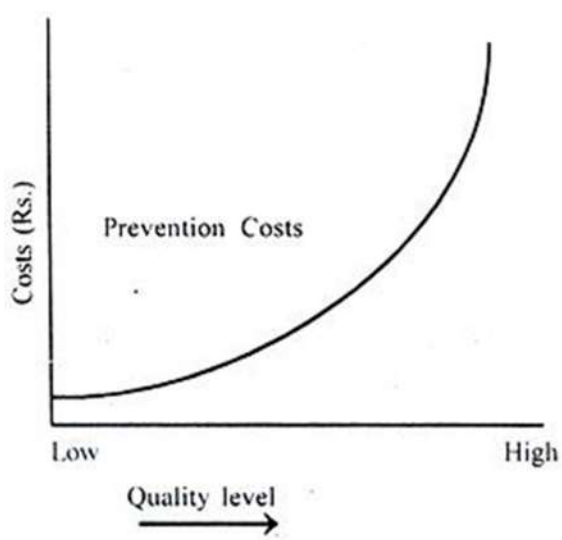
1.INTRODUCTION

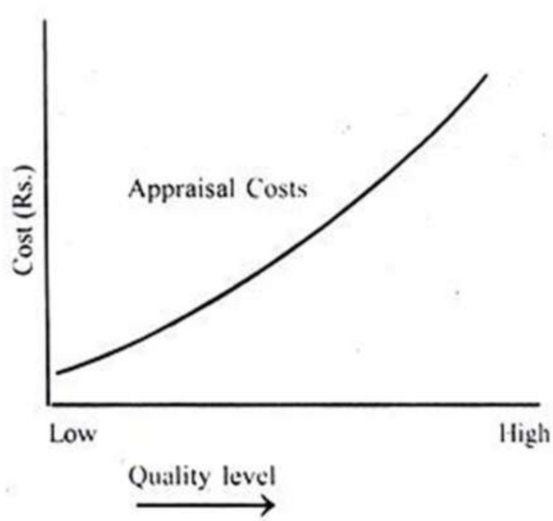
1.1 IS WARRANTY REQUIRED ?

The Iron Pillar located in Delhi, India, is a 7 m (23 ft) column in the Qutub complex, notable for the rust-resistant composition of the metals used in its construction. The surprise comes in learning its age, some 1600 years old, much older than one would expect for an iron column which, judging from other exposed iron, should have turned to a pile of dust long ago.

Looking at this ideal case, we can think of that warranty is a matter of no use and we can easily constitute a material which can resist the corrosive environment easily and the need of warranty is negligible. But it is a pillar of historic importance for its non corrosive character only.

In practice if we are going to make such material which is supposed to long for a long time it will cost much more. Variation of cost (prevention and appraisal) to quality is shown in the figure below.





As we can see the graph is not linear it is an exponential curve

1.2 HISTORICAL PREVIEW

Warranty is an important element of marketing new products as better warranty signals higher product quality and provides greater assurance to customers. The concept of warranty has been around for almost as long as there has been trade and there have been many representations of warranty throughout history. It has existed in some form or another from the early civilizations (Babylo- nian, Assyrian, and Egyptian Eras, Ancient Hindu and early Islamic periods), through the European Period (Roman Era, Germanic, Jewish, and early English periods, and the early Russian Era), the Middle Ages, the Industrial Revolution and beyond. Until the sixteenth century, the general purpose of warranty was to protect the buyer from fraud and faulty workmanship. When trade policy reversed around the dawn of the industrial revolution to favor the manufacturer, it was not a pressing issue since products were still produced locally by people known personally to buyers. Products were still relatively simple and easily evaluated, and any dissatisfaction was addressed directly to the manufacturer, with word of mouth travelling fast in local and tight knit communities. As communities grew, so did the acceptance of caveat emptor or ‘‘let the buyer beware’’. For further details of warranty evolution over this long period.

Late in the nineteenth century, standardized product warranties became more common, although many were extremely limited in coverage. As deceit became more widespread, consumers began to see warranties as indicators of poor quality, with manufacturers offering contracts with no intention of honoring them, and no legal incentive to do so. This was the basis of the exploitation theory of warranty. According to this theory, the warranty terms are developed for the manufacturer’s benefit, while the consumer has few rights and bears the risks. Buyers who believe this theory often feel that if a product is sold, it should last a certain amount of time, and the warranty is seen to serve the manufacturer by adding to the price of the product; i.e., by offering a service which should be provided anyway. Because a warranty is offered, it is reasoned, these buyers feel that the manufacturer does not have confidence in its own product.

In 1914, to counter this trend, the Federal Trade Commission (FTC) was established, which set forth codes to govern the sale of goods. By 1952, all but one state in the United States had introduced the Uniform Commercial Code (UCC) which specified the obligations of those parties involved in the sale of goods. This code also covers both explicit and implied warranties.

Before 1975, consumers were still at the mercy of manufacturers for several reasons. Warranties did not provide notice of consumer rights, disclaimers were couched in legal jargon, administration of warranty was confusing and ineffective, remedies were impractical for defective items, and excessive and unjustified claims often resulted from consumer frustration and hostility (Burton, PWH).

The Magnuson–Moss Warranty Act aimed to provide American consumers with information, improve the quality of warranties and to provide procedures for consumer remedies. The readability of warranties has been found to have increased slightly, however the act's standard of ''simply and readily understood'' is still a fair distance off (Burton, PWH).Another aim of the act was to ensure that warranty was a good indicator of reliability, leading to the signal theory of Warranty.

As products become more complex, and less easily evaluated by consumers, warranties are used to indicate the product's performance and reliability. The product performance and the warranty terms together determine the costs incurred by the manufacturer, so it follows that a longer warranty period will result in more costs unless the product performance is of a correspondingly higher quality. This theory [1] proposes that if a manufacturer offers a better warranty than a competitor, then the reliability of the product should also be better to reduce costs associated with warranty claims.

Due to this signaling characteristic, warranty is an important product feature and can be used by marketing to promote sales. Warranty had been viewed as both an insurance policy and a repair contract. This has given rise to a third theory of warranty, the investment theory. Under this theory, the warranty is seen as an investment by the buyer to reduce the risk of early failure. Manufacturers are insured against having to rectify problems caused by inappropriate use while the buyer is covered for repair costs of premature failures. The aim is to extend the useful life of the product by specifying responsibilities of the manufacturer and the buyer.

1.3 WARRANTY CONCEPT

A warranty is a manufacturer's assurance to a buyer that a product or service is or shall be as represented. It may be considered to be a contractual agreement between the buyer and manufacturer entered into upon the sale of the product or service. A warranty may be implicit or it may be explicitly stated. In broad terms, the purpose of a warranty is to establish legal responsibility among the two parties (manufacturer and buyer) in the event that an item fails. An item is said to fail when it is incapable to perform satisfactorily its intended function when properly used. The contract specifies both the performance that is to be anticipated and the redress available to the buyer if a failure occurs.

