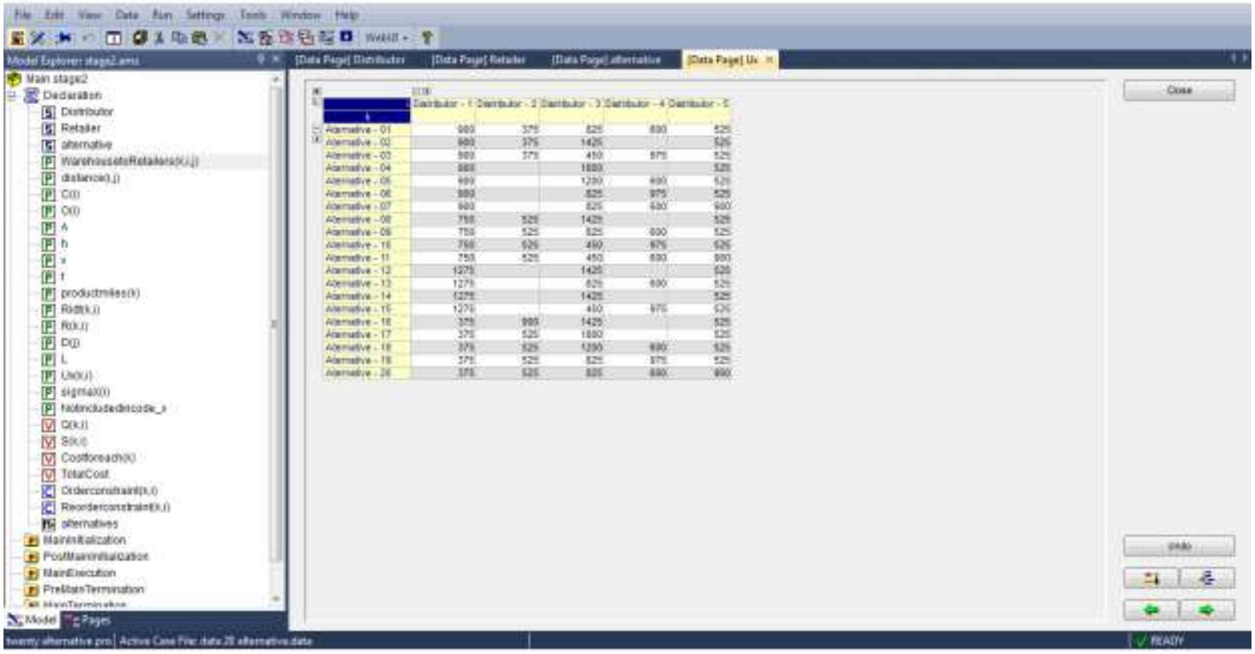










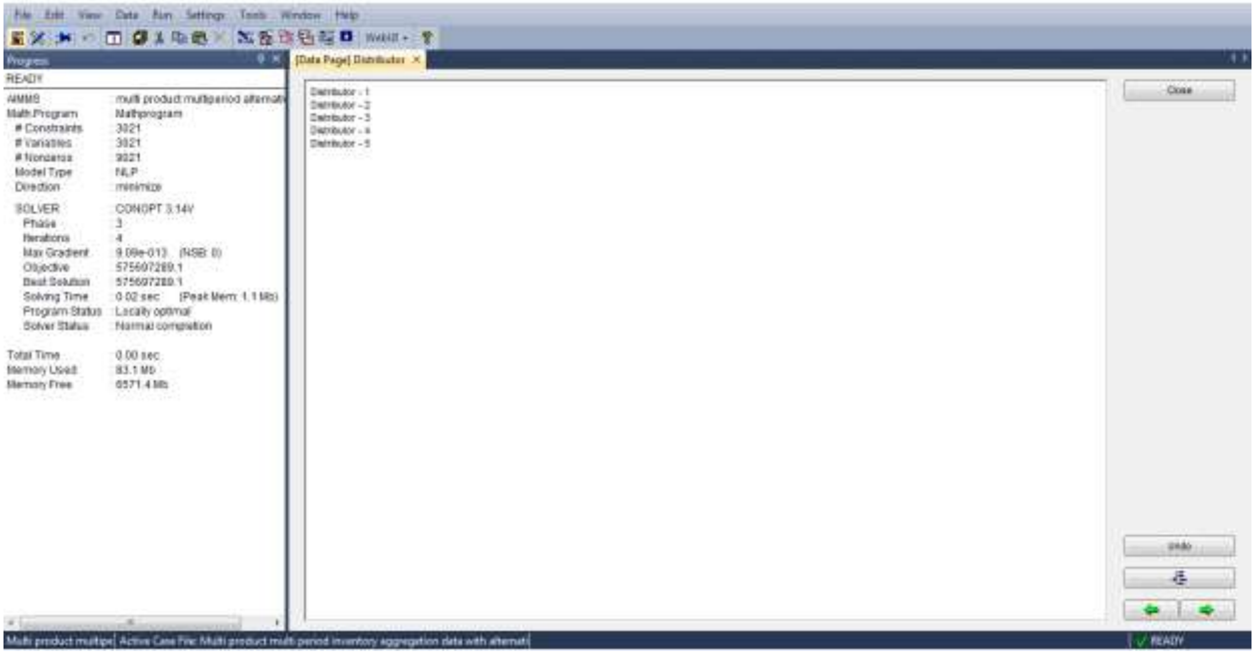

File
Edit
View
Data
Run
Settings
Tools
Window
Help
[Data Page] Distributor
[Data Page] Retailer
[Data Page] alternative
Main stage2
Declaration
Distributor
Retailer
alternative
Close
Undo
Model
Pages
READY

