RELIABILITY ASSESSMENT FRAMEWORK FOR A MULTI-STATE MULTI-COMPONENT SYSTEM

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by

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APRIL 2018

This work is dedicated to

My Parents & Loved Ones,

&

My Teachers

DEPARTMENT OF MECHANICAL ENGINEERING

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY
JAIPUR, RAJASTHAN (INDIA)

Certificate

This is to certify that the thesis entitled **“Reliability Assessment Framework for a Multi-State Multi-Component System”** is being submitted by **Ms. Niketa Jain (ID: 2013RME9037)** to the Malaviya National Institute of Technology Jaipur for the award of the degree of **Doctor of Philosophy** is a bonafide record of original research work carried out by her under our supervision and guidance. This thesis work, in our opinion, has reached the requisite standard.

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Declaration

I hereby certify that the work which is being presented in this thesis entitled **"Reliability Assessment Framework for a Multi-State Multi-Component System"** in fulfilment of the requirement of **Doctor of Philosophy** and submitted to the Malaviya National Institute of Technology Jaipur is an authentic record of my own work carried out at the Department of Mechanical Engineering under the supervision of **Dr. Ajay Pal Singh Rathore, Professor**, Department of Mechanical Engineering, Malaviya National Institute of Technology Jaipur; **Dr. Rakesh Jain, Professor**, Department of Mechanical Engineering, Malaviya National Institute of Technology Jaipur and **Dr. Om Prakash Yadav, Professor** and **Interim Chair**, Department of Industrial and Manufacturing Engineering, North Dakota State University, Fargo, ND, US. The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree. The content of the thesis has been checked for plagiarism and has been found under acceptable limit.

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Niketa Jain

Abstract

Most of the traditional reliability models consider binary perspective assuming that any system and its components mostly acquire two states: complete functioning or failure. Although, this assumption reduces many complex system modelling problems to a more comprehensible form; it fails to address the real life scenario where most of the systems degrade and undergo several intermediate states before reaching to a complete failure state (Kapur, K., 2006). Several researchers (Liu. Y. et.al. 2013; Li. X. et.al.2015) have proposed approaches to assess the reliability of complex systems with multiple components aligned to a function simultaneously. Each study has focused on distinct parameters to capture the reliability and sustainability of these intricate systems. There have been efforts to study the action of multiple competing failure processes on a multi- component system along with simultaneous impact of shocks acting on the system.

Few of the latest works have also proposed multi-state system reliability models based on their architecture and the behaviour of various components arranged in the system. Most of these studies have assumed individual components to be independent of each other. Apart from this there are very few studies which consider the functional criticality of these components which eventually impact the overall reliability of the system. This creates the need of a comprehensive approach which is capable to handle the functional dependency of components in a multi component system along with keeping their criticality order in its scope.

Another scope of work which literature brings out is that every individual component in a multi component set up does not deteriorate due to a single failure process. In reality, each component experiences damage due to more than one kind of degradation process. Therefore, it generates the need of a comprehensive approach which simultaneously handles the functional criticality order of multiple components in a multi component system along with the effect of multiple competing failure processes acting on each component.

The proposed approach extends the previous research by distinctively taking into account a multi-state system with *n* components. Initially, each component is assumed to be degrading due to a single type of failure process and all the components are functionally dependent on each other. Also, every single component degrades due to a cumulative effect of degradation due to age of the component as well as a correlative degradation effect of other components associated to it.

Degradation of every component from perfect functioning to a lower state is modelled using the Markov process which assumes that the next state of the component depends only on its current state and that time between transitions from existing state to a lower state follows the stationary exponential distributions. A priority order is assigned to every component based on its criticality or functional requirement in the system. This relates to the real life situation where all the components of a complex system are not equally critical for the system to perform its intended function. Each component i is assumed to have (M_i+1) performance levels, where 0 is complete failure and M_i is perfect functioning state. The system follows a gradual and hierarchical pattern of degradation i.e. individual component will degrade from a higher state M_i at any time instant '*t*' to a lower state M_i-1 at another successive time instant '*t+1*'. Further, an instantaneous degradation rate matrix is formed for all critical components to show its deterioration from state i to a lower state j. This degradation rate is used to assess the probability of single component moving to any lower state. A combined approach is given further to show the interactive behaviour of these system components with each other. This approach thus, shall be able to determine the overall system reliability considering the functional and critical dependence of individual components over each other.

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Notations

n	Number of components in the system
θ_r	Repair Cost
φ_r	Replacement Cost
t_k	Transition time instant at k^{th} transition
Λ_x	State transition probability matrix for x^{th} component
C_{mf}	Manufacturing cost of the system
C_w	Total warranty budget for the system
R_v	Reverse Logistics Cost
G_c	Goodwill cost
R	Threshold limit of system's acceptable working efficiency
M_x+1	Discrete levels of functional performance for x^{th} component of the system
M_x	Perfect functioning state for x^{th} component
$M_x - k$	k^{th} degraded functional level for x^{th} component
y_w	Number of components under warranty claim
t_{s_0}	System purchase time
t_{s_1}	Time of first service
t_{s_2}	Time of second service
t_{s_w}	Total warranty time
n_s	Number of subsystems in the system
p_{ij}^{x}	Probability of transition from state i to any lower state j for x^{th} component
G_{M_x}	Sum of all transition probabilities when x^{th} component is in state M
$G_{M_{x-k}}$	Sum of all transition probabilities when x^{th} component has degraded to k^{th}
L	Standard life of system
θ_{r_y}	Repair cost of y^{th} component after time t
φ_{r_y}	Replacement cost of y^{th} component

Abbreviations

ISO	International Organization for Standardization
MSMC	Multi-State Multi-Component
CCF	Common Cause Failure
MCF	Multiple Competing Failures
MSPM	Multi-State Physics Model
CCG	Common Cause Group
PCCF	Probabilistic Common Cause Failure
DCCF	Deterministic Common Cause Failure
CC	Common Cause
COCF	Correlated Common Cause Failure
SSI	Stress-Strength Interference
DCFP	Dependent Competing Failure Processes
MEM	Micro-Electro-Mechanical components
TSTP	Total System Transition Probability
PM	Preventive Maintenance
TCC	Total Cost to Company

Chapter 1
Introduction

1.1. Introduction

"It is imperative in the design process to have a full and complete understanding of how failure is being obviated in order to achieve success. Without fully appreciating how close to failing a new design is, its own designer may not fully understand how and why a design works. Thus the design that succeeds can actually provide less reliable information about how or how not to extrapolate from that design than one that fails. It is this observation that has long motivated reflective designers to study failures even more assiduously than successes." — **Henry Petroski**

Ever since the design industry has started, product failure has been one of the major focus areas for the design community. The rapid change in market dynamics has led to ever-growing customer expectations. This change has given great power to the consumers to choose from many identical and competing products that exist in the market. Customers are willing to pay the high price only if they get assurance of a high quality product that will perform satisfactorily over the useful life of the product (Murthy, 2007). In order to meet such challenging customer expectations in terms of product performance, it becomes necessary for the design engineers to locate prospective areas of product failure and redesign to manufacture a product with high reliability (Bhamare, Yadav, & Rathore, 2007).

Reliability is a broad term used to define the ability of a product to perform its intended function, under given environmental and operation conditions, for a stated period of time (ISO 8402). The term 'product' here could mean any component, sub-system or system under consideration.

The need for reliable products was first sensed in both commercial and military sectors in early 1950s. Since then enormous progress has been made in the area of reliability engineering. Before 1950s, the focus was either on quality control or on machine maintenance problems. Literature suggests that before World War II reliability was intuitive in nature and the basic concept of reliability was born during this time period (Bhamare et al., 2007).

A good and reliable design is capable of estimating the inherent reliability of a product or process and pinpoint potential areas for reliability improvement (Rausand and Hoyland, 2004). It is a strenuous job for design engineers to eliminate all failures from a design, therefore, it becomes necessary to identify the *'most likely'* stages of failure in a product life-cycle and take appropriate actions to mitigate the effects of those failures (Huang & Askin, 2004; Kamrad et al., 2005; Kapur, 2006; Minderhoud, 1999). Realistically, all failures cannot be eliminated from a design, so another goal of reliability assessment is to identify the critical causes that may lead to failure of system or product. These causes are termed as *'critical-to-failure'* stages and are very important in order to assess accurate product reliability (Wang et al., 2013; Wang et al., 2011).

The aim of this study is to design and develop a methodology to estimate the life cycle of any product or system based on its design and architecture. Predominantly, the proposed study helps the designers to identify the *'critical-to-failure'* stages and take appropriate actions to increase the product life. The proposed methodology serves as a foundation for development of a structured approach for reliability assessment of a system with complex system architecture and further use the obtained results to determine service time selection schedules.

1.2. Overview and Motivation:

With an increase in the design complexity and number of components & sub-systems, the system reliability assessment also get complex. The tedious enumeration process, existence of different states of the system and difficulty in uncertainty analysis of complex systems are some of the issues that make system reliability analysis problems complex (Bhamare et al., 2007).

In conventional reliability assessment methods, the performance levels of systems have been classified into two categories: *perfect function* or *complete failure* (Kapur and Lamberson, 1977; Hoyland and Rausand, 1994). Although this assumption reduces the effort to model and estimate complex systems' reliability, it fails to address the realistic scenario where nearly all multi component systems degrade progressively and operate at wide range of performance levels known as '*states*' before they reach a state of complete failure. This assumption directly impacts the evaluation of actual system condition and performance level at a definite time instant. Majority of the existing models have computed system reliability at a holistic level but fail to consider the interactions at component and sub-system levels (Liu & Kapur, 2008; Summers, A. E., Ford, K., & Raney, 2007; C. Wang, Xing, & Levitin, 2014; Liudong Xing & Amari, 2008). Hence, there is significant need for an effective reliability assessment model which incorporates different functional stages at both systems as well as at component level. Figure 1.1 shows the block diagram for a multi-component system with each component following different performance function.

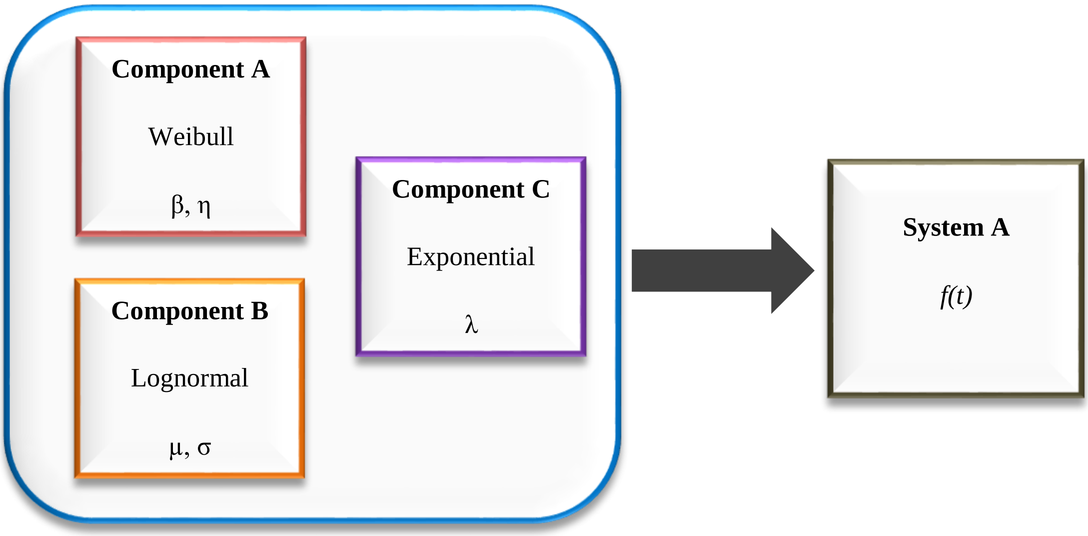

Figure 1.1: Block diagram for multi-component system reliability

The scale of complexity goes up for multi-state systems with an increase in number of components. Such multi-state systems with multiple number of components are known as multi-state multi-component or MSMC systems. It becomes extremely difficult to trace the root causes of failure for MSMC systems. The identification of root causes of a system failure helps in improving its design and reliability along with better customer experience. Therefore, it is important to formulate an approach that not only comprehends the multi-state functioning of any system and its components but also helps to identify the degraded or failed component. Once these degraded or failed components are identified, they can be repaired or replaced depending on their condition (Wang et al., 2014)

In MSMC system reliability, the primary focus of reported models is confined to the '*type*' of failure processes that cause system deterioration (Mitra, Saxena, & McCluskey, 2000; Summers, A. E., Ford, K., & Raney, 2007; C. Wang et al., 2014; L

Xing, Boddu, Sun, & Wang, 2010). Such models fail to examine the contribution of individual component to enable better system performance. This limitation creates potential scope for an approach which addresses both the dependent behavior of system components as well as the priority of every component in the system hierarchy. The dependent nature of every component plays a vital role for complex systems, where physical association between the components may cause faster propagation of degradation of each component or in few cases may even lead to system failure. For instance, consider a laptop where degradation of battery shall have a negative and high impact on the overall system performance, whereas the breakage of plastic body will not strongly impact the function of system and its associated components. The proposed methodology focuses on such functional dependency of mutliple components in a multi-state system environment.

The proposed study focuses on assessing system reliability assuming that any system and its associated components do not perform at same levels and different state transitions of individual component cumulatively constitutes the system state. The proposed approach has an added advantage over other approaches as it includes the critical importance of every component in a system hierarchy and thus enables to analyse the effect of degradation of critical components in the overall system reliability. The final and key aspect of the model is its ability to investigate the dependent nature of each component thus giving a more precise system deterioration probability. This is achieved by not only estimating the effect of independent degradation of any component but also taking into consideration the impact of other components associated with it. The dependence among components is captured by categorising the components in various sub-systems based on their critical hierarchy order. The following section discusses the purpose and objective of this study.

1.3. Research Objectives:

Literature review shows that majority of the research work done in the field of reliability assessment of complex system has focused either on independent nature of components or dependent nature through capturing *s*-dependence among the failure processes acting on the system. Mostly, the nature of estimation is generic having more focus on reliability assessment at system level and type of failure acting on the same. Additionally, there are no reported work on MSMC system reliability assessment which focus on the critical ranking of components in the system hierarchy. Moreover, very few researchers have estimated system reliability considering dependent nature of individual components. Thus, there is an acute necessity for an integrated methodology in the system reliability domain which is capable of evaluating MSMC system failure at root levels of any system.

As highlighted earlier MSMC system reliability assessment can be achieved at realistic level only when both physical and functional association of individual components in the system are taken into consideration (Jain et al., 2017; Lin et al., 2016; Xing et al., 2014). This study acknowledges that system reliability for any complex level may vary depending on intermediate performance states of system and its components and the deterioration of *'critical-to-failure'* components may lead to an accelerated degradation of the system.

The main aim of this study can be stated as, '**To develop a model for multi-state multi-component system (MSMC) reliability assessment with simultaneous focus on differing states of performance of any complex system and its components.'** The problem has been broken into following three specific research objectives:

i. To develop a model for reliability assessment of a multi-state multi-component (MSMC) system considering the criticality order and the dependent nature of individual components of the system.
ii. To identify components leading to accelerated deterioration of system reliability using transition state probabilities of components.
iii. To develop a preventive maintenance and warranty framework using the proposed MSMC system reliability assessment model.

The findings of the current research on will be helpful for researchers and practitioners to carry out a more effective and realistic assessment of multi-state multi-component systems. A brief discussion of the objectives of the study and methodologies used to tackle these objectives is given in the next section.

1.4. Research Methodology:

The aim of this section is to explicate the research methodology adopted in the present study. It describes the research plan, mathematical modeling, solution approach, numerical tests, and other procedures that are appropriate for achieving the research objectives mentioned in the previous section. The overall research plan is illustrated in Figure 1.2. An extant literature pertaining to fundamentals of multi-state multi-component system reliability is explored.