New products can be divided into the following three categories[2]:

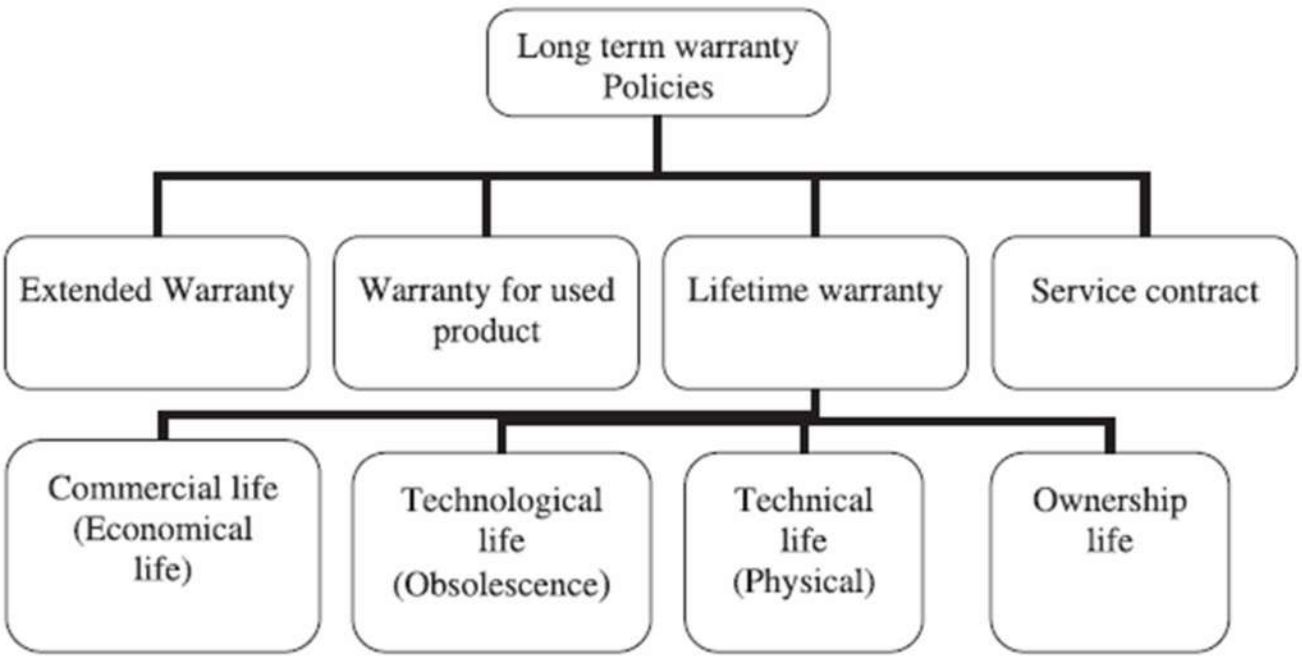
1. Consumer durables (e.g., household appliances, cars) bought by individual households as a single item.

2. Industrial and commercial products bought by businesses for the production of services (e.g., equipment used in a hospital to provide medical care, aircraft's used by airline operators) or products (e.g., components bought by a manufacturer). These are bought either individually (e.g., a single X-ray machine bought by a hospital) or as a batch of L ($L > 1$) items (e.g., batteries bought by a car manufacturer, fleet of trucks bought by a car rental agency).

3. Government acquisitions (e.g., new fleet of tanks or jet fighters) involving new and evolving technologies. As such, they are characterized by a high degree of uncertainty in the product improvement process. (Note: A government is also a large buyer of ‘‘standard’’ industrial and commercial products but these do not involve product improvement as part of the warranty.)

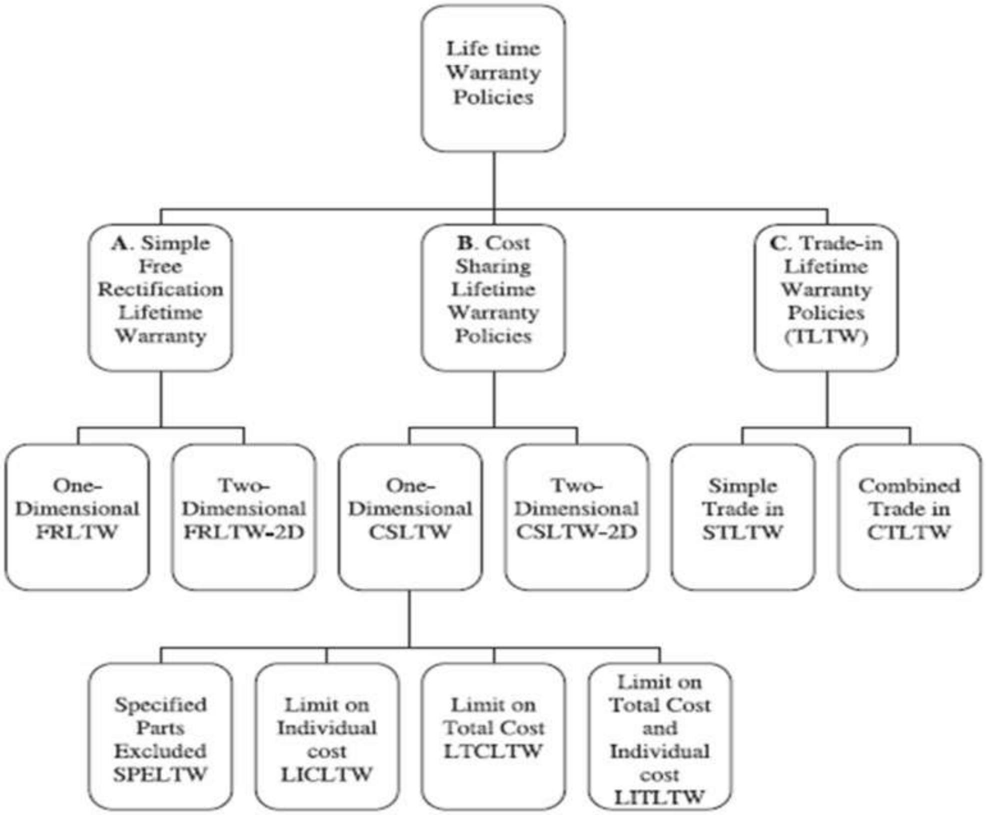
1.4. LONG TERM WARRANTY POLICIES

Long-term warranties offered by manufacturers/dealers include extended warranty, warranty for used product, long-term service contract and lifetime warranty policies [3]
and is shown in Fig.



1.5 LIFETIME WARRANTY POLICIES

The main complexity in this area is the uncertainties with functional life (lifetime) and subsequently the coverage periods. Another complexity is the uncertainty of servicing costs over longer uncertain periods. A taxonomy of all these policies is shown as follows:



1. FRLTW Policy

Policy 1. FRLTW (one-dimensional) with no cost to buyer/customer: Under this policy the manufacturer/dealer takes the accountability to rectify all defects and failures of the sold product due to design or manufacturing or quality control problems over the defined lifetime of the product.

Rectification can be a replacement, repair or in a few cases refund. Unlike normal warranty, the coverage period for a lifetime warranty is uncertain and randomly variable.

Policy 2. FRLTW on both age and usage (two-dimensional case) with no cost to buyer /customer [FRLTW-2D]: Under this policy the manufacturer/dealer rectifies all defects and failures of the sold product due to design, manufacturing or quality control problems over the age and usage of the product whichever comes primary within defined lifetime of the product. Here the coverage terminates at an age or usage due to the ownership change, technological obsolescence, useful or commercial reason.

2. CSLTW policies

Under this policy, the customer and the manufacturer/ dealer share the repair cost over the uncertain coverage period. The basis for the sharing can vary as indicated below. In line with, we propose four one-dimensional CSLTW policies. These are:

Specified Parts Excluded Lifetime warranty (SPELTW),

Limit on Individual Cost Lifetime Warranty (LICLTW),

Limits on Total Cost Lifetime Warranty (LTCLTW), and

Limit on Individual and Total Cost Lifetime Warranty (LITLTW).

These policies are described briefly as follows:

3. SPELTW: Under this policy, the components of the product are grouped into two disjointed sets, Set-I (for inclusion) and Set-II (for exclusion). The manufacturer/ dealer rectifies failed components belonging to Set-I at no cost to the buyer over the defined lifetime of the product. The costs of rectifying the failed components belonging to Set-II are borne by the customer (note: the rectification of failed components belonging to Set-II can be carried out either by the dealer or a third party).