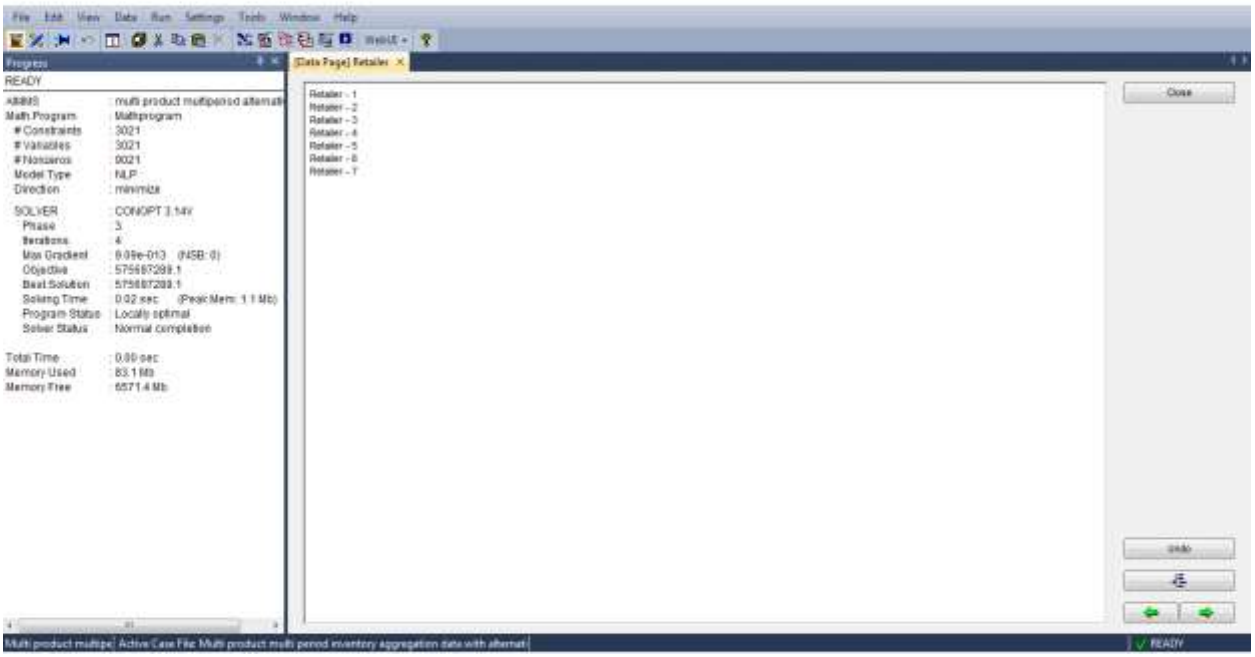
Appendix-II: AIMMS Codes and screen shots for multi-product, multi-period risk pooling inventory control model

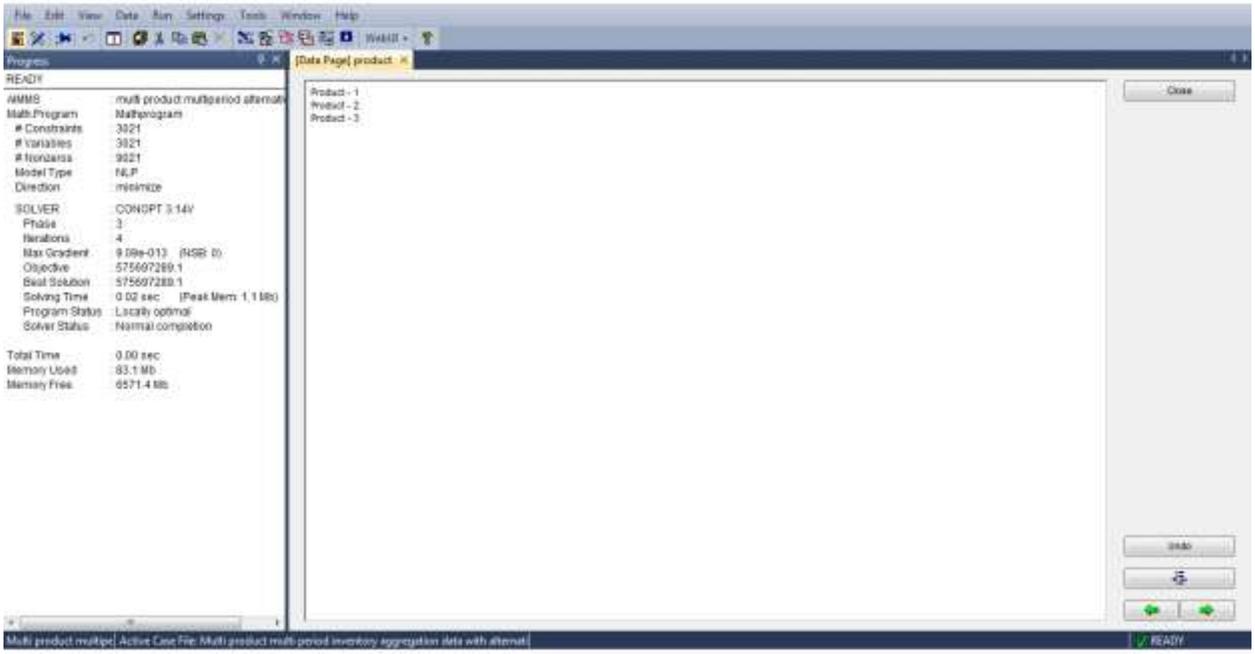
```
Set Distributor {
        Index: i;
        Definition: ElementRange(from:1,to:5,prefix:'Distributor - ');
    }
    Set Retailer {
        Index: j;
        Definition: ElementRange(from:1,to:7,prefix:'Retailer - ');
    }
    Set product {
        Index: k;
        Definition: ElementRange(from:1,to:3,prefix:'Product - ');
    }
    Set PlanningHorizon {
        Index: l;
        Definition: ElementRange(from:1,to:5,prefix:'Planning Horizon - ');
    }
    Set Alternatives {
        Index: m;
        Definition: ElementRange(from:1,to:20,prefix:'Alternative - ');
    }
    Parameter Alphaitoj {
        IndexDomain: (m,i,j);
    }
    Parameter Distance {
        IndexDomain: (i,j);
    }
    Parameter C {
        IndexDomain: (i,k);
    }
    Parameter O {
        IndexDomain: (i,k);
    }
    Parameter A {
        IndexDomain: (i,k);
    }
    Parameter H {
        IndexDomain: (l,i,k);
    }
    Parameter v {
        IndexDomain: (i,k);
    }
    Parameter t;
    Parameter LT;
    Parameter D {
        IndexDomain: (l,j,k);
    }
    Parameter R {
        IndexDomain: (m,l,i,k);
        Definition: sum(j,Alphaitoj(m,i,j)*D(l,j,k));
    }
    Parameter RDistance {
        IndexDomain: (m,l,i,k);
        Definition: sum(j,Distance(i,j)*Alphaitoj(m,i,j)*D(l,j,k));
    }
    Parameter ProductMiles {
        IndexDomain: (m,l,i,k);
```