This study of literature helps in conceptual understanding of a multi-component system's performance at intermediate stages before it reaches a complete failure state. Secondly, the methodologies for estimating reliability of a MSMC system are identified. Based on the other issues synthesized from the extant literature regarding MSMC system reliability, dependent and critical nature of components and identified methodologies for modeling and assessing MSMC system reliability, an integrated mathematical model is proposed and demonstrated.

The second chapter of literature review helps in building up an understanding of the fundamentals of multi-state system reliability, causes of system failure, independent and dependent behaviour of system components and the important issues regarding multi-state multi-component system reliability assessment. This further helped in identification of research gaps and research objectives. Figure 1.2 explains the steps of research approach followed in this study:

Setting objective and scope: In this step, research objectives are set on the basis of the findings of literature review and the research gaps. For this purpose, investigations related to MSMC system reliability assessment along with component dependency and component criticality are carried out.

Development of MSMC system reliability assessment model: The study proposes a a model of a MSMC system reliability assessment with simultaneous consideration of component interaction and functional criticality order of each component in the system. The dependence among components is captured by categorising the components in various sub-systems based on their critical hierarchy order. Appropriate solution methods and resources are identified and employed to get the solutions of the proposed model using MATLAB environment.

Action Plan for Preventive Maintenance and Warranty Decisions: A suitable model to assess MSMC system reliability is identified at initial level which is further extended to accommodate the managerial application of MSMC system reliability assessment. For this purpose, a preventive maintenance framework is developed to identify suitable time of service for various parts of a MSMC system.

Case study: A case study adopted from the work done by González Díaz et al., (2010) is presented to demonstrate the application of the proposed MSMC reliability assessment model. This complete research plan is provided in Figure 1.2.

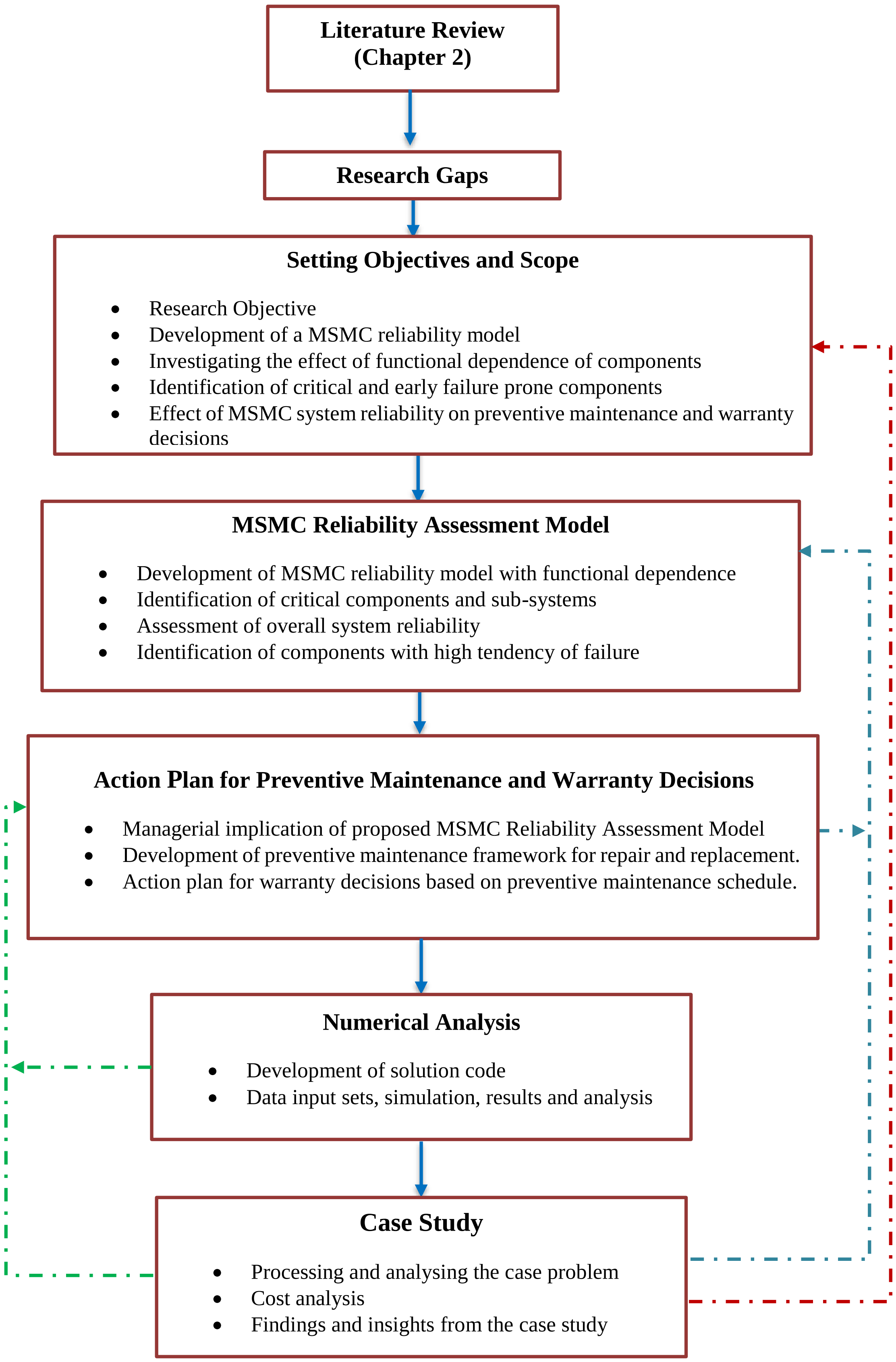

Figure 1.2: Research Plan

1.5. Thesis Outline

The thesis is organised into six chapters. Chapter 1 highlights the aims and objectives of the research along with the research methodology followed in this study.

Chapter 2 presents a comprehensive literature review of system reliability and system failure causes. The main contribution of this chapter is that it critically analysed different reliability assessment methods for a multi-state multi-component (MSMC) system and various failure causes that lead to system failure. This critical analysis of reliability assessment methods for MSMC systems and failure causes leading to system breakdown is done is done by following a structured literature review process. In this approach used for literature review, appropriate research articles are located, identified, collected and analysed in a systematic manner.

Chapter 3 presents the methodology used to model the functional dependence of individual components in a multi-state multi-component environment and hence assess the total system reliability based on the independent and dependent nature of system's components. To perform this, the chapter illustrates appropriate mathematical modeling paradigm for handling the component interaction with the other associated components. The developed mathematical problem is technically computed in a MATLAB environment using a set of test problem data for new product development.

Chapter 4 aims at giving the practical application of the proposed mathematical model for preventive maintenance and warranty decisions. A structured framework is proposed to relate the financial aspect of warranty and preventive maintenance to service time selection decisions. This chapter concludes by providing the integration of the developed MSMC system reliability assessment model to design a preventive maintenance framework and use the obtained service time selection decisions for better warranty policy formulation.

Chapter 5 presents the case study of an automotive manufacturing company operating worldwide. An example of existing product is used to understand the application of the model and framework proposed in chapter 3 and chapter 4. It is established with the case study that the proposed approach is an effective method for evaluating the reliability of a complex system along with determination of faulty components for which early repairs or replacements are required. The chapter also throws some light on how an early determination of deteriorating components helps the company to save the cost related to warranty decisions.

Chapter 6 concludes the thesis with a discussion of the contributions made, limitations of the present research as well as the future research directions.

Chapter 2
Literature Review

2.1. Introduction

Reliability analysis considering multiple possible states is known as multi-state (MS) reliability analysis. Multi-state system reliability models allow both the system and its components to assume more than two levels of performance. Through multi-state reliability models provide more realistic and more precise representations of engineering systems, they are much more complex and present major difficulties in system definition and performance evaluation. MSS reliability has received a substantial amount of attention in the past four decades.

This chapter provides a state-of-the-art review of system reliability and failure causes. The purpose of this chapter is twofold; first to perform a critical analysis of failure causes and reliability assessment of a multi-state system and second, to identify research gaps from the findings of literature review. For this purpose, a structured literature review approach as shown in Figure 2.1 based on Prakash, Soni, & Rathore (2017); Tranfield, Denyer, & Smart (2003) and Webster & Watson (2002) has been adopted.

The chapter is organised in four sections: Section 2.2 presents the methodology followed to conduct the literature review process. It explains the procedure followed for article selection and content classification from the available online database. Section 2.3 provides a summary of the significant research contributions in multi-state system reliability domain. Section 2.4 focuses on the research gaps derived from the literature review.

2.2. Review Methodology:

In this section, the issues of time horizon of review, journal selection, article selection, article classification and analysis of articles obtained from the extant literature will be discussed. Article selection and content analysis is broadly divided into three stages of (i) *Article Sampling*; (ii) *Content Diversification* and; (iii) *Synthesis* as shown in Figure 2.1.

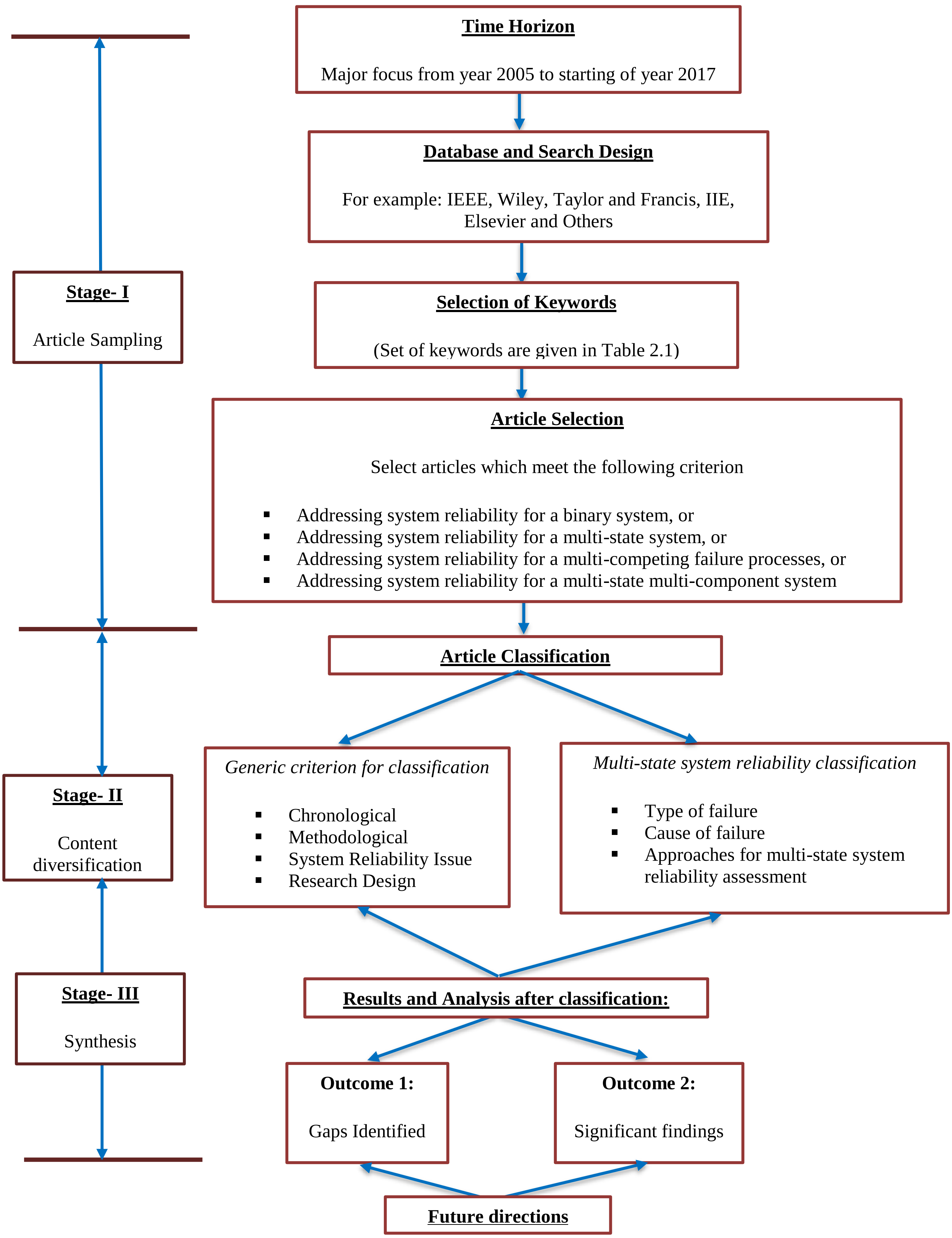

Figure 2.1: The stages and steps followed to conduct literature review and content analysis (adopted from Prakash et al., 2017)

2.2.1. Stage-I: Locating and Sampling of articles

The initiation of system reliability as separate field of reliability engineering has started in the late 70's. The available literature shows that significant work on multi-state system reliability began in the mid 80's (Kyung C. Chae & Clark, 1986; Thomas, 1986). A period from year 2005 to early months of 2017 is chosen for collecting literature using a broad set of primary and secondary keywords as given in Table 2.1. Most of the academic online database or publishers such as Elsevier, Science Direct, Springer, IEEE, Taylor and Francis, Wiley, IIE etc. were explored to collect relevant journal articles and conference proceedings.

Table 2.1: Keywords used for searching articles

S. No.	Constructs	Keywords
1	System Reliability	Binary; Multi-state; Multi-component; Multi-state Multi-component (MSMC);
2	Failure Type	Common Cause Failure (CCF); Correlated Failure; Multiple Competing Failure (MCF);
3	Solving Technique	Markov; Copula; Analytic hierarchy process;
4	Warranty & Maintenance	Warranty Cost; Reliability; Warranty Policy; Maintenance; New Products

An initial pool of 861 research articles was searched from the available online database for system reliability assessment. A systematic selection process is then followed for elimination and scrutiny of obtained research articles from the online database. Figure 2.2 shows a two-step screening process for article selection. The first level is the preliminary stage where articles are selected based on their relevance in title and abstract. The next level involves a detailed filtering based on the review of abstract and conclusion.

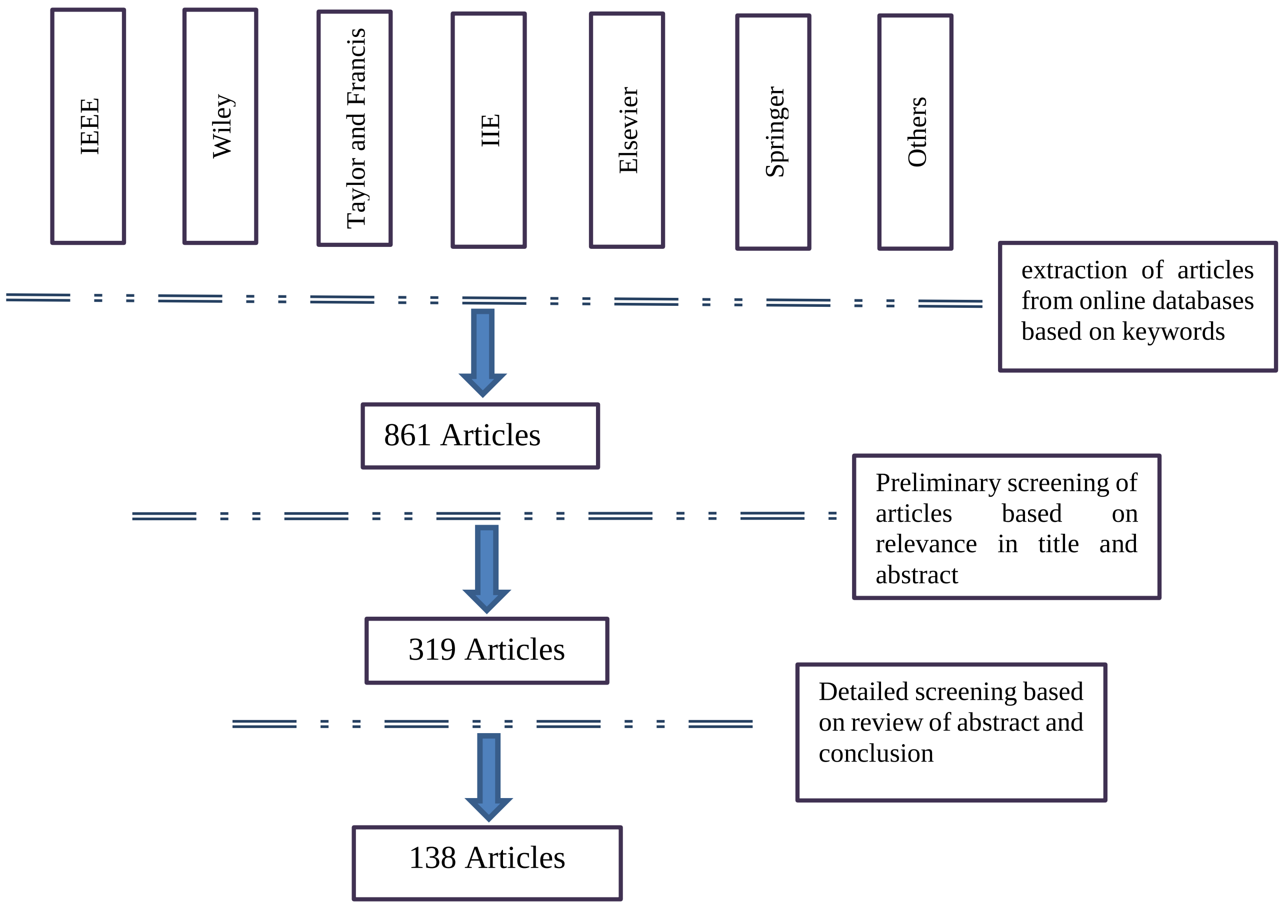

Figure 2.2: Scrutiny process for selection of articles

After the detail screening, a total of 138 articles were obtained for content analysis, diversification and synthesis.