4. LICLTW: Under this policy, if the cost of a rectification on each occasion is below the limit cI, then it is borne completely by the manufacturer/dealer and the customer pays nothing. If the cost of a rectification exceeds cI, the buyer pays all the costs in excess of cI (i.e. cost of rectification-cI). This continues until the termination of warranty based on defined lifetime.

5. LTCLTW: Under this policy the manufacturer/dealer's obligation ceases when the total repair cost over the lifetime exceeds cT. As a result the warranty ends at an uncertain lifetime or earlier if the total repair cost, at any time during the lifetime, exceeds a prefixed cut off cost cT.

Here, the warranty coverage is indecisive not only for uncertainty in exceeding total cost limit but also for the uncertainty of lifetime as defined in the policy.

6. LITLTW: Under this policy, the cost to the manufacturer/dealer has an upper limit (cI) for each rectification and the warranty ceases when the total cost to the dealer (subsequent to the sale) goes above a cut off cost cT or the termination of the product life due to the defined reasons, whichever occurs first. The customer pays all the costs in excess of cI.

Trade in with lifetime policies

The two types of trade in with lifetime warranty policies proposed here are as follows:

7. STLTW: Under this policy, the customer has an choice to get a replacement at a reduced cost of trade-in for the used one. In this category of warranty the failed used product under warranty is repurchased by the manufacturer/ dealers. The repurchased price would be a proportion of the original purchased price depending on the age of the product, i.e. the repurchased price (trade in price) Pt ¼ Po(a/E(L)), where Po is the original purchased cost and a is the age of the product at the time of trade in. E(L) is the expected lifetime of the product. However in actual life trade-in price can be negotiable.

8. CTLTW policy: Under this policy the failed or defective product is rectified at no cost to the customer/ buyer up to a certain time w and if the product fails any time beyond w over the rest of the lifetime the failed product is repurchased by the manufacturer/dealer at a reduced price . Clearly, under this policy the coverage time is divided into two terms. These are: (1) up to w, the warranty condition is similar to that of the free rectification warranty and (2) from the age w to the termination of lifetime follows the simple lifetime trade-in policy.

Modeling product failures for FRLTW policy

Here, the breakdown rate of product is an increasing function of age (as assumed). Since the quantity of failed components rectified at each failure is very small relative to the total number of components in the item, the rectification action can be viewed as having a negligible impact on the failure rate of the product as a whole. In other words, the failure rate after a repair is nearly the same as that just before the failure. Such a repair action is called a minimal repair . This type of failure can be modeled as the Non- Homogeneous Poisson's Process (NHPP). In this case L(t) is the failure rate associated with the failure distribution for the product. When product failure is distributed as F(t) following NHPP with density function f(t), then the failure intensity L(t) can be assumed as follows:

$$F(t) = 1 - \exp(-(\lambda t)^{\beta})$$

and

$$f(t) = \frac{d[F(t)]}{dt} = \lambda\beta(\lambda t)^{\beta-1}\exp(-(\lambda t)^{\beta}).$$

From Eq. (1), $\Lambda(t)$ is given by

$$\Lambda(t) = \frac{f(t)}{1 - F(t)} = \frac{\lambda\beta(\lambda t)^{\beta-1}\exp(-(\lambda t)^{\beta})}{1 - (1 - \exp(-(\lambda t)^{\beta})} = \lambda\beta(\lambda t)^{\beta-1}$$

with the parameters $\beta > 1$ and $\lambda > 0$.

Modeling lifetime

Since the lifetime of the product is uncertain, the warranty coverage period L is random variable (unlike basic warranty coverage, the upper limit of the coverage period is a random variable with mean ma). Failures might occur randomly within the region 0–L. Conditioned on L = a, one can model this lifetime as a random variable with a distribution function H(a) with H(l) =0 and H(u) = 1. l and u are the lower and upper limits of termination of lifetime L at statutory base. h(a) is the probability density function of a associated with H(a) and

$$h(a) = \frac{dH(a)}{da}$$

One form of H(a), which is analytically tractable, is the following truncated exponential distribution

$$H(a) \text{ is } \frac{e^{-\rho l} - e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}}$$

$$h(a) = \frac{\rho e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}}.$$

The mean value of useful life of the sold product can be expressed by

$$\mu_a = E(a) = \frac{(le^{-\rho l} - ue^{-\rho u}) + (e^{-\rho l} - e^{-\rho u})/\rho}{e^{-\rho l} - e^{-\rho u}}.$$

In real life distribution of lifetime coverage might not be possible to model using a particular distribution and can be modeled using probability mass function.

2. LITERATURE REVIEW

2.1. WARRANTY COST ANALYSIS

We discusses about the research on warranty policies and related topics that many researchers [4, 5, 6-8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18] have done in the literature by several different categorized groups. Descriptions of various types of warranty policies and mathematical models can be found in Blischke and Murthy [5, 19].

2.2. ONE DIMENSIONAL AND TWO DIMENSIONAL WARRANTY

One dimensional warranty is feathered by the warranty period, which is defined in terms of a single variable. Single variable could be time or age or usage. In the case of two dimensional warranties, there are two dimensions to express a warranty policy. One represents time and the other represents item usage. As a result, many different types of warranties can be defined based on the characteristics of warranty policies [5]. Many researchers have studied the cost analysis based on two dimensional warranty [20, 21, 22, 23, 24, 25, 26, 27, 28]. Yun and Kang [28] examined new warranty servicing strategy, considering imperfect repair with a two-dimensional warranty. Baik *et al.*[20] study two dimensional failure models for a system where degradation is due to age and usage with minimal repair. Many of the products have one of two attributes with some exceptions, for example, a vehicle. Several researchers have studied the warranty policy based on the automobile industry's data. Compared to one-attribute warranty, two-attribute warranty is more complex . Chun and Tang [29] propose several decision models that estimates the expected total cost incurred under various types of two-attribute warranty policies. Kim and Rao [11] considered two-attribute warranty policy for non-repairable items and the item failures are described in terms of a bi-variant exponential distribution. Jiang and Ji [30] study multiple attribute value model based on four attributes such as cost, availability, reliability and lifetime analysis. Samatli-Pac and Taner [16] developed and investigate different repair strategies for one- and two-dimensional warranties with the objective to minimize manufacturer's expected warranty cost using QRP. Other researchers [20, 21, 22, 23, 24, 25, 26, 27, 28] had also developed warranty models by considering two-dimensional warranty strategies.

2.3. RENEWING WARRANTY AND NON RENEWING WARRANTY

Under renewing warranty, the product which fails during its warranty period is replaced by a new one at a cost beard by the manufacturer or at a pro-rated cost to the user and the warranty is renewed. Under a non-renewing warranty, the manufacturer guarantees a satisfactory service only during the original warranty phase. Renewable warranty is usually given to the non repairable and inexpensive products such as home appliances and so on. Compared to the renewable warranties, the period of non-renewable warranties is longer. So this might be one of possible reasons why such policies are not in trend as non-renewable ones for warranty issuers. Jung *et al.* [31] investigate the optimal replacement policies following the expiration of warranty such as renewing warranty and non-renewing warranty. Chukova and Hayakawa [6,7] estimate the warranty costs over the warranty period under non-renewing and renewing warranty policies over the life cycle of the product. Sahin and Polatoglu [32] verify that the cost rate function is pseudo-convex under a fixed-maintenance period policy under non-renewing and renewing warranty policies. Chen and Chien [33] examine a model to study the effect of PM carried out by the buyer on items sold under a renewing FRW.