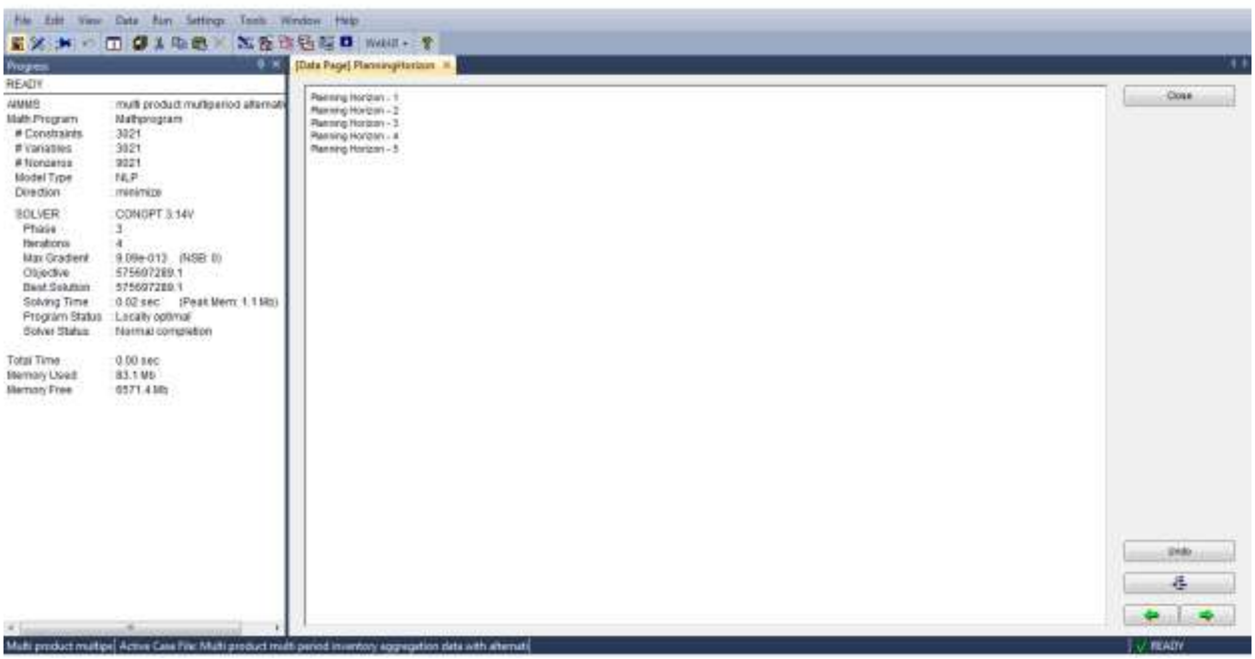
```
        Definition: sum(j,Distance(i,j)*D(l,j,k));
    }
    Parameter mu {
        IndexDomain: (m,l,i,k);
        Definition: R(m,l,i,k)*LT;
    }
    Constraint Orderquantity {
        IndexDomain: (m,l,i,k);
        Definition: Q(m,l,i,k)>=1;
    }
    Constraint Reorderpoint {
        IndexDomain: (m,l,i,k);
        Definition: Ro(m,l,i,k)>=mu(m,l,i,k);
    }
    Variable Q {
        IndexDomain: (m,l,i,k);
        Range: nonnegative;
    }
    Variable Ro {
        IndexDomain: (m,l,i,k);
        Range: nonnegative;
    }
    Variable Costofreach {
        IndexDomain: m;
        Range: nonnegative;
        Definition: {
            sum(k,sum(i,sum(l,A(i,k)*(12*( R(m,l,i,k)/ Q(m,l,i,k)))+( Q(m,l,i,k)/2+(
Ro(m,l,i,k)- mu(m,l,i,k))* H(l,i,k)* v(i,k)+12* RDistance(m,l,i,k)*t+ C(i,k)+12*
O(i,k)* R(m,l,i,k)))));
        }
    }
    Variable Totalcost {
        Range: free;
        Definition: {
            sum(i,sum(k,sum(l,sum(m, A(i,k)*(12*( R(m,l,i,k)/ Q(m,l,i,k)))+(
Q(m,l,i,k)/2+( Ro(m,l,i,k)- mu(m,l,i,k))* H(l,i,k)* v(i,k)+12* RDistance(m,l,i,k)*t+
C(i,k)+12* O(i,k)* R(m,l,i,k))))));
        }
    }
    MathematicalProgram Mathprogram {
        Objective: Totalcost;
        Direction: minimize;
        Constraints: AllConstraints;
        Variables: AllVariables;
        Type: Automatic;
    }
    Procedure MainInitialization {
        Comment: "Add initialization statements here that do NOT require any library
being initialized already.";
    }
    Procedure PostMainInitialization {
        Comment: {
            "Add initialization statements here that require that the libraries are
already initialized properly,
            or add statements that require the Data Management module to be
initialized."
        }
    }
    Procedure MainExecution {
        Body: {
            solve Mathprogram;
        }
    }
```