2.2.2. Stage-II: Content Diversification

The objective of second stage is to categorise obtained 138 articles using a two-fold classification schemes proposed by Badhotiya et al. (2017). The two-fold classification scheme are:

- *Generic Classification*
- *Content-based classification*

Generic Classification: the articles are classified on the basis of generic factors such as chronological, methodological, research design etc. This scheme categorises the articles according to the research approach employed based on three constructs viz. publication type, year and publishing outlet. A total of 117 peer reviewed journals and 21 peer reviewed conferences are analysed for content diversification and synthesis. The graphs shown in Figure 2.3 and Figure 2.4 demonstrates the increasing trend and growth of research articles in the field of system reliability over the last four decades.

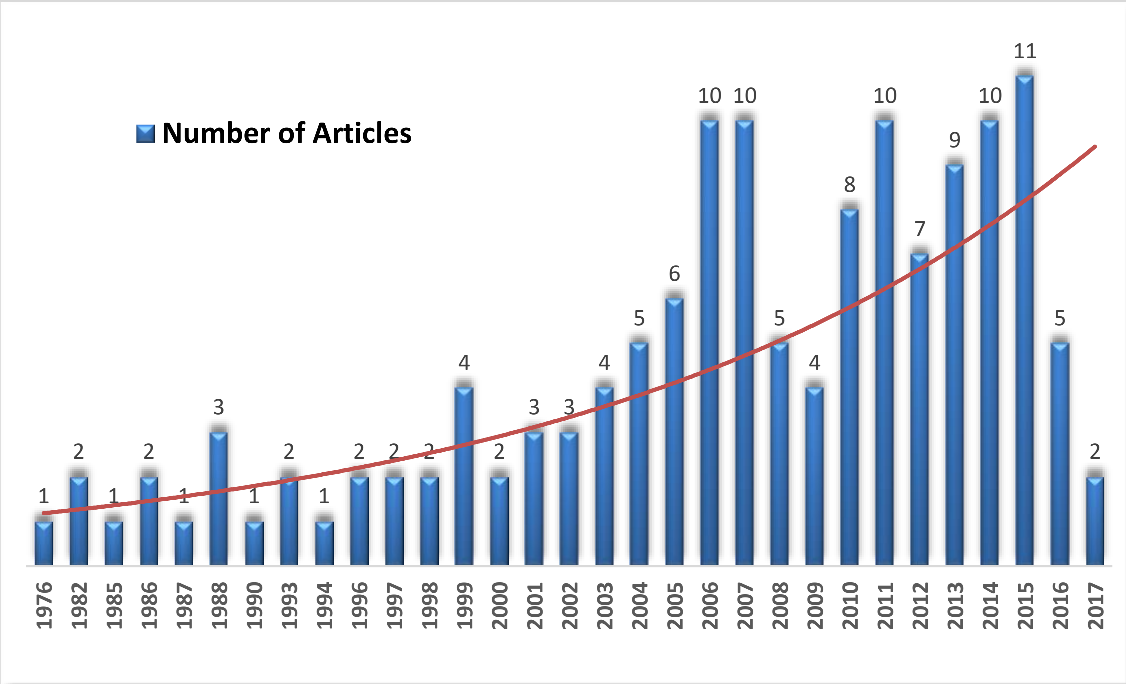

Figure 2.3: Distribution of articles over the years

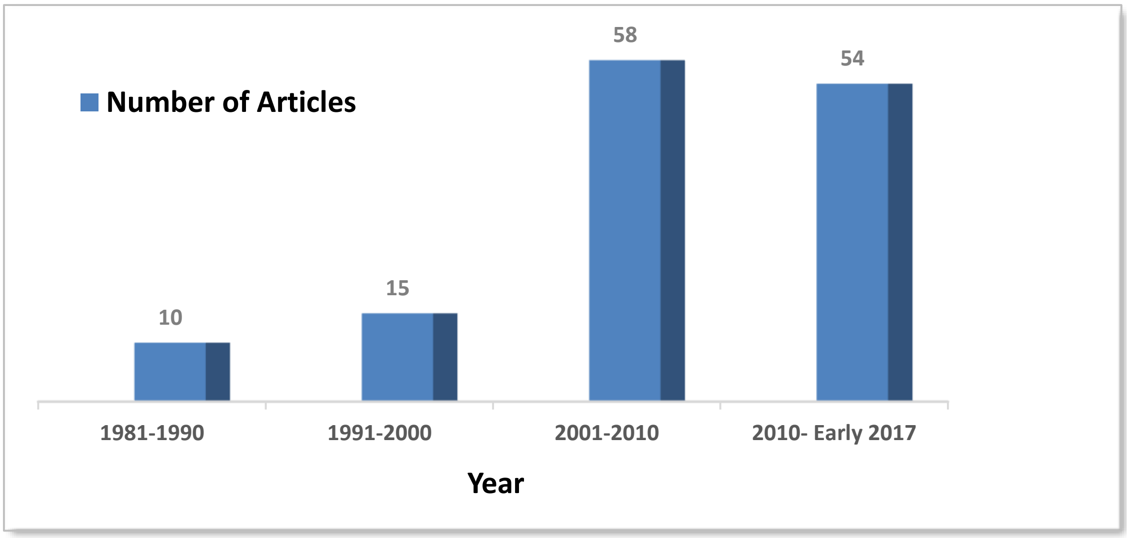

Figure 2.4: Growth of articles over last four decades

Figure 2.5 exhibits the of distribution of research articles based on their publishing outlet.

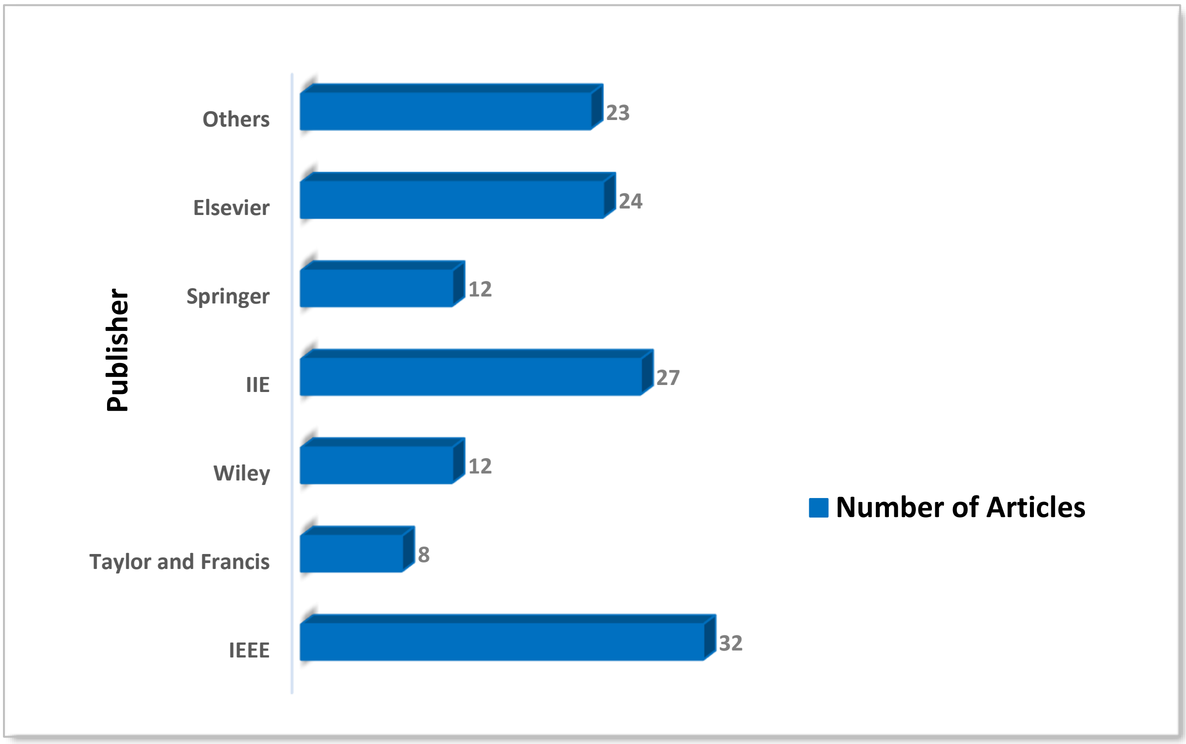

Figure 2.5: Distribution of articles across Publishers

***Content-based classification*:** The articles in this section are categorised as per the multi-state system reliability issue addressed in the article such as type of failure, cause of failure, number of components, criticality order etc. (Figure 2.6).

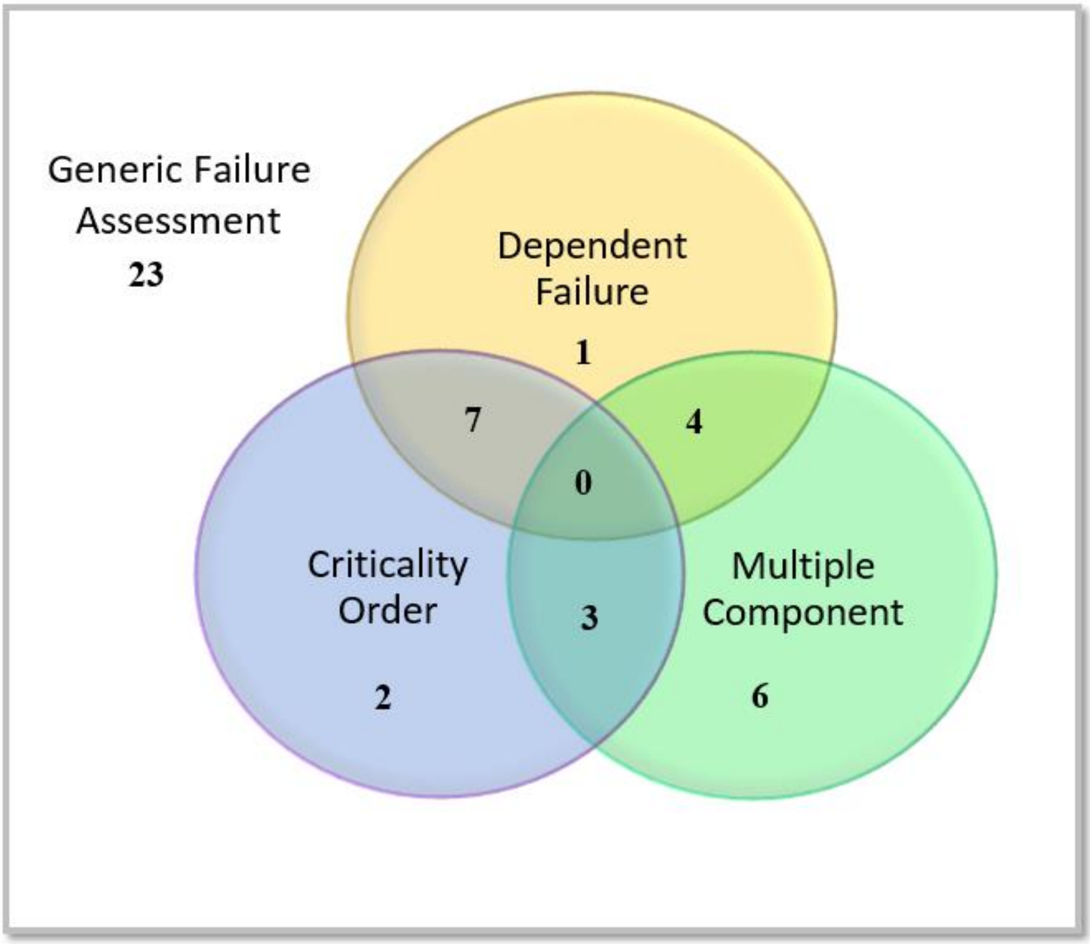

Figure 2.6: Contextual classification of multi-state system reliability

2.2.3. Stage III: Synthesis:

In this stage, the information extracted from review of 138 articles is analysed and synthesized to identify the growth, transition pattern and research gaps for multi-state system multi-component system reliability domain.

2.3. Literature Analysis and Findings:

Several key issues were identified regarding multi-state system reliability assessment based on comprehensive review of extant literature. The articles that helped to answer the following questions were selected to identify research gaps and research objectives:

- *Why is it important to consider multiple performance states of any system rather than assuming it to function at only two sates: 0 and 1?*
- *How does a correlation of components in any multi-state system influence the overall system reliability?*

- *Why it is important to consider the critical functionality of each component in a system and what is its contribution in system reliability evaluation?*
- *How the dependent nature of one component on the other influences the degradation of the system at a holistic level?*
- *Lastly, what are the practical applications and advantages of analysing system reliability by including the critical, correlated and dependent nature of components?*

An intensive review of over 138 research articles from peer reviewed journals and conference proceedings was carried out to explore issues regarding evaluation of multi-state multi-component system reliability.

2.3.1. System reliability assessment based on different failure causes:

In multi-state system reliability, failure of any system and its components can be categorized in two groups depending on the failure cause:

- ***Type 1:*** *failure due to a shared root cause or common cause.*
- ***Type 2:*** failure due to correlation between components.

2.3.1.1. Common Cause Failure:

The type I failure models are referred as Common Cause Failure models (CCFs). The reliability of any system or component is highly compromised by the presence of CCFs, as they significantly contribute in increasing the overall joint failure probabilities (Mitra et al., 2000; Summers, A. E., Ford, K., & Raney, 2007; C. Wang et al., 2014). Therefore, it is important to determine the effect of these common cause failures on system reliability.

The origination of Common Cause Failure is induced by both external as well as internal causes. A Common Cause Group (CCG) is formed consisting of components that have the same cause of failure. These CCFs may have either a deterministic or a probabilistic effect on their respective CCG. The failure of different components in a common cause group, which occur with different occurrence probabilities, can be categorized under a Probabilistic Common Cause Failure Group (PCCF). Similarly, the failure of components occurring with an assured failure probability falls under the

Deterministic Common Cause Failure Group (DCCF) (Liudong Xing, 2007; Liudong Xing & Wendai Wang, 2008). Considerable research efforts have been devoted to assess the effect of PCCFs and DCCFs on system reliability (Dai, Xie, Poh, & Ng, 2004; Vaurio, 2001; Liudong Xing, 2007; Liudong Xing & Wendai Wang, 2008).