2.4. WARRANTY PERIOD AND POST WARRANTY PERIOD

For the duration of warranty period, as mentioned above, there are several kinds of warranty polices such as FRW, PRW or CMW. However, for the duration of post warranty period, customers have to repair or replace the failure product at their own costs. Jung and Park [34] consider two categories of warranty policies such as renewing warranty and non-renewing warranty with warranty period and post warranty period. They develop the expressions for the expected maintenance costs for the periodic preventive maintenance during post warranty period. Jung *et al.*[26] study the optimal replacement policies for the duration of post warranty period considering the expected downtime per unit time and the expected cost rate per unit time. Jung [26] consider the optimal period for the periodic PM for the duration of the post warranty period which minimize the expected long-run maintenance cost per unit time.

2.5. WARRANTY RESERVE

Warranty reserve is one of vital factors which would be considered for the warranty policies. Therefore, numerous researchers [35, 36, 37, 38] have considered the warranty reserve for the cost analysis. Patankar and Mitra [38] examined the outcome of warranty execution on the expected warranty reserves of a linear pro rata rebate plan. Ja *et al.* [36,37] consider a policy where warranty is not renewed on product failure within the warranty period but the product is minimally repaired by the manufacturer with the warranty reserves.

2.6. RELIABILITY AND WARRANTY

The relationship along warranty policies and products' reliability is very strongly related. If the product's reliability is excellent, then the product's warranty could be extended. Or else, the product's warranty should be considered again. However, there are a number of exceptions. To increase a product's sales, a few providers extend the product's warranty period. They use the warranty policy as a marketing instrument. The reliability of product is determined by some important factors such as product's design, development, manufacturing stage and so on. It depends on the choice of suppliers and their cooperation in quality efforts as well. This implies that several vital factors must take into account the interaction between warranty and reliability. A company either gives a warranty that is far shorter than the expected life of their item or increases the cost to a very high level to envelop expected warranty costs. As a result, a product's reliability is one of imperative measures to investigate the warranty cost analysis [39]. On the other hand, Percy [40] presents some new ideas for improving a product's reliability by adopting Bayesian methodology.

2.7. MAINTENANCE POLICIES AND WARRANTY

Many researchers [41, 42, 43, 44,45,46,47,48,49] had published study on maintenance polices. Jhang and Sheu [46] derived the expected long-run cost per unit time for every policy. Sheu [47] consider a two-typed failures system which is area under discussion to shocks what arrive by a NHPP with the ARP and the BRP. Wang [48] reviews, classifies and compare a range of existing maintenance policies for both single-unit and multi-unit systems. Pham and Wang [49] too summarize different treatment methods and optimal policies on the imperfect maintenance. Jung and Park [50] develop the optimal periodic PM policies subsequently the termination of warranty.

The maintenance purposes are to minimize the maintenance allied operating costs, to maximize equipment availability and reliability or prolong equipment lifetime . For fading complex products, it is vital to perform preventive maintenance to achieve satisfactory reliability performance. Maintenance involves planned and unplanned activities carried out to keep a system at or restore it to an acceptable operating condition. Planned maintenance is generally referred as preventive maintenance though unplanned maintenance is labelled as corrective maintenance or repair .Two main preventive maintenance policies are block replacement policy and age replacement policy. Researchers propose these two kinds of preventive maintenance. Since then, a lot of study have been done regarding maintenance polices. Jhang and Sheu [47] find the expected long-run cost per unit time for each policy. Sheu [47] considers a two-typed failures method which is focused to shocks what arrive by a NHPP with age and block replacement policy. Wang [48] summarized, classified and compared different active maintenance policies for both single unit and multi-unit systems. Also, Pham and Wang [49] recap various treatment methods and optimal policies on the imperfect maintenance. Jung and Park [50] develop the optimal periodic preventive maintenance policies subsequently to the expiration of warranty.

2.8. MAINTENANCE COST ANALYSIS

Boland and Proschan [51] investigate a model for the minimal repair-periodic replacement policy and consider the difficulty of determining the period which minimizes the total expected cost of repair and replacement. Park *et al.* [52] consider the condition where each PM relieves stress temporarily and hence slows the rate of system degradation, while the hazard rate of the system remains monotonically increasing. Canfield [53] obtains the cost optimization of the PM intervention period by determining the average cost-rate of system operation. Wang and Pham [54] study availability, maintenance cost and optimal maintenance polices of the series system with n constituting components under the general assumption that each component is subject to correlated failure and repair, imperfect repair, shut-off rule and arbitrary distributions of times to failure and repair.

2.9. MAINTENANCE POLICIES AND WARRANTY

The maintenance purposes are to minimize the maintenance related operating costs, to maximize equipment availability and reliability or prolong equipment lifetime . For deteriorating complex products, it is vital to carry out preventive maintenance to achieve satisfactory reliability performance. Maintenance involves planned and unplanned activities carried out to retain a system at or restore it to an acceptable operating condition. Planned maintenance is generally referred as preventive maintenance while unplanned maintenance is labeled as corrective maintenance or repair. Two famous preventive maintenance policies are block replacement policy and age replacement policy. Barlow and Hunter [55] propose these two types of preventive maintenance. Since then, a lot of research have been done concerning maintenance polices. Jhang and Sheu [46] derive the expected long-run cost per unit time for each policy. Sheu [47] considers a two-typed failures system which is subject to shocks what arrive by a NHPP with age and block replacement policy. Wang [48] summarize, classify and compare various existing maintenance policies for both single-unit and multi-unit systems. Also, Pham and Wang [49] summarize various treatment methods and optimal policies on the imperfect maintenance. Jung and Park [80] develop the optimal periodic preventive maintenance policies following the expiration of warranty. Garbatov and Soares [44] plan the maintenance from an economic point of view so as to minimize maintenance costs but satisfying a minimum reliability level. Also, several researchers investigate the maintenance policies based on the Bayesian approach. Chen and Popova [21] propose two kinds of Bayesian maintenance polices. Additionally, a set of maintenance policies which consist of minimal repair and preventive maintenance is analyzed for the case of known and unknown failure parameters of the item's lifetime distribution.

3. METHEDOLOGY AND RESULT

These are the methods that the seller should consider before formulating a warranty statement. Before offering a warranty, the seller must understand

1) Performance and serviceability aspects of the product,

2) Customer marketplace

3) Legal issues

Extensive market research is done to understand the user's need and operating environment. Also appropriate legal counsel is consulted to make judgments and recommendations on the legal issues.

Warranty analysis is performed to know how much the warranty serviced will cost. Alternatively the producer likes to know the following:

1) The expected number of free replacements

2) The expected cost to the buyer

3) The expected profit to the manufacturer

The key warranty analysis steps are as follows:

1) Estimate the life distribution of the product

2) Understand the various types of warranty policies

3) Develop models that permit the required trade off analyses

All the disciplines shown above must be present before an informal warranty policy can be established. Market research begins as the product concept is being formulated. Initial reliability can begin with data from similar equipment. Models are defined as data are required for the product being manufactured.

Warranty analyses required the following:

1) Performance data from similar earlier equipment and from the product being manufactured.

2) Extensive in-house testing on critical subsystems, engineering models and early production models.

3) External trade testing at customer sites for reliability analysis and immediate feedback of early-life problems into the corrective action system.

4) Extensive market research to understand the user's operating scenario.

5) Benchmarking with competitors' products.

6) Tradeoffs among the different service strategies relative to site location, staffing and parts inventories.