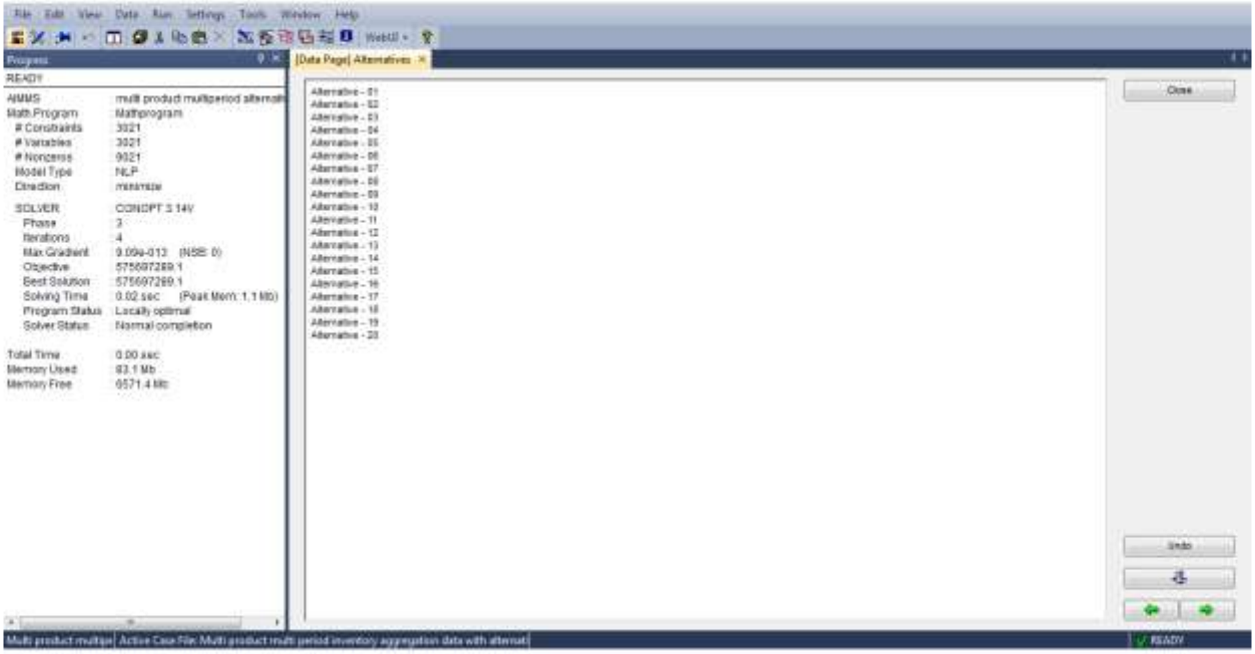
```
    Procedure PreMainTermination {
        Body: {
            return DataManagementExit();
        }
        Comment: {
            "Add termination statements here that require all libraries to be still
alive.
            Return 1 if you allow the termination sequence to continue.
            Return 0 if you want to cancel the termination sequence."
        }
    }
    Procedure MainTermination {
        Body: {
            return 1;
        }
        Comment: {
            "Add termination statements here that do not require all libraries to be
still alive.
            Return 1 to allow the termination sequence to continue.
            Return 0 if you want to cancel the termination sequence.
            It is recommended to only use the procedure PreMainTermination to cancel
the termination sequence and let this procedure always return 1."
        }
    }
}
```