The existing CCGs methods can further be classified into implicit and explicit approaches based on their solution methods (Figure 2.7). The explicit approach models the Common Cause (CC) occurrence as a 'basic event' assuming that all the system components are affected by the same common cause. Specifically, the explicit approach divides the system and its components into different CCG's depending on the type of common cause (CC) affecting them and gives an expanded model to estimate the total failure probability of the system (Dai et al., 2004; Vaurio, 2001). The implicit model evaluates the system without considering the effect of DCCFs at initial stage. Posterior, the contribution of DCCFs on the system is modeled using Universal Generating Function techniques (Tang & Dugan, 2004; Vaurio, 1998, 2001).

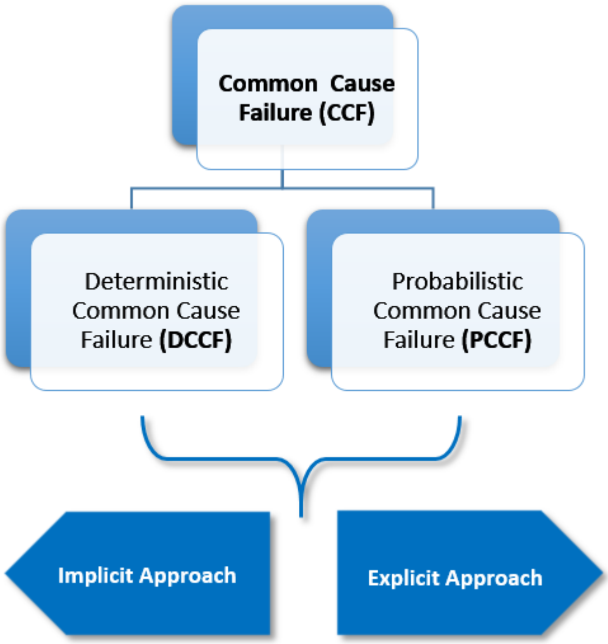

Figure 2.7: Classification of Common Cause Failure

Chae (1988) proposed a binomial failure rate model to handle PCCFs, where the occurrence of CC has a constant probability. However, the scope of this study is limited to systems with *s*-identical and *s*-independent components. Xing & Wang, (2008) suggested a more generalized method to analyse CCF occurrences where the system comprises of distinct components and each component has a non-identical component failure probability. The model is restricted to the conditional failure of various components which are s-independent. Moreover, these failures, originating due to the same common-cause only, consider the cases of external cause. An extension of this work was undertaken by Wang et al. (2014) investigating the impact of both external and internal PCCFs on system reliability. This study inspects both explicit and implicit methods thus offering a distinct advantage over existing models by allowing the component to be a part to multiple PCCFs each having a different failure probability. To structurally represent the system and give a description of PCCF behaviour, Xing et al. (2010) used fault tree and PCCF gates. The explicit and implicit methods developed by (Wang, Xing, & Levitin (2013) have no limitation on the time-to-failure distributions. These methods are illustrated by using three different distributions namely fixed probability, exponential distribution with constant failure rate and Weibull distribution.

The studies proposed using explicit and implicit methods for evaluating PCCFs have certain limitations in their solution approach. Both of these methods are confined to time-to-failure distributions that are complex in nature and hence ceasing their scope for PCCFs with failure cascading and loops. The explicit method is more simplistic in nature as it can handle both internal and external common causes using the same process. However, the computation for large scale systems becomes inefficient and in such cases, the implicit methods are more efficient since these methods are capable of handling various statistical relationships among common causes (C. Wang et al., 2014). These statistical relationships can be mutually exclusive, s-independent or s-dependent. Applications of both methods and effects of PCCFs on the system reliability are illustrated by detailed analyses of a suitable example in which systems are subjected to PCCFs (Fiondella, 2010). An extended study of PCCFs in systems with cascading failures or loops is available in (Fiondella & Gokhale, (2010). Also PCCFs in phased-mission systems involving multiple, consecutive, and non-overlapping phases of operations are studied in Liudong Xing, (2007). Wang et al. (2013) has considered the effect of PCCFs in competing analysis.

Apart from the CCF models, which are built from cause and effect analysis of any component and associated system, another extended classification in this field is done based on the failure correlation between more than one component. In many cases, multiple components in a system are subjected to failure at 'common time instant' and require modeling of simultaneous failure of such correlated components. The following paragraphs provide a brief discussion on such failure models with the ability to evaluate correlated failure processes or components for a multi-component system.

2.3.1.2. Correlated Common Cause Failure:

Modeling a failure caused by correlated components for a multi-component system is yet another important and vital aspect of system reliability. Evaluation of correlated common cause failure (COCF) becomes all the more necessary in cases where simultaneous functioning of multiple components and higher reliability are of prime importance and ignorance of which may have inimical effects on the system performance. Consequently, proposed models and techniques play a key role in handling the critical issue of risk imposed on multiple components due to correlation.

Barlow and Porschan (1975) have summarised many distinct statistical and probabilistic approaches to model correlated failures, which can be used to further assess the system reliability. The application of statistical multi-variate distributions is a common practice used in most of the elementary studies to understand the correlated failure phenomenon (Kotz et al., 2004; Lai and Xie, 2006; Singpurwalla, 2006). Another early study by Dhillon and Singh (1991) used the Markov method to analyse correlated failures in repairable and non-repairable systems. Few researchers have used implicit methods to investigate components subject to correlated failures with different life distributions (Chae & Clark, 1986; Liu & Kapur, 2008). Others have used multivariate Bernoulli distributions for reliability and sensitivity analysis of coherent system considering component reliability as a vector function of correlated success probabilities (Fiondella, 2010; Modarres, 2011). Fiondella & Gokhale (2010) proposed system reliability estimation with correlated components using Taylor series approximation methods. In this approach, system reliability with multivariate Bernoulli distribution has been converted to a multivariate normal distribution. In most of these techniques, a 2^n correlation parameters are used for a system with n components. As the system complexity increases with an increase in the number of components, it becomes even more difficult to estimate and compute the parameters of these models. Hence, model simplicity along with its performance and scalability are compromised to a great extent. On the other hand, the COCF models (multivariate Bernoulli distributions) use explicit technique with n^2 correlation parameters thus making it simpler than the CCF models.

Although a fair amount of research has been conducted to address the correlation between components, there is still a need for an approach that is capable of handling the critical importance of each component in MSMC system architecture. Most of the above mentioned research work has assumed the system and their components to work at the same levels of performance, which does not represent realistic scenario. In reality, the performance levels of all the components cumulatively define the state of the system.

The existing models focus more on modelling the *s*-dependence between the types of failure processes acting on the system. For a more realistic case, it is very important to analyse the system reliability from the perspective of component dependence where degradation of any individual component not only affects the component under consideration but also affects the other functionally related components of the system.

The proposed methodology is an attempt to address the identified limitations by focusing more on a realistic scenario of system failure and taking into account both critical hierarchy as well as dependency among components of a MSMC system.

2.3.2. Multiple Competing Failure Process:

Most of the conventional system reliability models work with a binary assumption of the system being either in completely functional state or in a failed state. However, in most of the situations every system has more than a single performance level. Such systems provide the intended function even at a degraded level. Hence, in analysing the reliability of any system or component it becomes very critical to assess and model them at various functional levels. A multi-state system reliability method gives a more pragmatic approach to handle such cases (Ramirez-Marquez et al., 2006). An extended challenge of multi-state systems is that more than a single failure process might be responsible for system degradation. For instance, most of the mechanical systems undergo a simultaneous action of fatigue, corrosion, wear etc. In some cases, these failure actions are competing against each other. These competing failure processes can be either dependent or independent.

An early research shows that an entire field has been dedicated to model such multiple competing failure processes. A considerable amount of effort has been made to accord with systems experiencing Multiple Independent Competing Failure Modes. (Huang & Askin, 2004) gave generalized stress-strength interference (SSI) reliability model to analyse the independent action of degradation and catastrophic failure on the system. An optimal replacement policy is proposed by Wang & Zhang (2005) for a system exposed to two type of random shock processes. Here, a δ-shock model and an extreme shock model describe the two processes. Li & Pham (2005) investigated system reliability under a scenario of simultaneous action of two degradation processes and a random shock process. An extended optimal replacement model is proffered by Chien et al. (2006) where systems experiencing the action of two random shock are studied. The model assumes that the primary failure can be fixed through repair whereas the secondary fatal failure needs a replacement action. Keedy & Feng (2012) suggested a probabilistic reliability and maintenance model for delayed and instantaneous failure modes. The action of each failure mode is assumed to be independent in nature.

A recent development in the field of multiple competing failure modes has come up giving a generalized term known as Multiple DCFPs (Multiple Dependent Competing Failure Processes). These approaches capture the dependent nature and simultaneous action of more than one failure process. Wang & Pham (2011) have given a preventive maintenance policy for systems exposed to dependent competing risks. The study considers the action of degradation wear along with two kinds of random shocks: (i) catastrophic shocks that may cause an abrupt system failure and (ii) non-catastrophic shocks that may lead to a sudden increase in the degradation level of the system. Wang et al. (2011) assessed system reliability under the simultaneous action of three-failure processes viz. fatal failure, failure due to shock and degradation. Huynh et al. (2012) provided with an age-based repair and maintenance model for systems exposed to two dependent competing failure processes. The system failure is derived from the coeval action of degradation and catastrophic shock process. Peng et al. (2010) introduced a maintenance and reliability model for two correlated failure processes. The continuous action of degradation was categorized under the term '*soft failure*' process and failure resulting due the action of fatal random shock was termed as '*hard failure*' process. Jiang et al. (2012) extended the above study by suggesting a hard failure with shifting threshold level. This shift in threshold limit was accounted due to the system exposure to various types of shock patterns. Song et al. (2014) developed maintenance model for a degrading system experiencing more than a single and sudden failure mode. A failure rate parameter has been introduced whose variation is dependent on the degradation level of the system, age of the system or both Rafiee et al. (2014).

A degradation process, mostly continuous in nature, for any system or component depends majorly on its application in different fields. Most of the Micro-Electro-Mechanical components (MEM's) experience degradation due to wear process. However, in some cases there might be an accelerated degradation due to external factors. Many of the mechanical systems also involve sudden jumps in the degradation process due to the action of random shocks along with a continuous fatigue process. An extensive literature study reveals that failure of any system may occur either due to soft failure or due to hard failure.

A Hard Failure Process can be accounted because of Fatal Shocks causing a catastrophic failure of the system (sudden death) whereas a Soft Failure Process is a result of both continuous degradation and occurrence of random shocks (non-fatal shocks; causing slow death to the system). Typically, there are five different types of random shock models: (i) extreme shock model: system failure occurs when the shock size exceeds beyond a threshold value; (ii) cumulative shock model: associated to system failure in which the cumulative damage accumulation due to shock goes beyond a critical; (iii) m-shock model: subjected to occurrence of system failure after 'm' number of shocks and this value has to be greater than a critical level; (iv) run shock model: failure of system is caused due to '*n*' successive run of shocks whose value crosses a threshold value; (v) δ-shock model: system failure occurs when arrival time between two sequential shocks is less than a threshold δ (Nakagawa, 2007; Liu et al., 2008)

Based on the existing literature, system failure can be subjected to three conditions as shown in Figure 2.8:

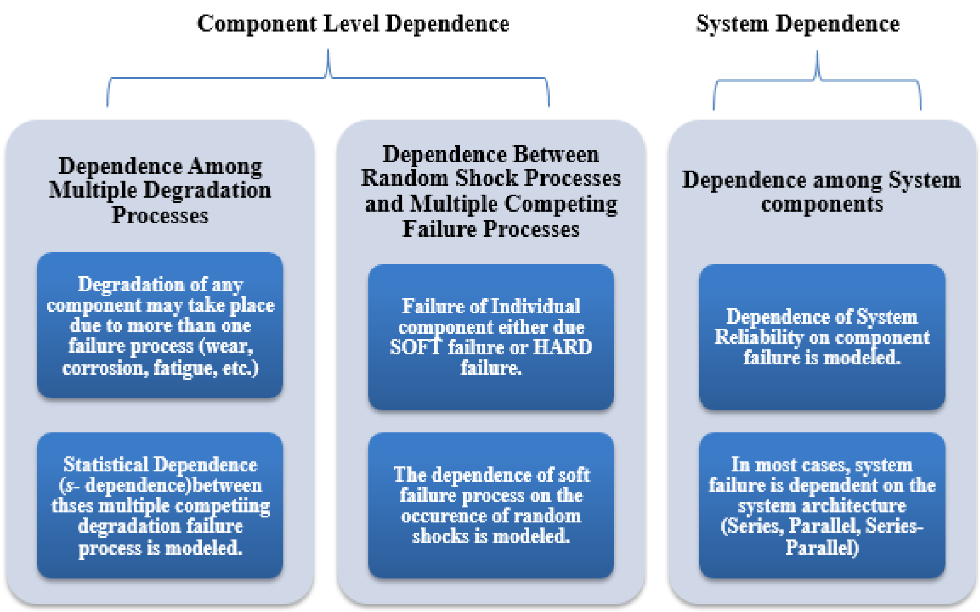

Figure 2.8: Conditions of system failure through MCFP's

There are more aspects of dependency between soft failure and hard failure in complex systems. A more interesting and challenging problem not addressed so far in the literature is multiple competing shock dependency. Most of the existing literature considers a random Poisson arrival of shock processes which is acting uniformly on the entire system, whereas in real life situation multiple shock processes may act simultaneously or dominantly on the complete system (Li & Pham, 2005; Peng et al., 2010). Especially in mechanical components, a system may experience multiple degradation as well as multiple shock failure processes at the same time along with dominance of certain kind of failure processes. This would directly affect the operating lifetime of any system. Study shows an extensive scope for models that can handle both multiple degradation as well as multiple shock processes simultaneously (Rafiee et al., 2014; Y. Wang & Pham, 2012; Z. Wang et al., 2011).

Most of the system failure models in reliability engineering are based on the assumption of independence. However, there are many practical situations when this assumption is inappropriate. Failure of any component or system can be subjected to various internal and external factors including human errors. Degradation and shock processes are notable among such failure causes. Understanding the relationships among these multiple failure processes belongs to the basic problems of System Reliability Analysis. Further degradation failure of any component or system may occur due to a combined effect of multiple degradation processes such as wear, fatigue, corrosion, etc. A prudent amount of work has been done to address these issues by capturing the *s*- dependence between various competing failure processes.

Song & Coit (2011) proposed a reliability estimation and preventive maintenance model for a complex multi-component system exposed to multiple dependent competing failure processes. This model is limited to series alignment of components with an age replacement technique. Li et al. (2011) modeled a reliability approach established on additive degradation process. The system comprises of *s*-correlated and *s*-dependent components in a series set up. Jiang et al. (2012) presented a reliability assessment model and a preventive maintenance model for s-dependent competing failure process with shifting failure threshold. The shift in failure threshold is due to the accelerated action of more than one failure process.

Several studies have been made which are specific for individual case problem. Wang & Pham (2011b) extended the existing work to a model a degraded system subjected to two types of competing failure processes: random shock and two kinds of degradation processes. The study includes both additive and multiplicative degradation path functions. The same study is extended to a higher level for a similar system along with the introduction of a 'time varying copulas' (Wang & Pham, 2012). A broader approach in this area is proposed by Xiang et al. (2013), where the sub-populations are experiencing a stochastic degradation process. Rafiee et al. (2014) has suggested a reliability model with changing degradation rates in return to the action of different types of shock processes.