TRADE Tradeoff issues

Warranty tradeoff analyses help the seller formulate a warranty strategy based on facts. Several factors must be studied and decisions made regarding their relative importance in an optimal warranty strategy. The factors that can be traded off by analysis are as follows:

1) Length of warranty

2) Type of warranty policy

3) Breadth of warranty coverage

4) Type of remedy

5) Service strategy

6) Administrative costs

7) Competitors' warranties

These factors are discussed as follows:

Length of Warranty

The length of the warranty should reflect the performance of the product. It is not wise to set the warranty duration at a usage value associated with high failure percentage of the product since too many claims would be possible. The life of the product needs to be understood in a statistical sense. Data can be gathered from a variety of sources such as subsystem and system testing, external trade trails, and historical databases of similar equipment. Appropriate meters need to be incorporated into the equipment design to record the life of the product. It is also important to obtain failure causes for each event.

The product being warranted will either be repairable or non repairable. The appropriate analysis for each category will be different.

Non repairable items

A non repairable item may be the total product or a separate warranty module. When analyzing data from a non repairable item, it should be verified that the data are from a homogeneous population and that there have been no design or material changes which would cause one item to last longer than another.

When studying a non repairable module in a repairable system, one should verify that the arrival pattern of the failures of these modules in the system does not exhibit either a trend toward decreasing times between failures or increasing times between failures. A random arrival is likely to occur when a failed module is replaced with an item of the quality of the original one. If this is the case, then it would be reasonable to assume that the module was exhibiting a renewal process in which the times between successive module replacements were independent and identically distributed with an unknown distribution.

The underlying life distribution for the product can be estimated using reliability models based on the Weibull or lognormal distribution. Goodness to fit tests should be applied to verify model validity. Confidence intervals should be stated for the parameters estimates. The expected number of renewals in a given time period can then be calculated assuming an ordinary renewal process starting at time=0.

Repairable items

The methodology for analyzing repairable systems is more complex than for non repairable systems. three analysis paths are described.

A trend test should be applied to the failure arrival data for each studied product in the sample. If the arrival pattern does not exhibit a trend toward decreasing times between failures or increasing times between failures, then it can be assumed that the equipment as a whole is exhibiting a renewal process. The entire product can be studied as if it were non repairable using the methodology described above.

If there is a trend, then the entire system can be studied using graphical repairable system methodology. Two such methods are the rate of occurrence of failures (ROCOF) and the mean cumulative number of failures (MCNF).The natural estimate of the ROCOF for the sample may be found by assuming that the underlying ROCOF for each equipment in the sample is similar and can be pooled. A graph of ROCOF verses time will provide information about the improvement or deterioration of the system. The non parametric graphical estimate of MCNF or mean cumulative cost can be used to evaluates can be obtained using a bootstrap method.

Finally, if there is a trend and it is desired to do a detailed system analysis, data need to be obtained for all major subsystem and components. Data on these individual modules can be fitted using appropriate reliability models. Then the overall system can be modeled using a series-parallel (redundancy) combination of the modules. To be thorough, the model should take into account such factors as the uncertainty in the distribution parameters' estimates, the concept of incomplete or new-better-than used repair, and the aging of parts in the system.This is best handled in a simulation model.

It should also be noted that the initial installs of a new product have the potential for more problems than later installs.

Types of Warranty Policies (non legal sense)

Knowledge of the product life distribution leads to understanding of the expected number of failure and/or remedies during a stated time period. A study of the types of warranty policies and associated cost models should lead the seller toward a warranty strategy which moderates incurring expenses. There are several types of warranty policies.

Four commonly used policies will be discussed here.

Free Replacement Warranty policy:- Under a free repair or replacement policy, the seller pays the entire cost of the remedy if the product fails before the end of the warranted period. Thus for long coverage times the warranty costs can be very large, and the number of replacement purchases over the product life cycle will be reduced, which in turn reduces the total profits.

In an ordinary free replacement policy, the remedied item has a warranty equal to the remaining length of the original warranty. Such a warranty assures that the buyer will receive as many free repairs or replacements as needed during the length of the original warranty. This type of policy favors the buyer at the expense of the seller.

Pro rata warranty policy

Under the pro rata warranty, if the product fails before the end of the warranty period, it is replaced at a cost which depends on the age or wear of the item at the time of failure. Typically a discount proportional to the remaining length of the warranty is given on the purchase price of the replacement items. The replacement item usually carries a warranty with terms identical to those on the original product.

The pro rata warranty policy is most often used for items which wear out and must be replaced at failure, rather than items which may be repaired. Examples are vehicle tires and batteries. A pro rata policy is more appealing to the seller but unattractive to the buyer since the buyer may have to purchase a new item at a cost, should the earlier item have a very short useful life. Thus the pro rata structure favors the seller at the expense of the buyer.

Combination Policy:- A combination policy contains both free and prorate periods. This policy has an initial free replacement period followed by a pro rata period during which the cost of the replacement item is calculated on a sliding scale. The relative lengths of the free and pro rata periods for the same overall warranty period can be studied on a cost basis.

In the combination policy, there is a consideration of whether the warranty is renewed only after each purchase or after each failure. Formulas for both buyer's cost and manufacturer's profits have been deprived under the assumption of a renewal process using the results of a renewal reward process for the long-run average costs.

A combination policy has a promotional appeal to attract buyers and at the same time keeps the warranty costs for the seller within a reasonable amount.

Fleet Warranty: A fleet warranty covers a population of items and therefore may be appropriate when a large number of identical items is being sold to a common buyer with the understanding that replacement parts will be provided by the manufacturer. In a fleet warranty, the manufacturer guarantees that the mean life of a population of items will meet or exceed some negotiated mean. If the mean fleet life meets or exceeds the warranty value, then no compensation is given by the manufacturer, even if individual items have very short lifetimes. If the mean life is less is less than the guaranteed value, compensation is given according to how much the specified mean life exceeds the observed mean.

Breadth of Warranty Coverage

The manufacturer usually assumes responsibility for defects in materials and workmanship. The entire product need not be covered. Warranties seldom include consumable such as filters and software diskettes, peripherals and interfaces not purchased from the manufacturer. Repair, maintenance, alteration, and/or modification of the product by other than manufacturer-authorized personnel are often reasons for voiding a warranty.

Types of Remedy

Failed items are typically repaired or replaced. The repair option is based on such factors as item complexity, original cost, and ease and efficiency of repair. Circuit boards are often repaired and placed back into spare-part inventory. Modules may be fabricated from rebuilt parts. The buyer must be notified in the warranty if used parts might be employed in a repair. Environmental concerns are promoting material recycling if repair is not feasible. Under a full consumer warranty, the buyer is entitled to a refund, less depreciation, if the defect cannot be remedied within a reasonable number of attempts.

Service strategy

The manufacturer needs to understand the buyer's operating environment when setting a service strategy. Some analysis is done to

- 1s the product's function critical to the buyer's business?
- Will the product be used 24 h per day, 7 days a week, or from 8 to 5 Monday through Friday, or on a casual basis?
- Is the buyer willing to do minor maintenance?
- Is the product designed so that it is easily serviced by a person with minimal training?
- If the equipment is portable, is the buyer willing to send it back to the manufacturer or take it to a service centre? How far are they willing to drive?

There are many possible service strategies, and each carries with it many logistics issues. Common scenarios are

- Mail-in return: Responsibility for the cost of packaging and shipping, TAT on the repair.
- Walk-in service centre: Staffed by manufacturer-seller or authorized personnel, hours of service.
- Onsite service: Provided by manufacturer-seller or authorized personnel, hours of service.
- Telephone assistance: Provided by manufacturer-seller or authorized personnel, level of training required, hours of service.

Telephone assistance is often a first step in screening the need for a site visit to remedy a problem.