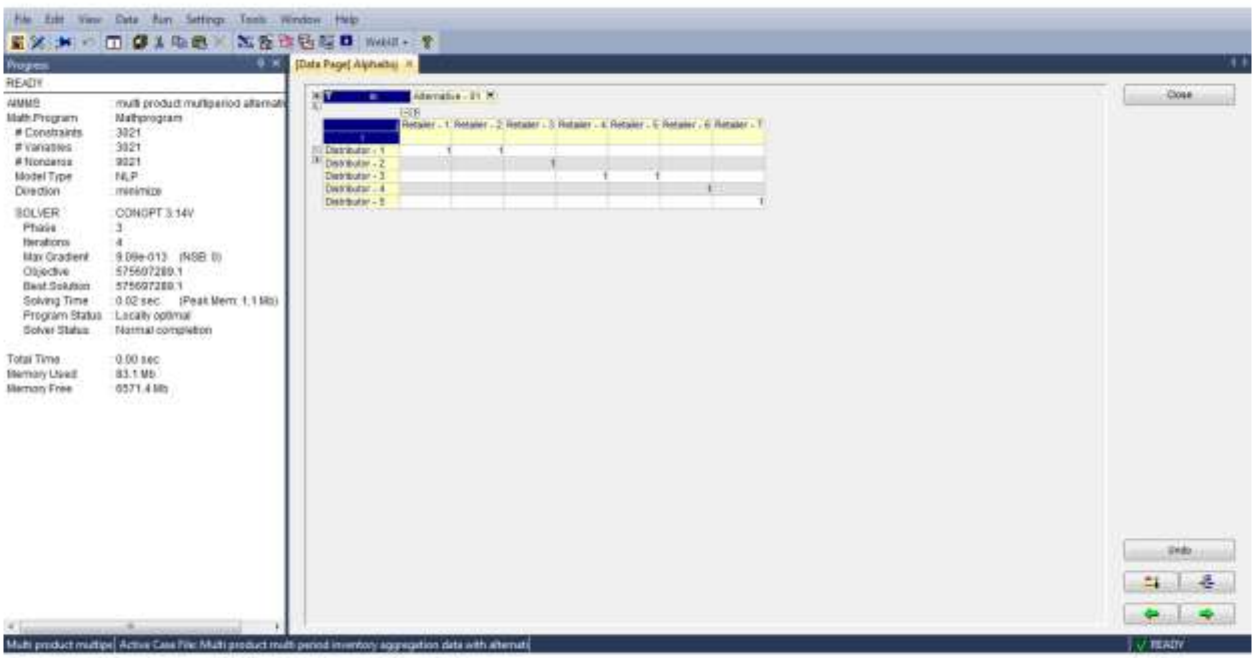


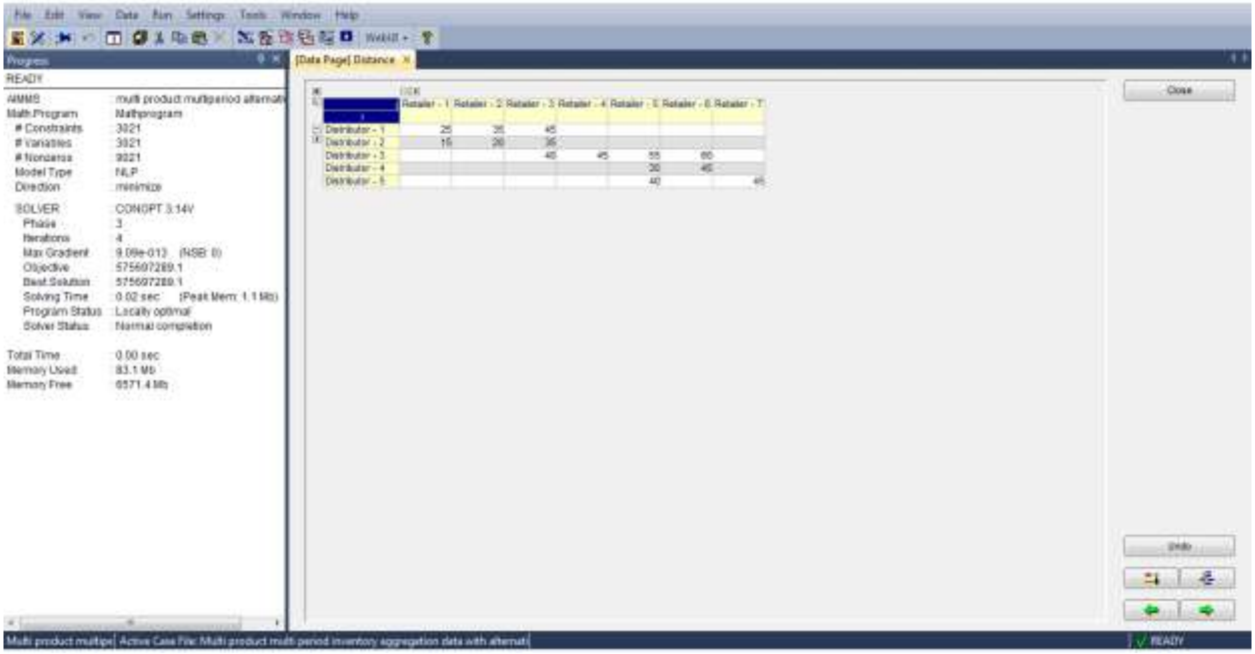


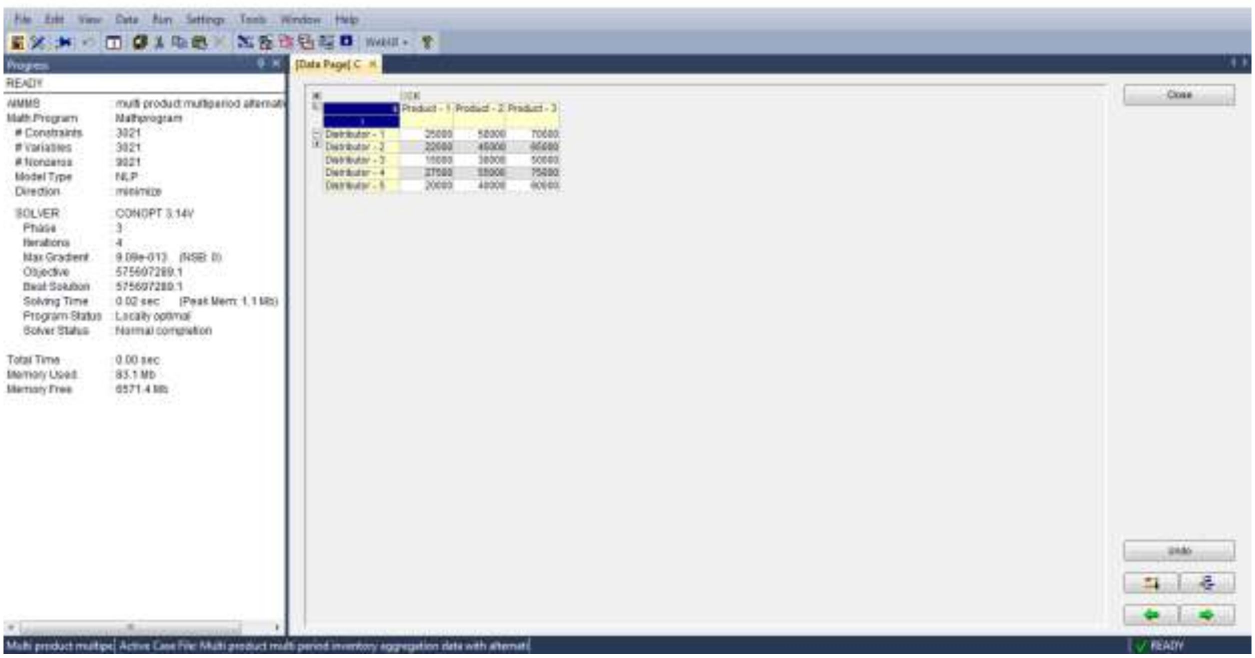

File Edit View Data Run Settings Tools Window Help
Progress
READY
AIMMS multi product multiperiod alternati
Math Program Mathprogram
Constraints 3021
Variables 3021
Nonzeros 9021
Model Type NLP
Direction minimize
SOLVER CONOPT 3.14V
Phase 3
Iterations 4
Max Gradient 9.09e-013 (NSB: 0)
Objective 575697289.1
Best Solution 575697289.1
Solving Time 0.02 sec (Peak Mem: 1.1 Mb)
Program Status Locally optimal
Solver Status Normal completion
Total Time 0.00 sec
Memory Used 83.1 Mb
Memory Free 6571.4 Mb
[Data Page] product
Product - 1
Product - 2
Product - 3
Close
Undo
Multi product multipe Active Case File: Multi product multi period inventory aggregation data with alternati
READY