2.3.3. Solution Approaches for system reliability assessment:

2.3.3.1. Statistical approaches:

The following section provides a brief discussion on various solution approaches used to address the multi-state system reliability. Most of the literature has focused on modeling the joint and marginal distributions for multi-state systems, however, very less work has been recorded to model the conditional probability for these multi-state systems and there is a strong need to develop a cumulative distribution function for more than one failure process (degradation failure processes and shock failure process) acting on them. Reliability function for any system or component has so far been modeled assuming the failure process to follow a linear degradation path function along with simultaneous action of a single kind of shock process. In real life situation, a system is prone to more than one competing shock processes accompanied by simultaneous and continuous action of the degradation process. Forthcoming research work can be focused to model dependence of such kind of multiple non-linear processes.

Various kinds of statistical tools and approaches have been used to record the s-dependence between multiple failure processes. Peng et al. (2011) defined a joint probability density function (pdf) to model the component degradation rates, conditional on some defined time. This model had several advantages and disadvantages depending upon the design or planning problem. However, that model does not accommodate shocks, and the most important issue is that it requires the determination of the functional form and estimation of associated parameters of the joint probability density function. Song et al. (2014) demonstrated the *s*-dependence among soft and hard failure processes in a multi-component system using a covariance function. The covariance function of the degradation of any two components is derived and proved to be greater than zero in the presence of shocks, thus proving the correlation and *s*-dependence of occurrence of two different events.

A universal generating function (UGF) based approach was proposed to find the dependence of a system's state with respect to component states. The model limits its study to the finite number of performance levels.

2.3.3.2. Copulas:

Another important and useful statistical tool used for modeling the *s*-dependence among components is known as Copulas. In the context of system reliability, multivariate distributions are obtained from modeling the joint behaviour of components using the marginal lifetime distributions of the components and the copula function. A copula separates the joint distribution onto two contributions: the marginal distributions of the individual variables and the interdependency of the probabilities. Another convenience is that the conditional distribution can be easily expressed from the copula. The copula function can provide with the degree of the dependence in each state, and also the structure of the dependence. Thus, Copula tends to give a better *s*-dependence model for different marginal distributions (Aleˇs KOZUB´IK, 2005;

Eryilmaz, 2014). Copula method has several advantages over the conventional and direct methods used to fit the joint probability distribution functions. Instead of using traditional marginal distributions, the copula method is utilized to establish the s-dependent structure among various degradation measurements. Copulas separately allow the modeling of the marginal behaviour, and the dependence structure.

Wang & Pham (2012) have used a time-varying copula to model the dependent competing risks for multiple degradation processes. A joint distribution function for the system reliability is modeled using a bivariate copula. Both constant and time-varying copula is used to test the system performance demonstrating the best fitting distribution for a particular time-varying copula. A suitable time-varying copula can further be used for parameter estimation for the degradation function featured as cumulative degradation embedded with random shocks. Further, reliability bounds for a multi-component system can be found using a multivariate copula based on the marginal distributions of multiple failure processes. Copula bounds can also be used to evaluate the system lifetime and hence find the dependence between n multi-state components.

Copula can give a joint probability distribution function for competing failures acting on individual component. Also, there is a need for priority based approach to find the system reliability for a case where either the shock process dominates or leads to catastrophic system failure or a degradation process leads the system failure.

2.3.3.3. Markov Models:

A powerful technique for analyzing complex probabilistic systems, based on the notion of states and transition between states is Markov Modeling. To formulate a Markov mode, system behavior is abstracted into a set of mutually exclusive system states. For example, the states of a system can be the set of all distinct combinations of working and failed modules of reliability model. A set of equations describing the probabilistic transitions from one state to another state and an initial probability distribution in the state of the process uniquely determine a Markov model. One of the most important features of a Markov model is that the transition from on state *i* to another state *j* only depends on the current state.

The use Markov models began in the early 90's for modeling system behavior for various discrete and continuous time operations. Sim & Endrenyi (1993) proposed a Markov model for continuously operating device with deterioration & Poisson failure. The distribution of the inter-arrival between successive degradation stages was assumed to be exponentially distributed with constant rates. Lam and Yeh (1994) Studied a multi-stage age dependent replacement policies for a multi-state system subjected to both deterioration and random shocks. Here the inter-arrival time between two successive states follows a continuous distribution with a finite mean. Xue & Yang (1995) modeled a lifetime distribution of the multi-state deterioration systems based on the continuous-time Markov process & semi-Markov Process. Li & Pham, (2005) used a Markov matrix for modeling a multi-stage degraded system with two competing failure along with random shocks. A framework for failure modeling of an electrical N-component was proposed by Rodríguez et al. (2015) using a Markovian arrival process.

The review of above and many more models suggest that Markov models are best suited to model the changing behaviour of any system or component from one state to another state. As stated above, the transition of system from current state *i* to another state only depends on the current state. That is, the way in which the entire past history affects the future of the process is completely summarised in the current state of the process.

If the state of any system is discrete, either finite or countably infinite, then the model is referred to as a Markov Chain. The proposed mathematical model for MSMC reliability assessment uses discrete time Markov Chain process to model the transition of individual component from *M* (perfect functioning) to *M-1* (lower state of functioning) state of performance.

Table 2.2 the summarises significant contribution of researchers in assessing multi-state system reliability using various solution approaches.

Table 2.2: Significant solution approaches for multi-state system reliability assessment

S. No.	Study	Scope	Comments
1.	Sim & Endrenyi 1993	Markov Model	• Proposed a model for continuously operating device with deterioration & Poisson failure. • The distribution of the inter-arrival between successive degradation stages was assumed to be exponentially distributed with constant rates.

S. No.	Study	Scope	Comments
2.	Lam & Yeh 1994	Semi-Markov Chain Model	• Studied a multi-stage age dependent replacement policies for a multi-state system subjected to both deterioration and random shocks. • Here the inter-arrival time between two successive states follows a continuous distribution with a finite mean.
3.	Xue & Yang 1995	Continuous-time Markov Model	• Modeled a lifetime distribution of the multi-state deterioration systems based on the continuous-time Markov process & semi-Markov Process
4.	Pham et. al. 1996	K-out-of-n:G systems	• Proposed a model to predict the reliability of such systems with components subjected to multi-stage degradation and catastrophic failure.
5.	Pham et.al 1997	Multi-stage Degraded System	• Modeled how to predict the availability & mean life time of a multi-stage degraded systems with partial repairs. • The transition rates were assumed to be constant.
6.	Zuo et. al. 1999	Mixture Model	• The whole population is divided into two independent sub-populations: • One sub-population is subjected to degradation & other is subjected to catastrophic failure.
7.	Li, W. and Pham, H 2005	Generalized model for multi-state degraded systems	• Unable to address the issues of maintenance and repair. • Modeled a Markov matrix for a multi-stage degraded system with two competing failure along with Random shocks. • The system state is decided jointly based on the state of each failure process and given in a matrix form. • Catastrophic failure is taken to follow a Poisson distributions
8.	Rathod et. al. 2011	Probabilistic Models of Linear Damage Accumulation for Fatigue Reliability Analysis	• The damage accumulation phenomenon is considered to be linear. • Fatigue life cycle is taken as a Normal curve, which is not always possible in real life conditions.

S. No.	Study	Scope	Comments
9.	Zhu et. al. 2012	Probabilistic Model of Non-Linear Damage Accumulation for Fatigue Reliability Analysis	• Fatigue life cycle is taken as a Normal curve, which is not always possible in real life conditions.
10.	Shijie Wang 2013	Reliability Model	• A joint probability distribution is given to describe the correlated failure mode acting on a system. • A linear regression model is used for the same.
11.	Gao et.al. 2014	Modified Nonlinear Fatigue Damage Accumulation Model Considering Load Interaction Effects	• Load-interaction effects under multi-level loading conditions and random loading conditions are required to be addressed.
12.	Song et.al. 2014	Multi-component System Reliability Model	• A Multi-component system reliability model is developed for complex multi-component systems with each component experiencing multiple failure processes due to simultaneous exposure to degradation and shock loads. • The two competing failure processes are mutually competing and *s*-dependent.
13.	Lin & Li 2015	Multi-state Physics Model	• Proposed a multi-state physics model (MSPM) for component reliability assessment by including semi-Markov and random shock processes. • The relative increment in transition rates after each shock process has been captured.

2.3.4. Relation between Reliability and Warranty

In the purchase decision of a product, buyers typically compare characteristics of comparable models of different brands available in the market. When competing brands are nearly identical, it is very difficult, in many instances, to choose a particular product solely on the basis of the product-related characteristics such as price, features, product quality, and finance offered by the manufacturer, and so on. In such situations, post-sale factors – warranty, support level, maintenance, spare parts cost and their availability, etc., are important in the choice of the product. Of these, warranty is a one of the most influential factors that is known to the buyer at the time of purchase (Murthy, 2006). "A warranty is seller's assurance to a buyer that a product is as has been represented. It may be considered to be a contractual agreement between the buyer and manufacturer entered into upon the sale of the product or service." (Blischke and Murthy, 1996).

A warranty of any type, since it involves an additional service associated with a product, will lead to potential costs beyond those associated with the design, manufacture and sale of the product. These costs, in fact, are unpredictable future costs, which typically range from 2% to as much as 15% of net sales (McGuire, 1980). As a result, warranty has a significant impact on the total profits for a manufacturer. Failures over the warranty period are closely linked to product reliability. The reliability of product is determined by decisions made during design, development and manufacturing stages (Murthy, 2006).

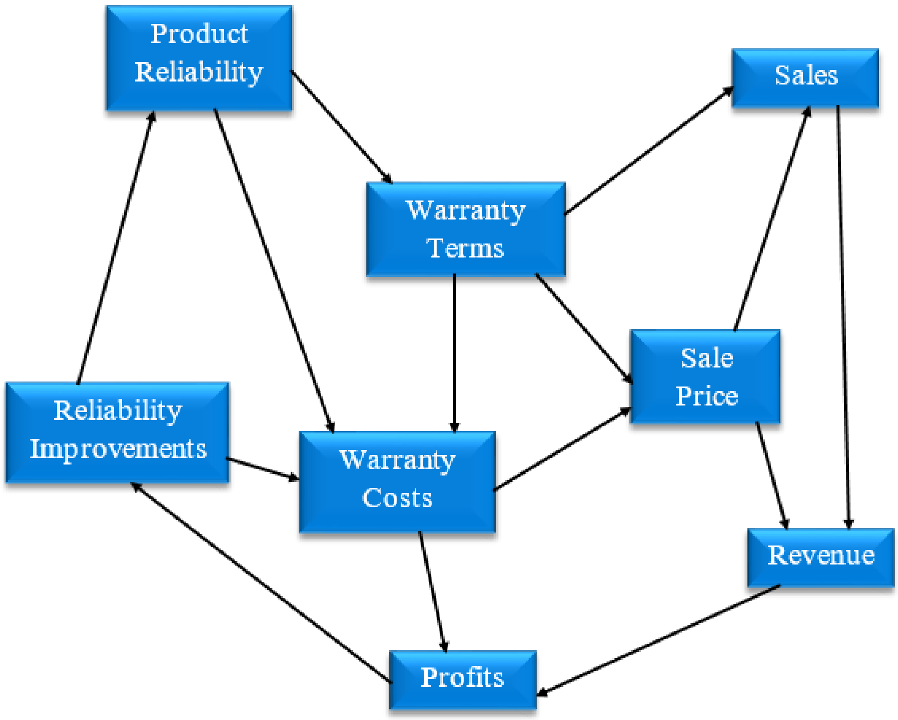

Figure 2.9: Key elements for reliability-warranty management (adopted from Murthy, 2006)

The warranty cost strongly depends on the reliability of the product which in turn depends upon several factors, some of which are controlled by the manufacturer, such as the decisions made during the design and development stage (Figure 2.9). Some of the factors are related to the consumer such as the product usage pattern and the operating environment and maintenance. To address the issues related to warranty, there is a need to develop a framework which addresses the critical parameters which affect the decisions related to warranty and integrate the issues in such a way that the warranty cost can be minimized (Ambad & Kulkarni, 2013).

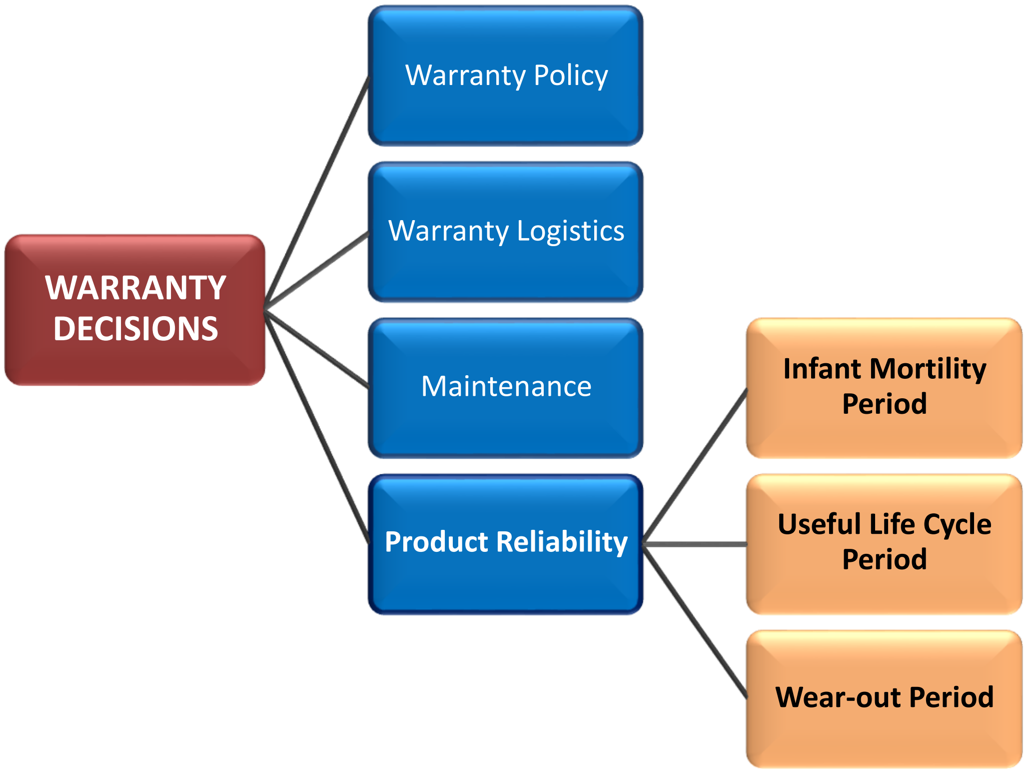

Figure 2.10: Factors affecting warranty decisions

This implies that reliability related decisions must take into account the interaction between warranty and reliability (Figure 2.10). Most of the companies either give a warranty that is far shorter than the expected life of the item or increase the cost to a very high level to cover expected warranty costs (Murthy, 2006). Neither approach is appropriate in the current competitive marketplace. Firms that fail to reduce the warranty costs, relative to their competitor, can find themselves out of business in such an environment. Warranty is a commercial issue and reliability is a technical issue. As such, reliability decisions must be done in a framework that incorporates both these issues. A life cycle approach perspective is needed for making reliability-warranty decisions (González-Prida Díaz & Crespo Márquez, 2014). This framework proposed in chapter five discusses the link between warranty and reliability, and how the life cycle and failure prediction of any system and its components can help the manufacturer to decide more precise and cost efficient warranty plans.

Reliability of any product or system deals with the estimation, prevention and management of product life with least uncertainty and risks of failure. A failure can occur early in an item's life due to manufacturing defects or at a later time in its life due to degradation which is dependent on age and usage (Ambad & Kulkarni, 2013).

Products degrade with age and/or usage and fail when they are unable to carry out their normal functions. Reliability theory deals with various issues such as the understanding of the degradation mechanism, the design of reliable products and the operation of unreliable products (Murthy, 2007). It becomes difficult to determine the reliability of any product with complex design, system architecture and with increasing number of components and systems' states of performance.