Administrative Costs

All warranty plans have administrative costs. Records must be kept indicating the start of each item's warranty. For consumer products, the buyer is often responsible for keeping the sale slip. Databases detailing the defect type and location, the defect remedy, and the amount of usage at the time of the problem, must be built, maintained and analyzed.

Competitors' Warranties

A product's warranty must be competitive with those stated for similar products. If the optimal warranty policy for your product is broader than what is being offered by the competition, then the warranty could become the selling point. If this is not the case, then the product must be analyzed using warranty scenario set forth in the competitors' strategy. The cost implications should be studied, and areas for design improvement identified.

Trade-off Process

Basic to any trade off analysis is the definition of the purpose of the analysis. For example, we wish to know which the best warranty duration for a product is given that the other trade-off factors have been evaluated and the optimum levels set. The candidates are then identified. All identified candidates must be viable; that is to say, they must make sense in the context of competition and must be implementable. If the competition is offering a 12-month warranty does not make good business sense in the context of completion even though a 6 month or an 18 month warranty is implementable.

What is the basis for comparison or the figure of merit to be used to rank the alternatives? Is it cost, reliability, or availability? The figure of merit could be ability to compete while holding the profit line. This is an extremely important step in the trade off process. Many well-intentioned engineers have spent much time developing and performing a trade off analysis only to learn at the end that their chosen figure of merit did not adequately discriminate between or among the candidates; therefore a meaningful ranking of the candidates was not achievable.

The model development should be geared to the availability of input data to support it. It is of little value to have a very detailed set of algorithms in the model and having insufficient data definition at that point of time to support it. The complexity of the mathematical-statistical model should be directly proportional to the level of definition of the input data. Since we are interested in comparing the figures of merit of the candidates to yield a ranking, it is necessary to consider elements in the model only where differences are expected to exist between or among alternatives.

The next step is singularity the most vital in the entire process. Since most models are evaluators, not optimizers, the step of input data estimation is crucial to the evaluation of the candidates. If the input data does not reflect the differences among the alternatives, then the model outputs cannot reflect the differences. To secure reasonable estimates for input data, a team of knowledgeable people representing the necessary disciplines is required. For example the reliability discipline but also relative to the product itself. These criteria apply to all member of the team.

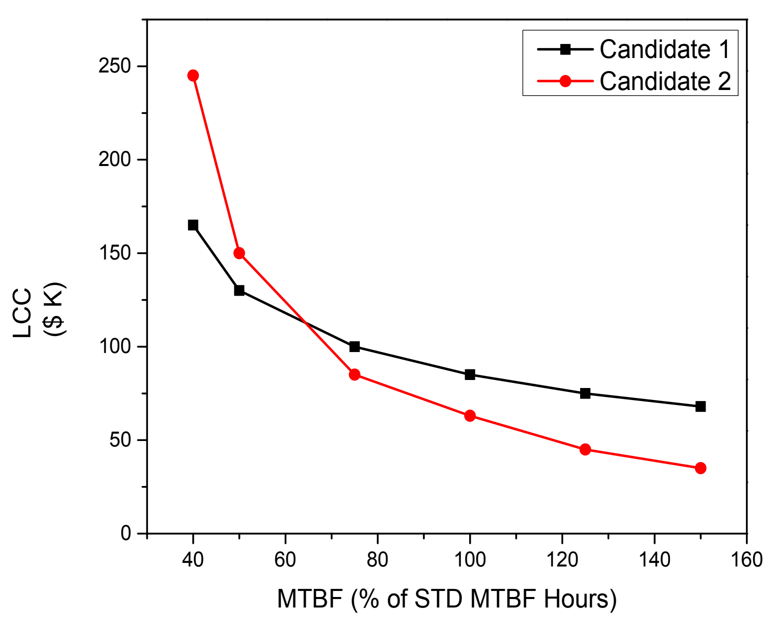
One of the team members' responsibilities is to document the rationale for their estimates. It is important to know when the estimate came, especially when the ranking is quite close and a further examination of the input data is estimated can make or break the validity of the analysis.

Having developed the model and secured the input data, we can now perform the baseline analysis. The importance of the baseline analysis cannot be overemphasized. An initial stake in the ground must be established. The stake is the baseline analysis. The baseline analysis provides a frame of reference from which to establish the sensitivities of the driving input parameters. The baseline analysis represents the most likely result based on the most reasonable inputs from the team. The likely baseline remains the baseline until the design and support considerations of the product change through trade off analyses and a new baseline is established.

With the results of the baseline analysis in hand, the ranking of candidates can be achieved. The figure of merit established in our third step should clearly indicate whether high is good or low is good, and the candidates can be ranked accordingly. In interpreting the results, care should be taken to determine the percentage differences between or among the figure of merit for the candidates. This will be especially crucial in the final decision step.

With the results of the sensitivity analysis performed in conjunction with the baseline analysis, we can perform sensitivity analysis with regard to the trade off analysis results- the initial ranking. By varying the driver parameters from the baseline analysis plus and minus-say, 25% and a on rerunning the trade off analysis, we can see the effect on the ranking. This can be accomplished as a uni-variant or multi-variant operation the ultimate being a simulation approach. What we have done is identify the thresholds on the input data drivers where the candidate ranking change. Because of the estimative nature of the total process, sensitivity analysis is essential to help us determine under which condition candidate A is better, under which conditions candidate B is better and so o. Rarely does one candidate always prevail under all values of the input data drivers.

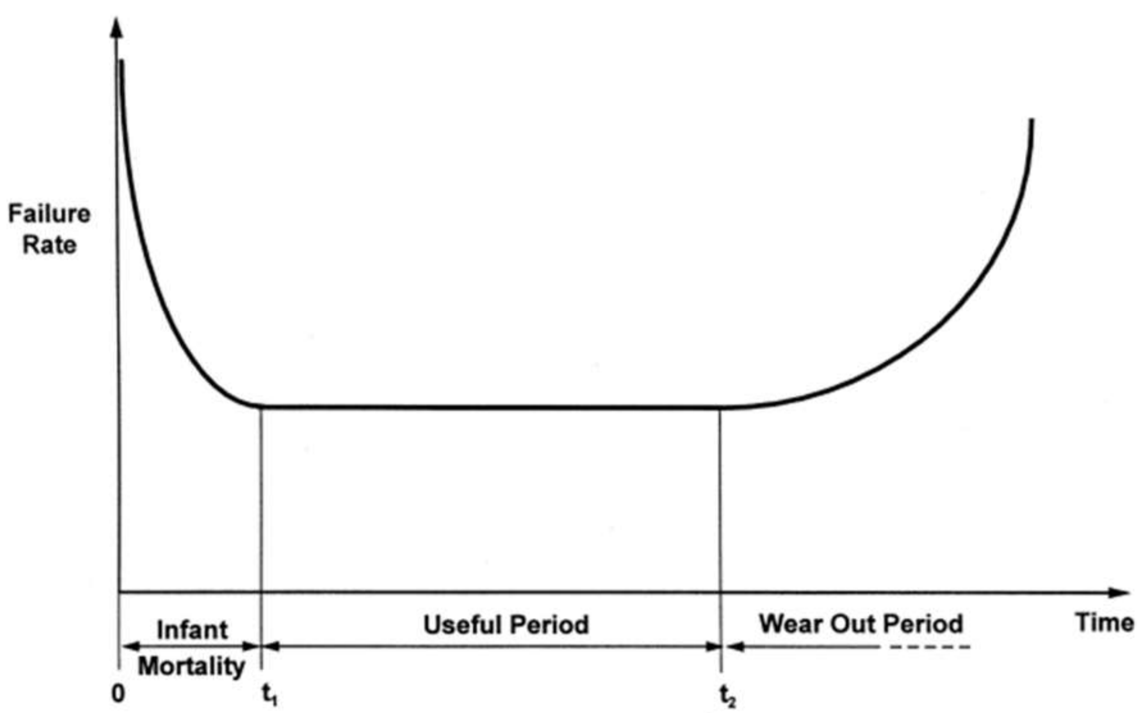
Finally, the final step- the decision process: To select the best candidate, we must study the results of the trade off sensitivities that led to the final ranking. If possible, a plot of the relativities that led to the final ranking. If possible, a plot of the relative ranking as a function of the driver parameter values should be made. In this way a judgment can be made as to what region of the plot is most likely to occur with respect to the input driver value. To illustrate the graph shows the result of a trade off analysis where the life-cycle cost (LCC) of two candidates is plotted as a function of MTBF.