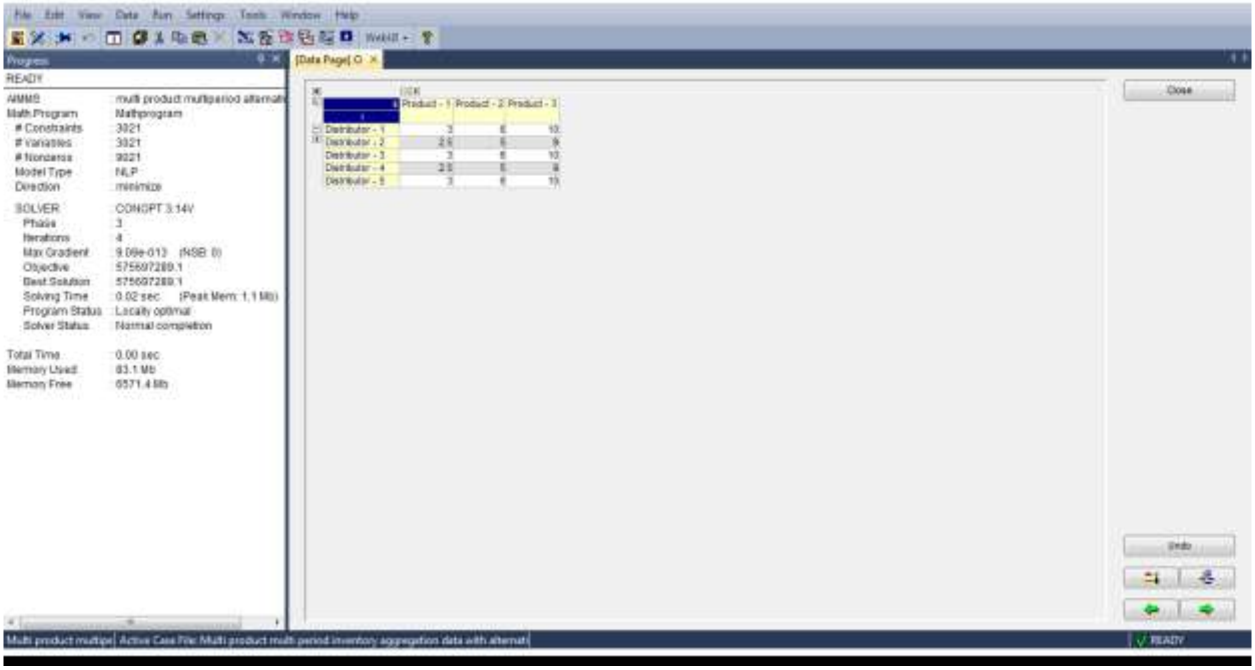

File Edit View Data Run Settings Tools Window Help
Progress
READY
AIMMS multi product multiperiod alternati
Math Program Mathprogram
Constraints 3021
Variables 3021
Nonzeros 9021
Model Type NLP
Direction minimize
SOLVER CONOPT 3.14V
Phase 3
Iterations 4
Max Gradient 9.09e-013 (NSB: 0)
Objective 575697289.1
Best Solution 575697289.1
Solving Time 0.02 sec (Peak Mem: 1.1 Mb)
Program Status Locally optimal
Solver Status Normal completion
Total Time 0.00 sec
Memory Used 83.1 Mb
Memory Free 6571.4 Mb
[Data Page] PlanningHorizon
Planning Horizon - 1
Planning Horizon - 2
Planning Horizon - 3
Planning Horizon - 4
Planning Horizon - 5
Close
Undo
Multi product multipe Active Case File: Multi product multi period inventory aggregation data with alternati
READY