2.4. Research Gaps:

Based on literature review few research gaps are identified and discussed below:

1. Most of the existing research have focused on assessing multi-state system reliability following the assumption that transition of both system as well as its components from a higher state of performance to a lower state of performance is simultaneous in nature. Whereas, transition of individual component of any system cumulatively determines the system state. Thus there is a strong need for a study which distinctly considers the different performance states of a system at various levels.
2. The literature review also shows that majority of work on multi-state reliability assessment has focused on capturing independent nature of components and failure processes. The dependency in system reliability has only been addressed by s-dependent models for multiple competing failure processes acting on the system. Hence, there is necessity for an approach that captures the physical association among system components.
3. There is hardly any reported literature that has addressed the issue of component criticality to estimate system reliability. In most of the multi-component multi-state systems, there are few components that contribute more critically in system functionality as compared to the other components of the system. The failure of these components may lead to system break-down or complete failure. Thus it is crucial to consider the 'criticality order' of these components while estimation of multi-state multi-component (MSMC) system reliability.
4. Additionally, the models proposed in literature are limited to estimate MSMC system reliability and are not capable of identifying the cause of failure or the erroneous component. This identification of faulty component

is important to avoid accelerated degradation of the entire system along with reducing the overall maintenance cost of both system and its components.

5. A substantial need for a methodology that combines reliability to maintenance and repair aspect of system is lacking in the existing study. As previously stated, reliability is an important quality characteristic from both designer's and customer's point of view, it becomes all the more important to evaluate the effect of reliability assessment on maintenance and repair schedules of a system.

The flowchart for literature review process and research gap identification of this study is presented in Figure 2.11.

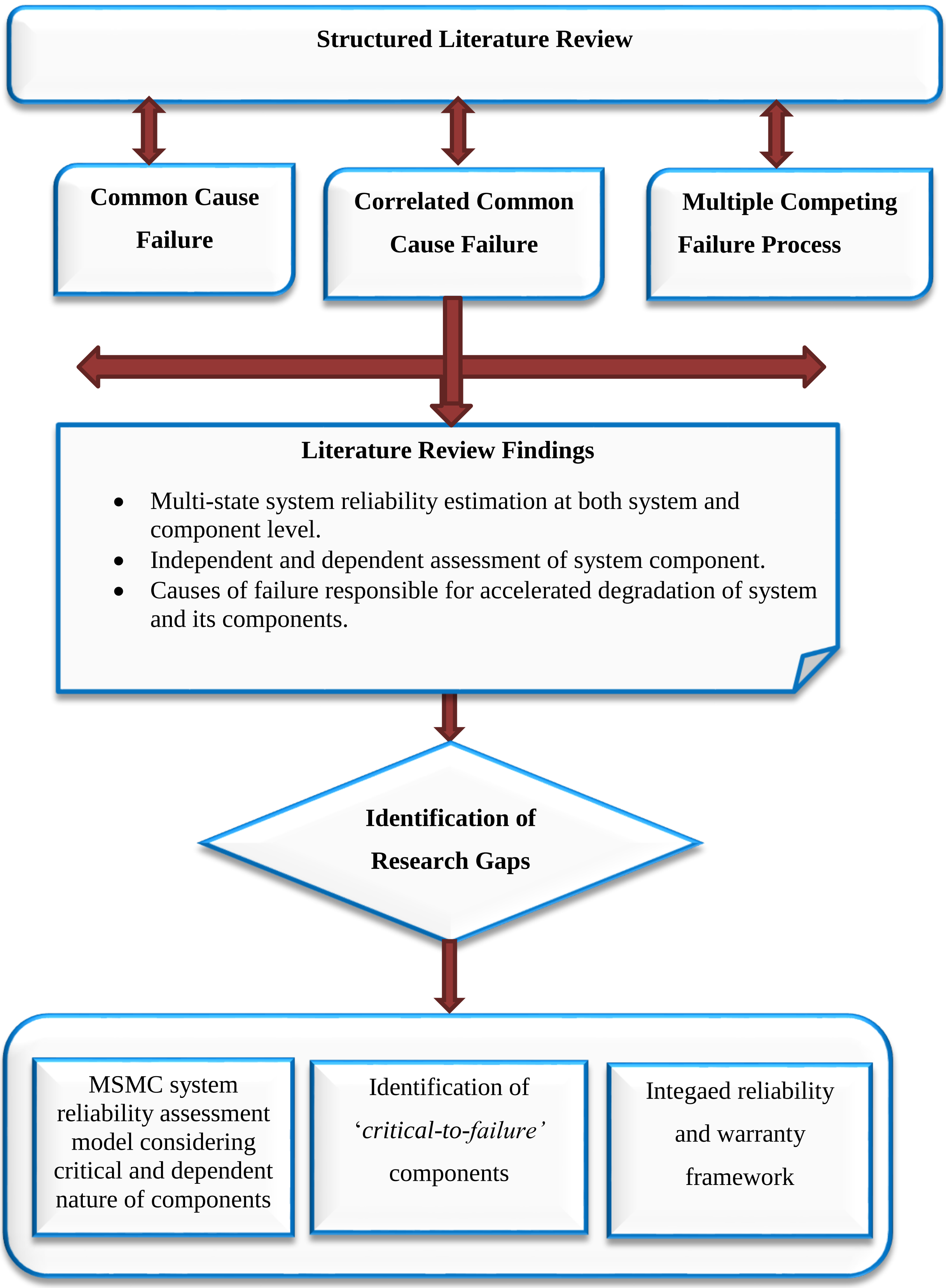

Figure 2.11: literature review process and research gap identification

Chapter 3
Reliability Assessment of MSMC Systems

3.1. Introduction:

System reliability is a *"time oriented"* quality measure and is very important from both customer as well designer's point of view. Therefore, it is necessary to evaluate system reliability like any other quality characteristic. The classical system reliability models assume that any system and its components can only have two possible sates of performance, either working or failed. However, this assumption is not realistic as most of the engineering systems have the ability to perform at multiple intermediate states before they fail completely. Especially in today's real world, different number of performance states needs to be considered for accurate reliability assessment of complex systems. One of the key issues in reliability assessment of these complex systems is to identify critical areas that can lead to an accelerated system deterioration. It is observed that most of the reported reliability models for multi-state systems are limited to the assumption that the deterioration of system components is independent of each other (Wang et al., 2014; Xing et al., 2010; Xing & Wang, 2008). This assumption fails to address the reliability of real life systems and its components. Hence, there is strong need of an approach that captures the dependent nature of system components and thus estimate more accurate reliability for multi-state multi-component (MSMC) systems.

The following sections demonstrate the mathematical formulation, implementation and result analysis of the proposed model to assess MSMC system reliability with simultaneous consideration of dependent and critical nature of components.

3.2. Model Overview

The reliability assessment approach proposed in this chapter captures a realistic scenario of multiple states of performance at both system and component level for a MSMC system. The degradation of system components is modeled with a discrete-time Markov process, which assumes that state transition from a current state i to the next state j is independent of previous state transitions. The probability of transition of system from current state to the next state only depends on the current state i. Due to this property of Markov process it is considered to be a '*memory less*' process. Figure 3.1 shows the change in transition states of any system or component from M_x state of perfect function to 0_x state of complete failure. It should be noted here that division of time units may vary from system to system depending on its life cycle (t_{max}). For example, in some cases the maximum life cycle of a system may be as large as 10, 00,000 life cycles and for other systems t_{max} could be as small as 10,000 cycles. In this study, the time division is done at system level. The variation in states at component level is captured by different transition probabilities allocated for each component.

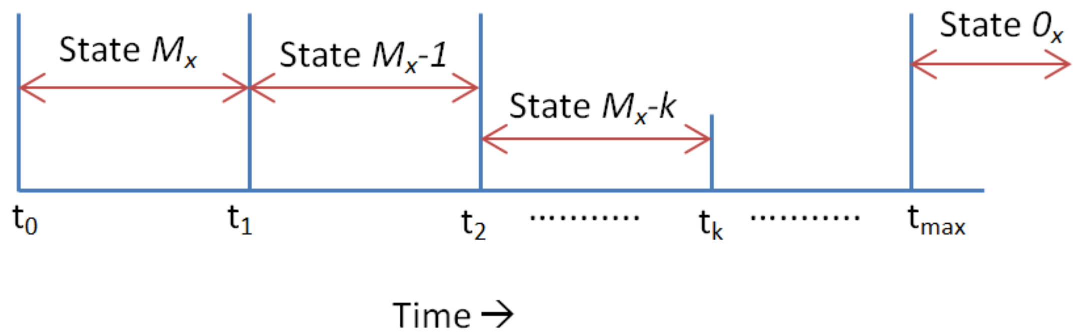

Figure 3.1: State Classification (adopted from Liu & Kapur, 2008)

The system is primarily divided in two levels: the sub-system (n_s) and component level. Every sub-system is selected based on the *'functional output'* given by it. Each sub-system includes the components that enable the sub-system to perform its expected function. Further, each component is assigned a *'priority order'* in the hierarchy of criticality based on 'function' performed by that component in sub-system's efficient working.

Further, a state transition probability matrix Λ_x is generated for every component either using a simulation process or using designer's expertise depending on whether the system under consideration is a new product without any prior knowledge of its degradation behaviour or the system is an existing product with availability of degradation data from past. In the case example discussed in section 3.4, the system is a new product and the state transition matrices Λ_x are generated using simulation process by considering design expert's knowledge of *'expected degradation process'* to be followed by individual component. Based on the type of degradation process each component follows, an individual state transition matrix is generated and assigned to it. Once these matrices are generated, they are used as inputs for assessing the independent probability of each component to move to any state M_x-k at any time instant t_k.

The structure function used to establish the probability of any component x to be in state M_x- k at any time instance t_k derives its base from reliability measures given by (Liu & Kapur, 2008) for multi-state systems. This structure function is an extension of the previous study by (Kapur, 2006) to develop structure function equations for *binary systems*. This is very general and is more realistic than the earlier approaches where the number of the states for the system and its components is the same. The approach proposed in this study evaluates degradation of any MSMC system at two levels: (i) *component level* and (ii) *sub-system level*, which ultimately provides the system failure probability. The following section elaborates the proposed methodology:

3.3. System Description

To conduct this study, we have considered a system with n components C_1, C_2, C_3 …C_n ; as shown in Figure 3.2. As system reliability is the only concern here, system maintenance and repair are not taken under consideration. Therefore, the system only degrades with time and does not make transitions to higher states. To assess the impact of dependent behavior of system components over each other, the degradation of each component is divided into two categories: (i) *independent deterioration of each component due to the action of degradation process acting on it* and (ii) *dependent deterioration due to correlative effect i.e. degradation due to the dependent and interactive action of other associated components*.

All the system components are functionally dependent on each other and degradation of each component follow a discrete-time Markov chain process. A single kind of degradation process *'F'* is acting on individual component. The overall failure of the system is evaluated in terms of component degradation, which will finally be used to assess the system failure probability and hence give the idea of system performance and reliability. A ***state transition probability matrix*** $\wedge_x$ is generated to report the transition of each component from a current higher state to the successive lower state of transition.

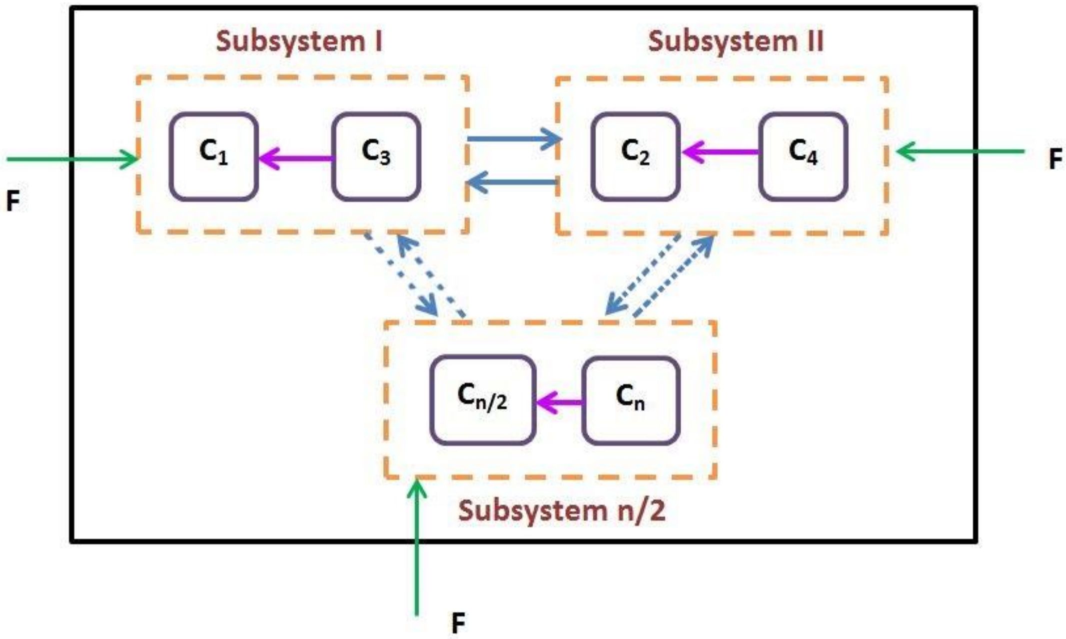

Figure 3.2: System with *n* functionally correlated components

3.3.1. Assumptions

1. Each component in the system consists of M_x+1 state, where 0 is complete failure state and M_x is perfect functioning state.
2. No repair and maintenance is performed on the system.
3. With time the system only degrades and does not make transition to a higher state.
4. Only a single transition can occur in a single time instant ($t_{initial} = t_0$, next time instant $(t_1) = t_0+\tau$).
5. Every component follows a gradual and successive hierarchical pattern of degradation (i.e. states of degradation for any component x will be from $M_x \rightarrow M_x$ -1; M_x -1 $\rightarrow M_x$ -2… M_x -k $\rightarrow$ 1 & 1 $\rightarrow$ 0). Here x =1, 2 …n.
6. The criticality order of all the components is $C_1 > C_2 > \ldots C_{n/2} >> C_3, C_4 >> \ldots C_n$.

3.3.2. Proposed Model

The procedure to assess system reliability can be summed up in the following eight sections:

3.3.2.1. State Transition Probability Matrix (Λ_x):

A ***state transition probability matrix*** Λ_x is generated to study the degradation behaviour of individual component over a given period of time. The value of transition probabilities can be estimated for an existing product by using past failure data of individual component and determining its degradation path function whereas in case of a new product, where no such data is available, the expected failure path function followed by every component can be taken from a team of design experts. This failure path function is further used as input to generate the transition probability matrix of every component.

Each matrix Λ_x includes four discrete component states $\{M_x, M_x - 1, M_x - 2, 1_x\}$, with M_x being a state of *'perfect functioning'* and 1_x being a state of *'lowest performance'* before complete failure. Therefore, a distinct state matrix can be assigned to each component of a system made of *n* components, where each component performs at four discrete levels i.e. $\Lambda_x = \{M_x, M_x - 1, M_x - 2, 1_x\}$. The matrix below demonstrates a generic form of such state transition probability matrix for x^{th} component with four performance levels between M_x and 0_x.

$\Lambda_x =$

C_x	M_x	M_x-1	M_x-2	M_x-3	1_x
M_x	$p_{m,m}{}^x$	$p_{m,m-1}{}^x$	$p_{m,m-2}{}^x$	$p_{m,m-3}{}^x$	$p_{m,1}{}^x$
M_x-1	$p_{m-1,m}{}^x = 0$	$p_{m-1,m-1}{}^x$	$p_{m-1,m-2}{}^x$	$p_{m-1,m-3}{}^x$	$p_{m-1,1}{}^x$
M_x-2	$p_{m-2,m}{}^x = 0$	$p_{m-2,m-1}{}^x = 0$	$p_{m-2,m-2}{}^x$	$p_{m-2,m-3}{}^x$	$p_{m-2,1}{}^x$
M_x-3	$p_{m-3,m}{}^x = 0$	$p_{m-3,m-1}{}^x = 0$	$p_{m-3,m-2}{}^x = 0$	$p_{m-3,m-3}{}^x$	$p_{m-3,1}{}^x$
⋮	⋮	⋮	⋮	⋮	⋮
1_x	$p_{1,m}{}^x = 0$	$p_{1,m-1}{}^x = 0$	$p_{1,m-2}{}^x = 0$	$p_{1,m-3}{}^x = 0$	$p_{1,1}{}^x = F$
Time Interval (Units)	t_0 = *initial time*	t_1	t_2	t_3	t_{max}

Each matrix summarises random probabilities $\boldsymbol{p_{ij}}^{\boldsymbol{x}}$ to study the degradation of $\boldsymbol{x^{th}}$ component i.e. every $\boldsymbol{p_{ij}}^{\boldsymbol{x}}$ gives the probability of transition of $\boldsymbol{x^{th}}$ component from one higher state $\boldsymbol{i}$ to the next lower state $\boldsymbol{j}$.