In the baseline sensitivity analysis, MTBF was clearly the dominant driver; therefore we plotted the LCC of the two candidates as a function of MTBF. Looking at the plot, if the true MTBF is as low as 40 % of the baseline MTBF, then clearly candidate 1 is preferred since it has a lower LCC than candidate 2 (low is good). At a true MTBF of 50 percent of the baseline MTBF the gap is closing. At a true MTBF of approx 60 percent of the baseline MTBF the curves cross. From that point on, candidate 2 is preferable. Now it is judgment time. The key question is what will be the most likely true MTBF in the field. The best answer is first make an initial estimate of the MTBF for the baseline analysis. Based on his rationale for the baseline MTBF.

Objective

The objective is to come across the optimal system design and burn-in period, which minimize the total price incurred on the system by customer and manufacturer over its useful life. The useful life of the system is defined as the time over which the system is used to sustain a process which directly or indirectly generates revenues. It is assumed that the useful life of the system begins when the system starts working and hence does not include the burn-in period for the system.



No. of Replacement in time T for non-repairable products

For non-repairable products the sequence of failures with replacements constitute a renewal process and the expected number of replacements in time interval [0,T], M(T), is given by the following renewal equation

$$M(T) = F(T) + \int_0^T M(T-t)\,\mathrm{d}F(t)$$

F(t) is the cumulative failure distribution function.

No. of repairs in time T for repairable products

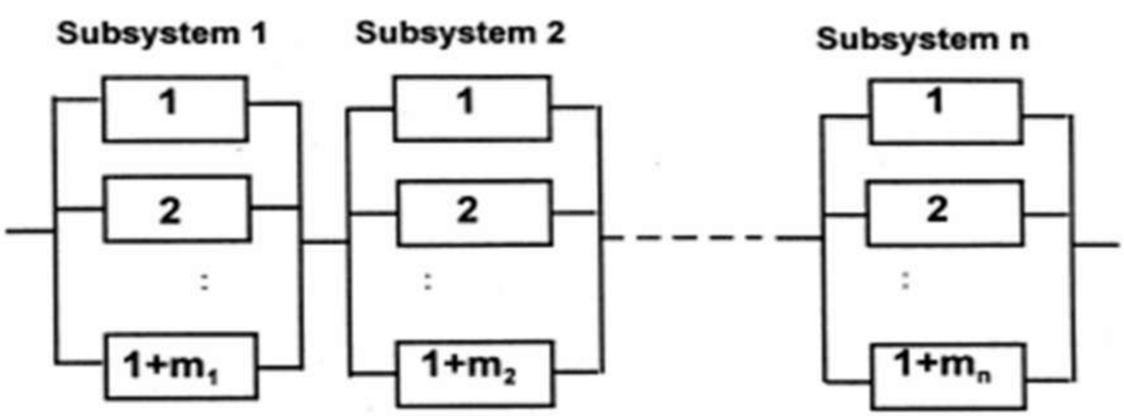
For repairable products assuming that the failure rate of the product remains unchanged , the expected number of repairs in [0, T], E[N(T)], is given by:

$$E[N(T)] = \int_0^T h(t)\,\mathrm{d}t$$

where h(t) is the failure rate function.

System

A series-parallel system is considered comprising of n subsystems in series. Subsystem j (j = 1, . . ., n) consists of (1 + mj) identical components connected in parallel (Fig.).



Manufacturing Cost

The manufacturing cost for components in subsystem j contains four costs [57]: C0j is the manufacturing cost per component without burn-in; C1j is the setup cost of burn-in per component; C2j is the cost per unit time of burn-in per component and C3j is the repair cost of the subsystem per failure during burn-in. Let Vj be the expected manufacturing cost for subsystem j with (1 + mj) components and a burn-in time b

$$V_j = (1 + m_j)(C_{0j} + C_{1j} + C_{2j}b) + C_{3j}\int_0^b h_j(t)\,\mathrm{d}t$$

Warranty Cost: The manufacturer is responsible for all repair or replacement costs during the warranty period [0, w]. For this policy, the expected cost of repair of subsystem j, Wj, during the period [b,b + w] can be written as:

$$W_j = (C_{3j} + C_{4j})\int_0^w h_j(b + t)\,\mathrm{d}t + \gamma$$

where C4j is the additional cost that arises when a failure occurs during the warranty period (e.g. handling and warranty administration cost) and γ is a one time warranty implementation cost independent of number of failures during warranty.

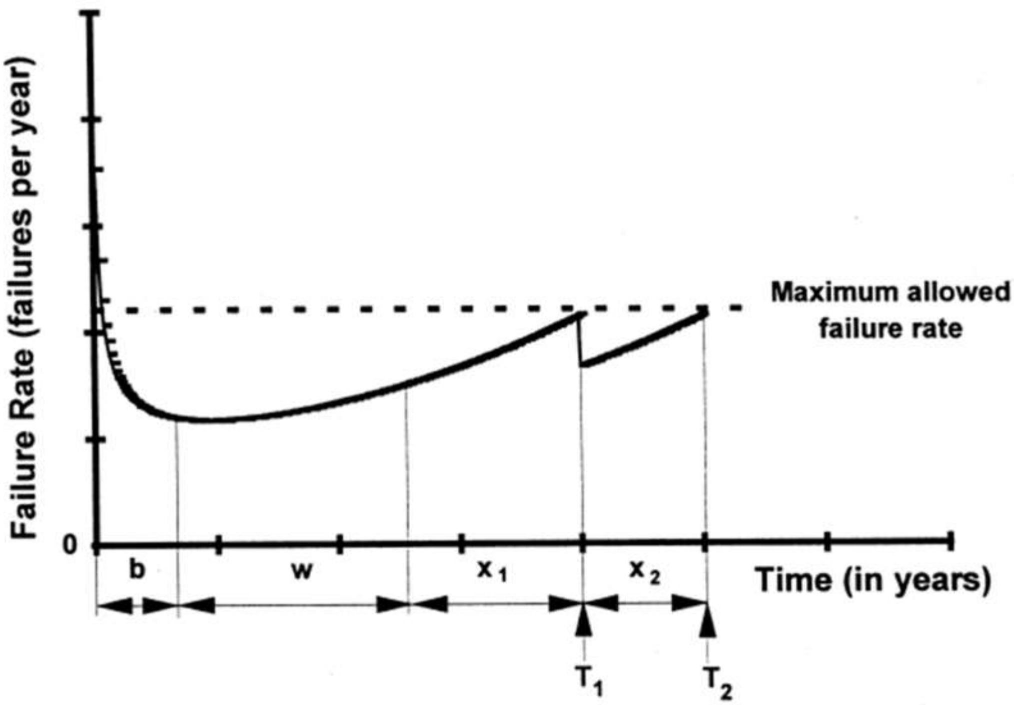
Post Warranty Cost

Post warranty cost is incurred by the customer and includes the cost due to PM and cost of minimal repair after the warranty period. When a PM is performed on the system, all components undergo PM. Thus the cost of a PM action for a subsystem j with (mj+1) components will be MCj(mj+1), where MCj is the cost of a PM action for a component in subsystem j.PM is modelled using the age reduction concept.