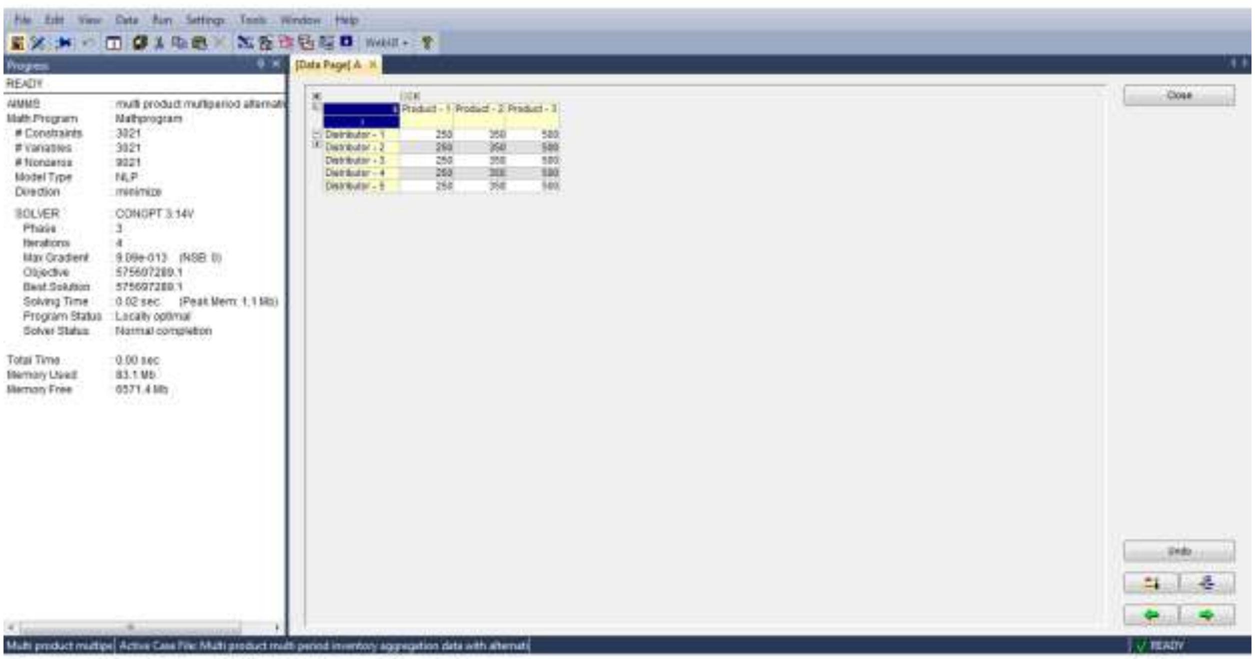


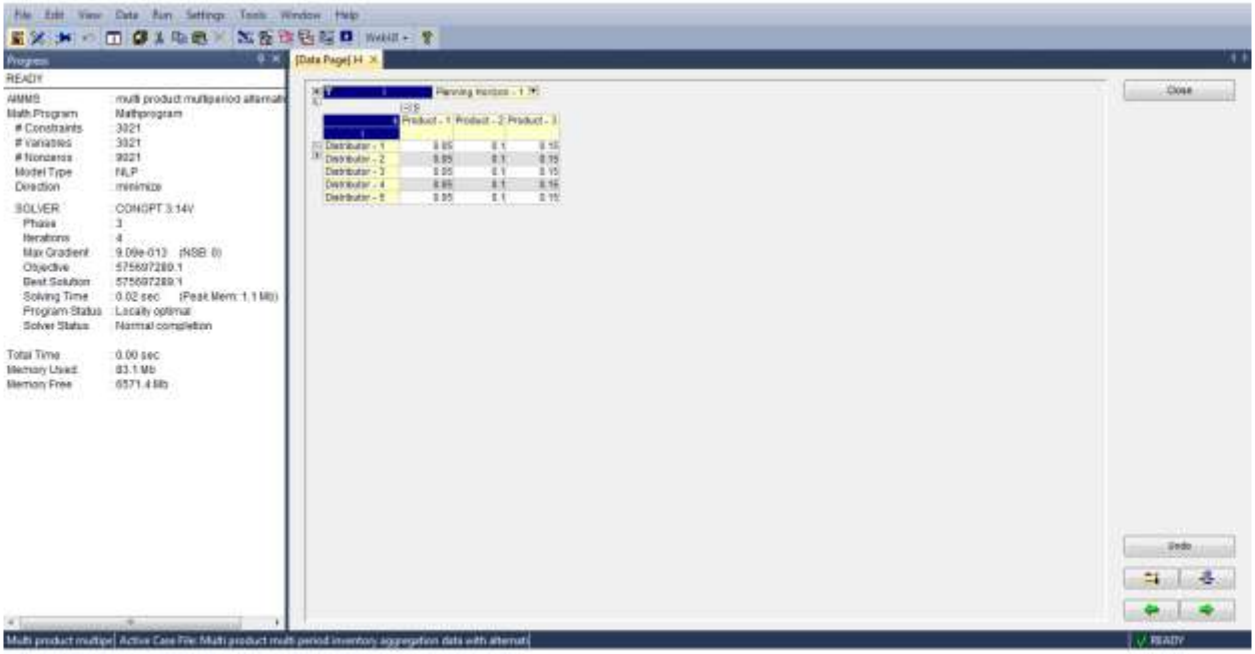


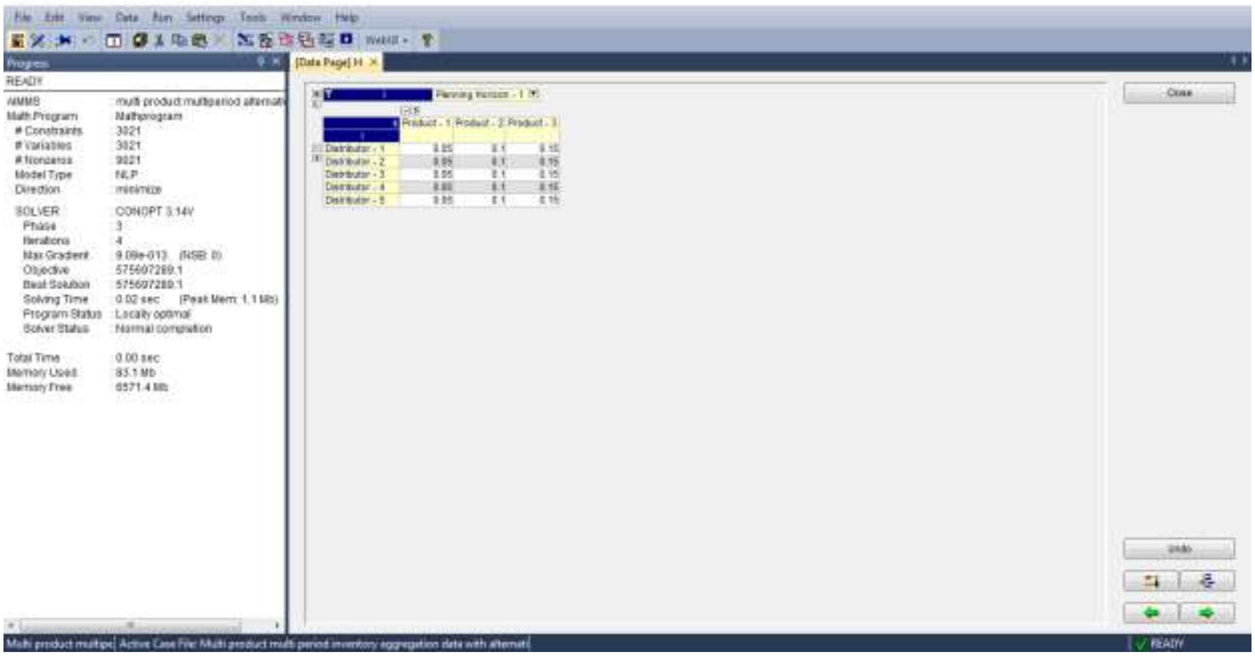

File Edit View Data Run Settings Tools Window Help
Progress
READY
AIMMS multi product multiperiod alternati
Math Program Mathprogram
Constraints 3021
Variables 3021
Nonzeros 9021
Model Type NLP
Direction minimize
SOLVER CONOPT 3.14V
Phase 3
Iterations 4
Max Gradient 9.09e-013 (NSB: 0)
Objective 575697289.1
Best Solution 575697289.1
Solving Time 0.02 sec (Peak Mem: 1.1 Mb)
Program Status Locally optimal
Solver Status Normal completion
Total Time 0.00 sec
Memory Used 83.1 Mb
Memory Free 6571.4 Mb
[Data Page] Distance
Close
Retailer - 1 Retailer - 2 Retailer - 3 Retailer - 4 Retailer - 5 Retailer - 6 Retailer - 7
Distributor - 1 25 35 45
Distributor - 2 15 20 35
Distributor - 3 40 45 55 60
Distributor - 4 30 45
Distributor - 5 40 45
Undo
Multi product multipe| Active Case File: Multi product multi period inventory aggregation data with alternati
READY


File Edit View Data Run Settings Tools Window Help
Progress
READY
AIMMS multi product multiperiod alternati
Math Program Mathprogram
Constraints 3021
Variables 3021
Nonzeros 9021
Model Type NLP
Direction minimize
SOLVER CONOPT 3.14V
Phase 3
Iterations 4
Max Gradient 9.09e-013 (NSB: 0)
Objective 575697289.1
Best Solution 575697289.1
Solving Time 0.02 sec (Peak Mem: 1.1 Mb)
Program Status Locally optimal
Solver Status Normal completion
Total Time 0.00 sec
Memory Used 83.1 Mb
Memory Free 6571.4 Mb
[Data Page] C
Close
Product - 1 Product - 2 Product - 3
Distributor - 1 25000 50000 70000
Distributor - 2 22000 45000 65000
Distributor - 3 15000 38000 50000
Distributor - 4 27500 55000 75000
Distributor - 5 20000 40000 60000
Undo
Multi product multipe| Active Case File: Multi product multi period inventory aggregation data with alternati
READY