The proposed work models degradation of individual component with Markov process which assumes that the next state of any system depends only on its current state and not on any previous state. Also the transitions between states follow a stationary exponential distribution. For the Markov process, the instantaneous state transition probability from state $\boldsymbol{i}$ to state $\boldsymbol{j}$ is also assumed to be discrete and is represented by $\boldsymbol{p_{ij}}^{\boldsymbol{x}}$, where $\boldsymbol{i} > \boldsymbol{j}$ and $\boldsymbol{i} \in [\, M_x, \; M_x - 1, .., 1]$.

The scope of this study is limited to system reliability and hence system maintainability is not the concern here. Due to this assumption the component shall only degrade with time and does not make any transitions to higher level i.e. the probability of any component to move from a lower state to any higher state is taken as zero. At any time-instant when component is further degrading from a state of low performance $\boldsymbol{i}$ = 1 to the next similar state $\boldsymbol{j}$ =1, then the component's performance is considered to be a '*complete failure*'. Also the component is assumed to follow a hierarchical pattern of degradation and hence it will not be able to degrade directly to any lower state per transition.

3.3.2.2. Probability of Component Transition Due to Age Effect:

To obtain the degradation probability of any component from one state to the next lower state we need to consider its age-effect. At any time $\boldsymbol{t_k}$ the probability of component $\boldsymbol{x}$ to degrade from a higher state $\boldsymbol{i}$ to next lower state $\boldsymbol{j}$ in hierarchy is given by eq^n 16:

$$\boldsymbol{P(D_{xf_c}) = \frac{p_{ij}}{G_{xi} - G_{xj}}\{exp(-G_{xj}.t) - exp(-G_{xi}t)\}} \qquad \textbf{(1)}$$

where,

$\boldsymbol{G_{xi}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{M}} \boldsymbol{p_{ij}}$ for a given value of $\boldsymbol{j}$;

$\boldsymbol{G_{xj}} = \sum_{\boldsymbol{j=1}}^{\boldsymbol{M}} \boldsymbol{p_{ij}}$ for a given value of $\boldsymbol{i}$;

$\boldsymbol{f_c}$ is the functional level of $\boldsymbol{x^{th}}$ component and

$\boldsymbol{t}$ is the time taken by component '$\boldsymbol{x}$' to degrade from $\boldsymbol{M_x}$ to $\boldsymbol{M_x}$ - $\boldsymbol{k}$ state.

$\boldsymbol{P(D_{xf_c})}$ will determine the probability of degradation of individual component on the basis of its age-effect.

3.3.2.3. Interaction at Component Level:

As stated above, the failure of any multi-component system is a combined outcome of (i) independent degradation of each component and (ii) the effect experienced due to degradation of other functionally associated components. Consequently, the dependent behaviour or the '*interaction effect*'' of critical system components on its associated components is assessed using Bayes' law.

For this purpose, the entire system is divided into smaller sub-system assemblies ($\boldsymbol{S_{x_s}}$) each having two components. This division is done such that every sub-system includes one component having a higher order of criticality as compared to the other component in the sub-system.

In this model, we have considered a system consisting of *n* components divided into *n/2* sub-system assemblies, *n* being an even value- S_1, S_2... $S_{n/2}$. As shown in Figure 3.2 each sub-system consists of one pair of components. For example, sub-system S_1 comprises of C_1 & C_3 and sub-system S_2 comprises of C_2 and C_4. While calculating the impact of components in each subsystem, we have focused on estimating the impact of degradation of critical component C_1 on its associated component C_3 when C_1 has already moved to a lower state, which is given as:

$$P(\boldsymbol{D_{3,1}}|\boldsymbol{D_{1,1}}) = \frac{\boldsymbol{P(D_{3,1}).\ P(D_{1,1}|D_{3,1})}}{\boldsymbol{P(D_{1,1})}} \quad \textbf{(2)}$$

Similarly, the impact of degradation of critical component C_2 on its associated component C_4 when C_2 has already moved to a lower state is given by:

$$\boldsymbol{P(D_{4,1}|D_{2,1})} = \frac{\boldsymbol{P(D_{4,1}).\ P(D_{2,1}|D_{4,1})}}{\boldsymbol{P(D_{2,1})}} \quad \textbf{(3)}$$

In the above equations (2) and (3), impact of degradation of C_3 on C_1 and impact of C_4 on C_2 will be very less as the critical hierarchy order of C_3 and C_4 components is too low as compared to C_1 and C_2. Hence the effect of degradation of C_3 on C_1 i.e.

$P(D_{1,1}|D_{3,1})$and the effect of degradation of *C4* on *C2* i.e. $P(D_{2,1}|D_{4,1})$ can be ignored for calcuation purposes.

Further, generalising the above probabilities for *n* components gives:

$$\boldsymbol{P}(\boldsymbol{D}_{n,1}|\boldsymbol{D}_{l,1}) = \frac{\boldsymbol{P}(\boldsymbol{D}_{n,1}).\ \mathbf{P}(\boldsymbol{D}_{l,1}|\boldsymbol{D}_{l,1})}{\boldsymbol{P}(\boldsymbol{D}_{l,1})} \qquad \textbf{(4)}$$

where, l = 1, 2, …, n/2.

In equations (2) and (3), $P(\boldsymbol{D}_{3,1}|\boldsymbol{D}_{1,1})$ and $P(\boldsymbol{D}_{4,1}|\boldsymbol{D}_{2,1})$ are the probabilities of degradation effect experienced by components C_3 and C_4 due to degradation of components C_1 and C_2 . Also $P(D_{1,1}), P(D_{2,1}), P(D_{3,1})$ *and* $P(D_{4,1})$ represent degradation probabilities of component C_1, C_2, C_3 and C_4 respectively.

3.3.2.4. Cumulative Sub-System Level Probabilities:

In previous sections we have modeled the impact of degradation of individual components and the impact of critical component degradation on the associated components in each sub-system. As mentioned earlier, the proposed approach calculates system reliability for any MSMC system by establishing reliability at two levels: component level and sub-system level. In this section, sub-system level transition probabilities ($\boldsymbol{P}(\boldsymbol{S}_{x_s f_s})$) are calculated to examine the degradation of each sub-system to the next lower level of performance. Here, $\boldsymbol{S}_{x_s}$ denotes the subsystem number, $\boldsymbol{f_s}$ is the level of deterioration of subsystem $\boldsymbol{S}_{x_s}$ and $\boldsymbol{P}(\boldsymbol{S}_{x_s f_s})$ is the transition probability that subsystem $\boldsymbol{S}_{x_s}$ will move to a lower state $\boldsymbol{f_s}$.

Deterioration of any sub-system can be accounted to a combined effect of degradation of critical component and the effect of this degraded critical component on the other associated components. Therefore, the cumulative probability of degradation of sub-system S_1 to the next lower level of performance can be given as:

$$\boldsymbol{P}(\boldsymbol{S}_{1,1}) = P(D_{1,1}) + P(D_{3,1}|D_{1,1}) + P(D_{3,1}) \qquad \textbf{(5)}$$

As stated in section 3.3.3, component C_3 is very low in hierarchy of criticality as compared to component C_1 and hence its effect on system degradation can be ignored for the ease of calculations. Therefore,

$$\boldsymbol{P(S_{1,1})} = P(D_{1,1}) + P(D_{3,1}|D_{1,1}) \quad \textbf{(6)}$$

Similarly, for sub-system *S2*

$$\boldsymbol{P(S_{2,1})} = P(D_{2,1}) + P(D_{4,1}|D_{2,1}) + P(D_{4,1}) \quad \textbf{(7)}$$

$$\boldsymbol{P(S_{2,1})} = P(D_{2,1}) + P(D_{4,1}|D_{2,1}) \quad \textbf{(8)}$$

Further generalisation for *n/2* subsystems gives cumulative probability as:

$$\boldsymbol{P(S_{l,1}) = P(D_{n,1}) + P(D_{n,1}|D_{l,1})} \quad \textbf{(9)}$$

where, l = 1, 2, …, n/2

3.3.2.5. Interaction at Sub-System Level:

The impact of deterioration of sub-systems on each other can be calculated in a similar way as done for components in section 3.3.3. The impact of degradation of sub-system S_1 on sub-system S_2 when sub-system S_1 has already moved to a lower level of performance is given by:

$$\boldsymbol{P(S_{2,1}|S_{1,1})} = \frac{P(S_{2,1}).\ \mathbf{P(S_{1,1}|S_{2,1})}}{P(S_{1,1})} \quad \textbf{(10)}$$

In above equation, $\mathrm{P}(\mathrm{S}_{1,1}|\mathrm{S}_{2,1})$ represents the impact of degradation of sub-system S_2 on S_1 which can be estimated using designers' expertise.

A similar pair-wise estimation of impact of the next sub-system S_3 on sub-system S_2 can be determined using equation 7.

$$\boldsymbol{P(S_{3,1}|S_{2,1})} = \frac{\boldsymbol{P(S_{3,1}).\ P(S_{2,1}|S_{3,1})}}{\boldsymbol{P(S_{2,1})}} \quad \textbf{(11)}$$

This pair-wise estimation will continue until the impact of $n/2^{\text{th}}$ sub-system on n^{th} sub-system is calculated.

This way all possible pair-wise scenarios can be considered to capture the dependent impact of one sub-system over other sub-systems. Equation 12 gives a generalised form for calculation of sub-system interaction:

$$\boldsymbol{P}\left(\boldsymbol{S}_{\boldsymbol{l},\boldsymbol{1}}|\boldsymbol{S}_{\frac{\boldsymbol{n}}{2},\boldsymbol{1}}\right) = \frac{\boldsymbol{P}(\boldsymbol{S}_{\boldsymbol{l},\boldsymbol{1}}).\ \boldsymbol{P}\left(\boldsymbol{S}_{\frac{\boldsymbol{n}}{2},\boldsymbol{1}}\middle|\boldsymbol{S}_{\boldsymbol{l},\boldsymbol{1}}\right)}{\boldsymbol{P}(\boldsymbol{S}_{\frac{\boldsymbol{n}}{2},\boldsymbol{1}})} \tag{12}$$

3.3.2.6. Cumulative System Level Probabilities:

The system level transition probabilities can be evaluated by taking cumulative effect of sub-system degradation on the overall system using equations (4), (5) and (6). To compute overall system transition probability, we propose to first estimate transition probability of each sub-system and its impact on other sub-systems. For example, probability of system degradation or failure when the degradation effect of sub-system S_1 is taken under consideration and given as:

$$P(I) = \boldsymbol{P}\left(\boldsymbol{S}_{\boldsymbol{1},\boldsymbol{1}}\right) + \boldsymbol{P}\left(\boldsymbol{S}_{\boldsymbol{2},\boldsymbol{1}}|\boldsymbol{S}_{\boldsymbol{1},\boldsymbol{1}}\right) \tag{13}$$

Similarly, the probability of system degradation or failure when the degradation effect of sub-system S_2 is taken under consideration is given as:

$$P(II) = \boldsymbol{P}\left(\boldsymbol{S}_{\boldsymbol{2},\boldsymbol{1}}\right) + \boldsymbol{P}\left(\boldsymbol{S}_{\boldsymbol{1},\boldsymbol{1}}|\boldsymbol{S}_{\boldsymbol{2},\boldsymbol{1}}\right) \tag{14}$$

Again, a generalised cumulative system probability is given as:

$$P(L) = \boldsymbol{P}\left(\boldsymbol{S}_{\boldsymbol{l},\boldsymbol{1}}\right) + \boldsymbol{P}\left(\boldsymbol{S}_{\boldsymbol{l},\boldsymbol{1}}|\boldsymbol{S}_{\frac{\boldsymbol{n}}{2},\boldsymbol{1}}\right) \tag{125}$$

where, L= I, II, …, n/2

3.3.2.7. Total System Transition Probability:

The overall system transition probability will be a summation of all the system level probabilities estimated using equations discussed in section 3.3.6. Hence, assuming $\frac{n}{2}$ sub-system assemblies, the ***Total System Transition Probability (TSTP)*** is given as:

$$P(I) + P(II) + \cdots + P(L) \qquad \textbf{(16)}$$

3.3.2.8. Comparison with the Threshold Limit (R):

The calculated value of TSTP can be compared with a given threshold limit *(R)* of the system. This comparison helps to analyse whether or not the system lies in the 'safe performance zone'. This threshold limit '*R*' can be estimated from the historical data of existing system performance; whereas in case of new products this value is obtained using expert's opinion. If the obtained TSTP value exceeds the given value of *R* then the system is considered to be in failed condition. It is important to note here that the value of TSTP should lie within a range of probability of 0 to 1 and in a situation otherwise; the system is considered to be in failed condition. Figure 3.3 demonstrates a framework of the proposed mathematical model.

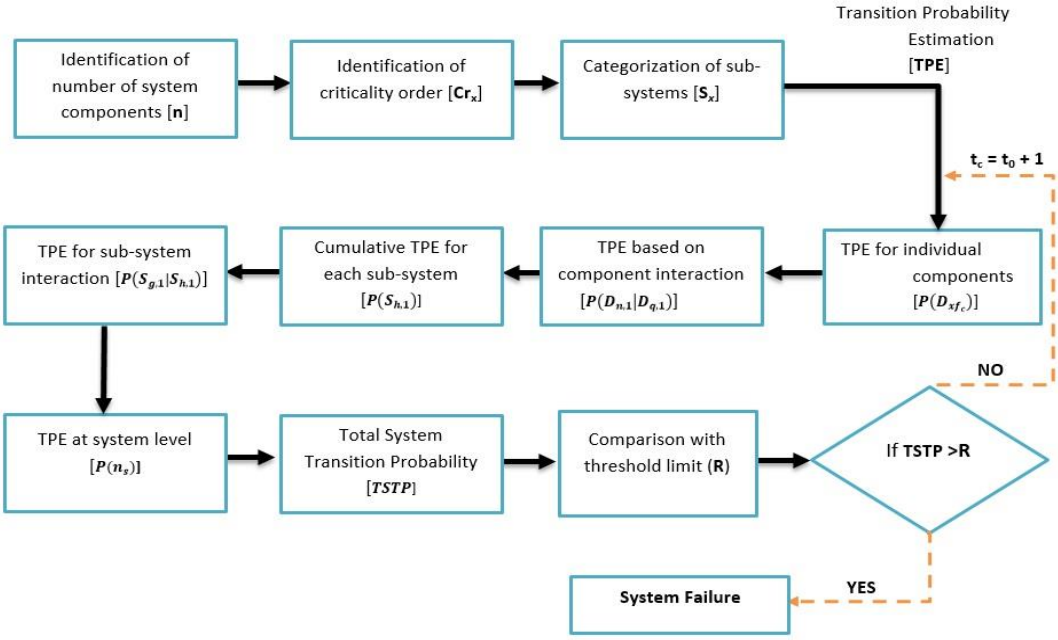

Figure 3.3: Reliability assessment framework for a MSMC system

The following section presents an algorithm procedure to assess the system reliability, taking into consideration the dependent nature of multiple components of a multi-state system.

3.4. Formulation of algorithm:

The algorithm stated below explains the procedure of assessing the system reliability with a system consisting of n components.

Set x *(number of components in the system)*,

R *(expected threshold limit for system performance)*,

Cr[x] *(criticality order of x components in the system)*.