According to this concept the PM action reduces the effective age T1 to T1/α, where α is an improvement factor due to PM, such that, 1≤ α ≤ ∞. Assuming that the failure rate function form of the system does not change after PM and the improvement factor is the same for all the components in all the subsystems, then for a given system design, a closed form of Ti (for i ≥ 2) can be expressed as a function of T1

$$T_i = T_1 \sum_{k=0}^{i-1} \left(\frac{\alpha - 1}{\alpha} \right)^k$$

30where, i= 2, 3, 4, ... The PM scheduling and system failure rate after each PM action are shown in Fig



- Objective Function:

$$\mathrm{AAC}_i = \frac{\mathrm{IC} + \sum_{j=1}^{n} (V_j + W_j + \mathrm{PW}_{ji})}{T_i - b}$$

AACi=upto time Ti average annual cost

IC =Installation costs

Vj=Manufacturing Costs

Wj=Warranty Cost of subsystem j

PWji= Post warranty cost of subsystem j

NUMERICAL EXAMPLE

The reliability function for a component in subsystem j is

$$r_j(t) = e^{-\int_0^T h_j(t)\, dt}$$

For failure rate curves, we used β = λ= 1 and k = 0.5. The failure rate curves were developed for different values of b and c. The reliability function used for components in subsystem j is

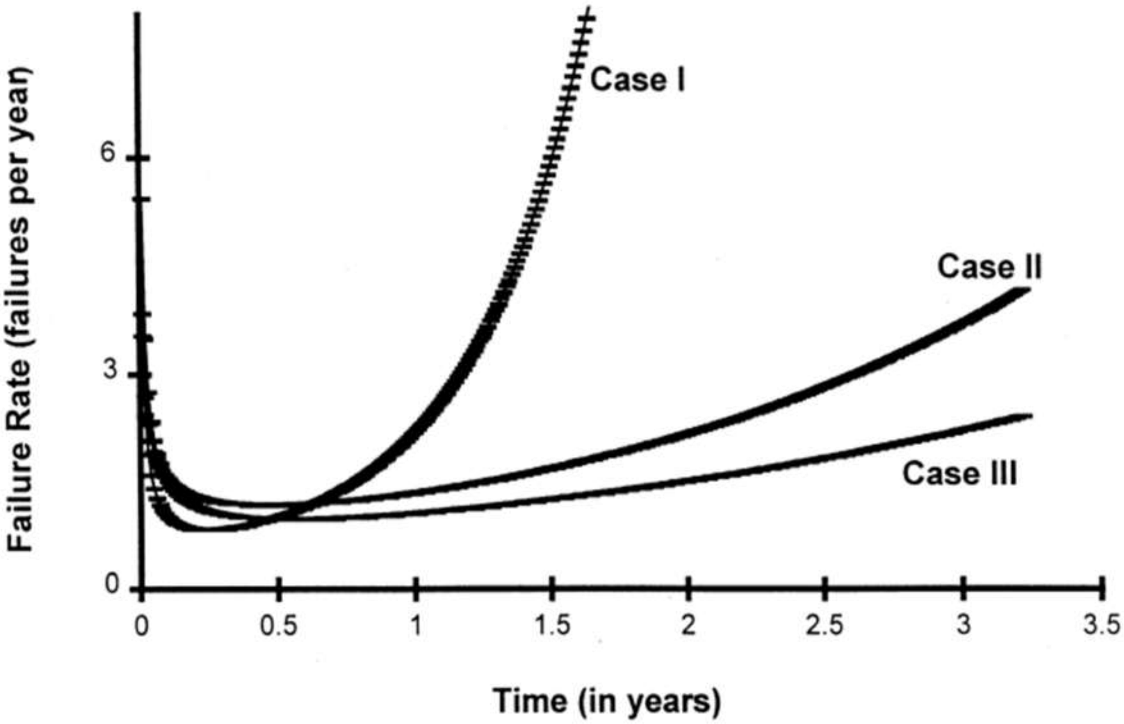
$$r_j(T) = e^{-0.5(T^{c_j} + e^{T^{b_j}} - 1)}$$

The various costs associated with system components are presented in Table below

j	Co	C1	C2	C3	C4	MC	Cm
1	100	1	1	1	10	20	3
2	150	1	1	2	10	20	4
3	200	1	1	2.5	15	20	5

- The installation cost of the system is 400. The one time warranty implementation cost, g, is equal to 10. The maximum allowed failure rate is two failures per year. The maximum number of components allowed in each subsystem is 15. A resource constraint is added with g1=10, g2=15, g3=20 and G = 200. The improvement factor α for all the components is 1.67.

Graph Obtained for each Case



Case I

In this case, the slope of the DFR (decreasing failure rate) region is steep and the length of the DFR region is very short. This is followed by a small period of constant failure rate after which the system deteriorates very quickly. Such characteristics result from a model of a simple and a highly deteriorative system with short product life.

Case II

In this case, the starting failure rate is very high followed by a useful period which has a gradually increasing failure rate and merges with the faster increasing failure rate region. Such characteristics can be related to simple mechanical products which are prone to early failures and gradually deteriorate over time.

Case III

This case represents the system for which the initial failure rate is very high and the system has a distinct DFR region which decreases gradually over a longer period than the last two cases. The system's useful life is long and the system deteriorates very gradually.

Such characteristics model complex systems which have a high failure rate initially, because of the complexity. However, if given a sufficient burn-in period which involves a lot of debugging, the system will perform under the maximum allowed failure rate for a longer time

CONCLUSION

The systems have been economically evaluated for different warranty periods from either a consumer's or manufacturer's perspective. The contribution of this dissertation is to focus on the developments of warranty cost models with various maintenance policies as well as the warranty policy with post warranty periods for single-component and multi-component systems.

Through various types of warranty cost models for, we want to distinguish this study from previous research in the following aspects. More specifically based on the proposed alter- and mixed- quasi-renewal processes, we develop several cost models and also derive reliability measures for various systems.

Warranty cost models are presented based on the quasi-renewal processes and exponential distribution. Cost analyses are conducted for various systems under the basic assumption that a repair service is imperfect. We develop warranty cost models, reliability, and other measures for several systems including multi component systems. We develop cost models by combining both warranty period and post warranty period and then derive the long run expected cost per unit time to find two decision variables including optimized maintenance cycle. The warranty services are separated into repair services and replacement services. Using the two-dimensional NHPP, we determine the threshold level for repair service time. In other words, as for the two kinds of warranty services, repair and replacement, if manufacturers can not finish the repair services within the threshold time, then they will have to provide replacement services instead of repair services to increase customers' satisfaction. So, we use two dimensional NHPP and obtain the expected warranty cost and the variance of the warranty cost.

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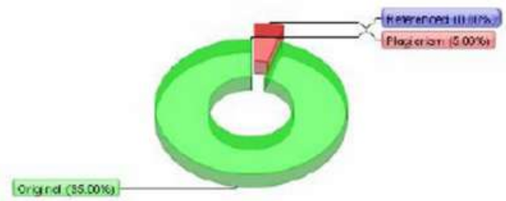

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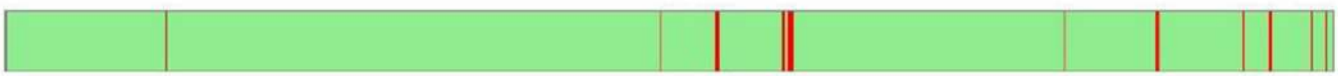

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