File Edit View Data Run Settings Tools Window Help
Progress
READY
AIMMS multi product multiperiod alternati
Math Program Mathprogram
Constraints 3021
Variables 3021
Nonzeros 9021
Model Type NLP
Direction minimize
SOLVER CONOPT 3.14V
Phase 3
Iterations 4
Max Gradient 9.09e-013 (NSB 0)
Objective 575697289.1
Best Solution 575697289.1
Solving Time 0.02 sec (Peak Mem: 1.1 Mb)
Program Status Locally optimal
Solver Status Normal completion
Total Time 0.00 sec
Memory Used 83.1 Mb
Memory Free 6571.4 Mb
[Data Page] O
Product - 1 Product - 2 Product - 3
Distributor - 1 3 6 10
Distributor - 2 2.5 5 9
Distributor - 3 3 6 10
Distributor - 4 2.5 5 9
Distributor - 5 3 6 10
Close
Undo
Multi product multipe Active Case File: Multi product multi period inventory aggregation data with alternati
READY


File Edit View Data Run Settings Tools Window Help
Progress
READY
AIMMS multi product multiperiod alternati
Math Program Mathprogram
Constraints 3021
Variables 3021
Nonzeros 9021
Model Type NLP
Direction minimize
SOLVER CONOPT 3.14V
Phase 3
Iterations 4
Max Gradient 9.09e-013 (NSB 0)
Objective 575697289.1
Best Solution 575697289.1
Solving Time 0.02 sec (Peak Mem: 1.1 Mb)
Program Status Locally optimal
Solver Status Normal completion
Total Time 0.00 sec
Memory Used 83.1 Mb
Memory Free 6571.4 Mb
[Data Page] A
Product - 1 Product - 2 Product - 3
Distributor - 1 250 350 500
Distributor - 2 250 350 500
Distributor - 3 250 350 500
Distributor - 4 250 350 500
Distributor - 5 250 350 500
Close
Undo
Multi product multipe Active Case File: Multi product multi period inventory aggregation data with alternati
READY





Appendix-III: List of publications

1. Peeyush Vats, Gunjan Soni and Ajay Pal Singh Rathore (2018), "A review of inventory control problems" **International Journal of Intelligent Enterprises**, Vol. 5, No. 3, pp. 213-230(DOI:https://www.inderscienceonline.com/doi/pdf/10.1504/IJIE.2018.093396)
2. Peeyush Vats, Gunjan Soni, Ajay Pal Singh Rathore and Om ji Shukla, "Grey based decision-making approach for the selection of distributor in a supply chain" **International Journal of Intelligent Enterprises, (Communicated)**
3. Peeyush Vats, Gunjan Soni and Ajay Pal Singh Rathore, "Risk pooling approach in inventory control model for multi-products in a distribution network under uncertainty" **Springer book series on Advanced Engineering optimization through intelligent techniques (Scopus Indexed, Accepted)**
4. Peeyush Vats, Gunjan Soni, Ajay Pal Singh Rathore and J K Purohit, "Risk pooling approach in multi-product multi-period inventory control model under uncertainty" **IEEE Explore (Accepted)**

Peer Reviewed Conferences

1. Peeyush Vats, Gunjan Soni and Ajay Pal Singh Rathore, "**Introduction to Multi Agent approach for Inventory Control in Supply Chain: A Review**" in 3rd International conference on Supply Chain Management at Udaipur on 28th Nov 2014 to 30th Nov 2014 organized by Indian Institute of Industrial Engineering (IIIE) Navi Mumbai.
2. Peeyush Vats, Gunjan Soni and Ajay Pal Singh Rathore, "**A Review of Some Inventory Control Models: Multi-objective Optimization**" 7th International Conference on Innovative Research in Engineering Science and Management (ICIRESM-16) 13th November 2016 at IETE Delhi.
3. Peeyush Vats, Gunjan Soni and Ajay Pal Singh Rathore, "**An Inventory Control Model for Multi product in Multi echelon Supply Chain**" in CPIE-2016 in the 4th International Conference on Production and Industrial Engineering organized by Production and Industrial Engineering Department of NIT Jalandhar INDIA, on 20th Dec 2016

4. Peeyush Vats, Gunjan Soni and Ajay Pal Singh Rathore, "**A two echelon supply chain network design optimization**" in ICETMIE- 2017 organized by North Cap University, Gurgaon on 13th-14th October-2017.
5. Peeyush Vats, Gunjan Soni and Ajay Pal Singh Rathore, "**A demand aggregation approach for inventory control in two echelon supply chain for multi-products**" on 27th April held at Anand International College of Engineering.
6. Peeyush Vats, Gunjan Soni and Ajay Pal Singh Rathore, "**Risk pooling approach in inventory optimization for multi-products in two stage supply chain under service level constraint**" on 3rd August 2018 to 5th August in the international conference of AEOTIT-2018 held at SVNIT Surat.
7. Peeyush Vats, Gunjan Soni, Ajay Pal Singh Rathore and J K Purohit, "**Risk pooling approach in multi-product multi-period inventory control model under uncertainty**" POMS-2018 Kandy Sri Lanka **(Communicated, Abstract Accepted)**

Appendix-IV: Biographical Profile of Researcher

Peeyush Vats is born in Shamli, Uttar Pradesh (India) on 30th March 1977. He did his Bachelor's in Engineering (B.E.) from Hindustan College of Science and Technology, Farah Mathura affiliated to Dr. Bhim Rao Ambedkar University Agra in year 2001. He completed his Master's in Engineering (M.E.) in Manufacturing Technology from National Institute of Technical Teacher's Training and Research Chandigarh, affiliated to Panjab University Chandigarh in year 2010. He is presently working Poornima College of Engineering, Jaipur and simultaneously he is presently pursuing Ph.D. from Malaviya National Institute of Technology Jaipur, India as a part-time candidate. He has over ten years of teaching experience at undergraduate levels in various private engineering colleges. His areas of research interests are optimization, risk pooling, supply chain management, operations management.