While TSTP<R, *do the following.*

Initialize the system by setting:

M_x+1, *(performance levels for each component where, M_x = initial state of perfect performance for x^{th} component)*,

and

time t = t_0 *(initial time)*

M_x-k *(is the k^{th} degraded functional level for x^{th} component)*

While m< M_x+1; *(initialize value of m=0; where 'm' is a counter variable for increment after every iteration.)*

Generate Λ_x *(a state transition probability matrix for each component)*

Calculate $P(D_{xf_c})$ using **(1)**

Calculate $P(D_{mf_c}|D_{m'f_c})$ using **(2 & 3)** *(component interaction probabilities for all the identified sub-systems with each component degrading to 'm' performance level)*

Calculate $P(S_{x_sf_s})$ using **(4 & 5)** *(cumulative sub-system level probabilities)*

Calculate $P(S_{x_s f_s}|S_{x_s' f_s})$ using **(6 & 7)** *(sub-system interaction probabilities for all the identified sub-system degraded to a performance level 'fs')*

Calculate P(I) & P(II) using **(8 & 9)** *(cumulative system level probabilities)*

Calculate TSTP using **(10)** *(Total failure probability at system level)*

If (TSTP > R)

Then exit.

Else

Set m=m+1;

3.5. Model Implementation and Simulation

The following section proposes a hypothetical case of new product with no past data available to study the degradation behaviour of system and its components. Let us consider a system with *four* components where each component is functionally associated to the other components. As stated in section 3.3, the transition of any component from a higher functional state to a lower functional state is due to the combined effect of degradation due to age as well as degradation due to the action of other associated components.

It is assumed that a single kind of failure process is acting on individual component and there are *four* discrete working levels for every component in the system before they reach a complete failure state. The system functions in such a way that component C_1 and C_2 are considered to be more critical than component C_3 and C_4. Therefore, the criticality order of the components can be defined as $\boldsymbol{C_1 > C_2 >> C_3, C_4}$.

On the basis of inputs taken from design team and software simulation analysis, a degradation path function for every component is identified. Using this path function as an input parameter, a state transition probability matrix (Λ_x) is generated for each component which gives the probability of transitions of individual component from its current state ***i*** to its successive lower state ***j***. This transition probability matrix is used to estimate the probability of degradation of individual component from the current state $\boldsymbol{M_k}$ to the next successive lower state $\boldsymbol{M_{k-1}}$ at time instant $\boldsymbol{t_k}$ **.** The following matrices give the probability of transition of C_1, C_2, C_3 & C_4 after a defined time interval.

The probability of C_1, C_2, C_3 and C_4 degrading from a perfect functioning state M_x (x being the component number) to the successive lower state of performance is given by equation 1 i.e. $\boldsymbol{P(D_{11})}$ = 0.5375, $\boldsymbol{P(D_{21})}$ = 0.0429, $\boldsymbol{P(D_{31})}$ = 0.3605, $\boldsymbol{P(D_{41})}$ = 0.1731. Further interaction probabilities are calculated using equations 2 and 3 to assess the impact of degradation of components on each other. Hence, the interaction probabilities are computed as $\boldsymbol{P}(\boldsymbol{D}_{3,1}|\boldsymbol{D}_{1,1})$= 0.0134 and $\boldsymbol{P}(\boldsymbol{D}_{4,1}|\boldsymbol{D}_{2,1})$= 0.0807. The next level of computation involves assessing the probability of degradation of each subsystem ($\boldsymbol{S_1}$ & $\boldsymbol{S_2}$) to its successive lower level of performance which is calculated using equations 4 & 5. The cumulative probabilities of degradation of each subsystem is given as $\boldsymbol{P(S_{1,1})}$**=** 0.5510 and $\boldsymbol{P(S_{2,1})}$**=** 0.1236. A similar approach is used to assess the interaction probabilities at sub-system level as followed in the case of component interaction. These sub-system level interaction probabilities shall take into consideration the dependent behaviour at sub-system level. Using equations 6, the probability of $\boldsymbol{S_2}$ moving to lower state given that $\boldsymbol{S_1}$ has also degraded to a lower is given as $\boldsymbol{P}(\boldsymbol{S}_{2,1}|\boldsymbol{S}_{1,1})$= 0.0022. Further, these probabilities are used to record the cumulative system level probabilities ***P***(***I***)= 0.5064 & ***P***(***II***) = 0.1214. The total system transition probability of the system is given as ***TSTP***= 0.6278. The given thresholds limit (*R*) for performance of the system is 60% and clearly, the *TSTP* value is greater than *R*. Hence, we can derive the conclusion that with the given impact of failure process on system components the system shall fail to perform its intended function and fail after degrading to the first performance cycle at time t_1.

Above simulation is done for a system with an expected life cycle period of t_{max} units. The probability matrix Λ_x which estimates the independent transition of every component from state *i* to the next successive state *j* are obtained with simulation process as we have no past knowledge of component and system's behaviour considering the system to be a new product. It is to be noted here that the life cycle and cycle intervals are taken at a system level and hence the transition interval is common for every component. Whereas, the transition probabilities changes for every component depending on varying degradation forces and the operating environment and thus constitutes to different states of performance even at same time interval. Figure 3.4 shows the comparative transition of each component for a life cycle period of t_{max}.

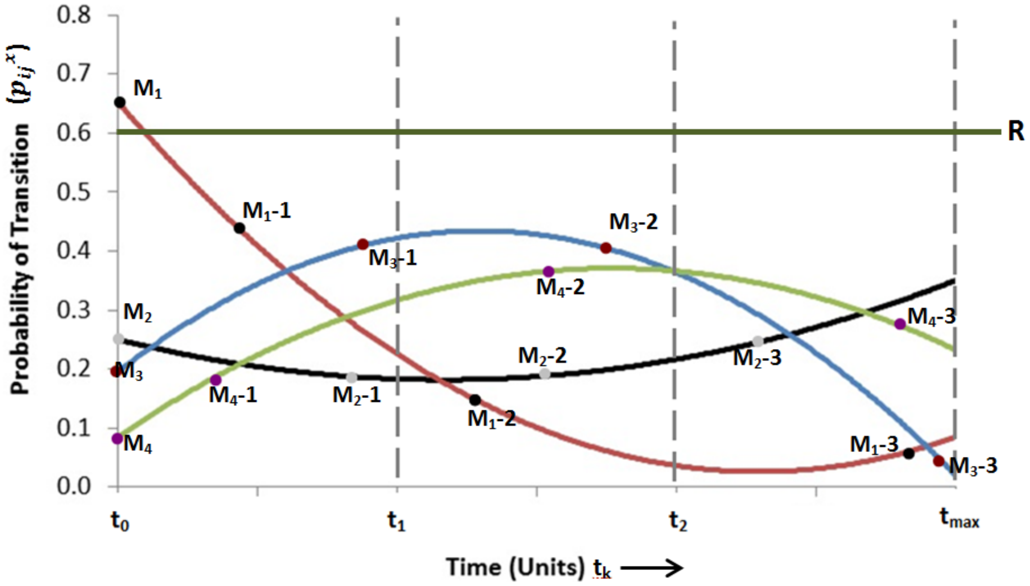

Figure 3.4: Variation in transition probabilities of different components for a definite cycle period

The graph explains variation in transition probabilities of each component for a defined and common cycle interval. Each transition probability ($\boldsymbol{p_{ij}}^{\boldsymbol{x}}$) denotes the probability of x^{th} component moving from one state to the next state. This variation can be increasing and decreasing in nature depending on the kind of environmental conditions and forces each component is exposed.

For instance, let us consider a cycle period of time t_1, the probability of transition of C_1 from its respective perfect state M_1 to the next lower state M_1-*1* is 0.3813, for the same cycle interval, the transition probability of C_2 moving from M_2 to the next lower state M_2-*1* is 0.0718. Similarly, the transition probabilities for C_3 and C_4 are shown by their respective curves in Figure 3.4. The variation in transition probability of every component at same time instant can be explained due difference in function performed by every component. As mentioned previously every component is exposed to different kind of environmental conditions and hence few of them may have high transition probability at initial time intervals and have decreasing transition probabilities once the component heads towards stable performance.

3.6. Results Analysis and Discussion:

The following section briefly discusses the results of proposed model and how the overall system transition probability can further be used to improve the design and maintenance schedule for any system and its components. As stated in previous sections, the given threshold limit *R* is identified either using the past performance data or using designer's expertise depending on whether the system is an existing one or a new one. In this case example, based on the input from design experts, the threshold value of *R* is considered as 60%. The system transition probability less than 60% indicates that the system is in good condition. It is considered to be in a failed condition if the estimated total system transition probability (TSTP) goes beyond the given threshold value (R). As seen in Figure 3.5, at time t_1 the TSTP value goes beyond the given threshold limit *R* of the system that essentially defines the failure time of the system.

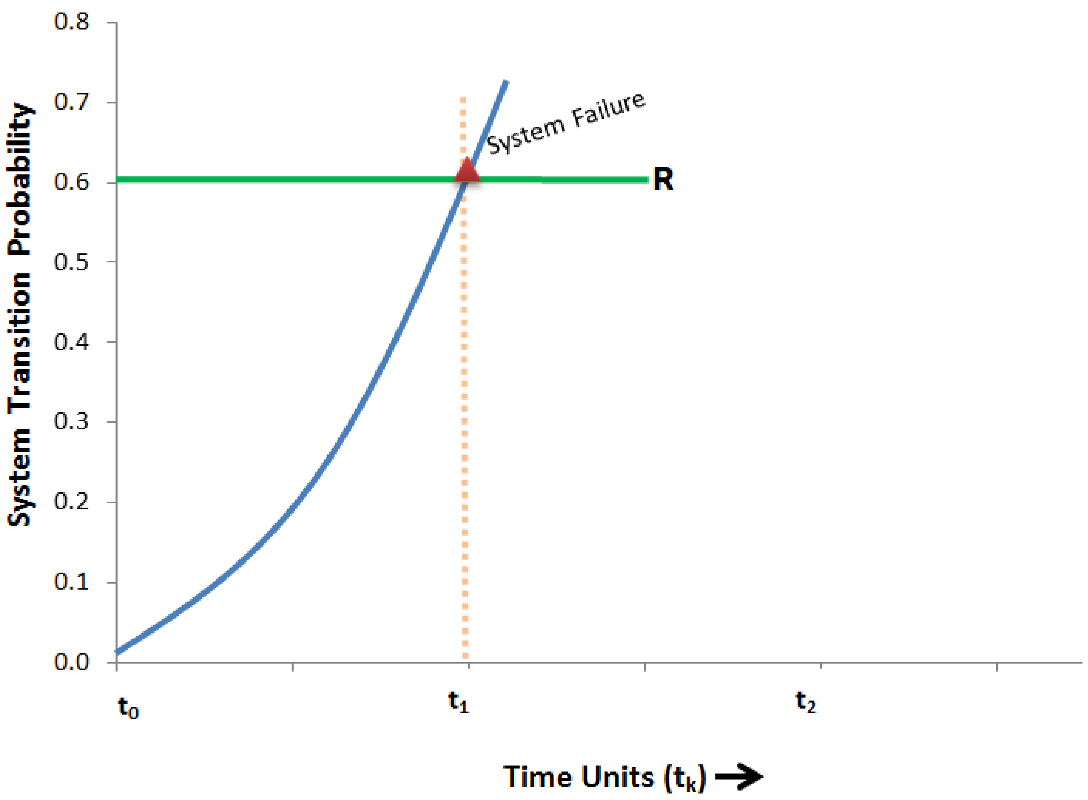

Figure 3.5: Total system transition probability exceeds the given threshold limit R

Hence, in case discussed in section 3.5, where the total system transition probability (TSTP) comes out to be 62.78% after a cycle interval time t_1 and the expected threshold limit R is given as 60%, the system is considered to be in failed condition. Thus we can conclude that the system possesses a tendency to fail after its first transition at t_1 time interval with a probability of 62.78%.

Once the overall system transition probability and the expected life cycle at which the system will fail are identified, it can be further used to determine a suitable maintenance and repair schedule for system as well as its components. The estimated system reliability can also help the design team to investigate areas of improvement in the existing design to improve the expected life cycle of any system.

Chapter 4
Preventive Maintenance and Warranty Plan

4.1. Introduction

Reliability of a product is one of the most important variables for both new and existing product design. Greater reliability implies more customer satisfaction and higher sales. Warranty is one of the few variables used in marketing the product’s reliability over a given period of time. Consumers will also incline to those products that promise a more sustainable performance. The competitive market scenario and numerous options open to buyers are resulting in complex product designs. Various buying options available to the customer are also affecting consumer’s purchase decisions. In such situations, post-sale factors – warranty, support level, maintenance, spare parts cost and their availability, etc., are important in the choice of the product. Of these, warranty is a one of the most influential factors that is known to the buyer at the time of purchase. Design complexity is further making it difficult to accurately evaluate reliability at both systems as well as components level. Therefore, it is important to precisely assess the reliability at each level of system design. The reliability of a system in use decreases with age due to product deterioration. This deterioration is affected by several factors, including environment, operating conditions and maintenance. The rate of deterioration can be controlled through preventive maintenance. Preventive maintenance (PM) over the warranty period has a greater impact on the warranty servicing cost. It is worthwhile to carryout maintenance as it affects the overall health of the product in future (Hussain & Murthy, 2003; Murthy & Jack, 2003; Park & Pham, 2012).

This chapter focus on the development of a structured approach to integrate the proposed MSMC system reliability assessment model with preventive maintenance schedule and warranty decision management. The subsequent section provides the background of relation between reliability and warranty management. Section 4.3 presents the integration of reliability assessment model with preventive maintenance schedule and further highlights the managerial implication warranty policy decision.

4.2. Background

Warranties are now recognized as an integral component of firms' strategic marketing plans. Chrysler, for example, effectively used warranty terms of its automobiles to generate greater consumer confidence in its products. Many other firms are likewise discovering that offering longer or more comprehensive warranties enables them to compete more effectively against foreign and domestic rivals (Agrawal, Richardson, & Grimm, 1996).

Research suggests that consumers believe that warranty terms are an important source of information regarding brand reliability along with financial and performance risk associated with purchase decisions (Bearden & Shimp, 1982; Shimp & Bearden, 1982). Boulding & Kirmani, (1993) reported the result of an experimental study in which consumers relied on warranty scope and manufacturer reputation to make inferences about brand reliability. In a nationwide survey, one out of every two consumers interviewed reported using warranty information to judge product reliability (U.S. Department of Commerce 1992). All of these studies suggest that consumers tend to regard more comprehensive warranty terms as indicative of superior product reliability. By relying on warranty terms to judge product reliability, consumers implicitly assume that warranty scope accurately reflects the relative reliability of brands.

Although both experimental and survey-based research indicate that consumers infer brand reliability on the basis of warranty information and that manufacturers regard the communication of warranty terms as important in their marketing efforts, the degree to which existing reliability approaches are successful in signalling warranty terms remains unclear (Agrawal et al., 1996). One needs to build model that link system reliability to warranty cost estimation (Jiang & Murthy, 1997; Murthy, 2007).

As mentioned earlier, offering warranty results in additional warranty servicing cost and this cost depends on several factors. These include the warranty terms, usage environment and intensity and the reliability of the product. The reliability of the product, in turn, is influenced by design and manufacturing decisions that are part of the technical aspects of the business. This implies that warranty offered (a commercial issue) and product reliability (a technical issue) are very closely linked and affect each other. Most manufacturing businesses treat the two separately with reliability treated as a pre manufacturing decision and warranty as a post-manufacturing decision. The interaction between the two is very critical for success of both new and existing products to improve business performance. As such, the two must be integrated as part of the product development process. The product life cycle provides the framework to link these two issues in an effective manner by integrating the technical and commercial implications.

The reliability of the product over its life cycle may vary considerably—a typical scenario is as shown in Figure 4.1 (from Blischke and Murthy, 2000). During the design stage, product reliability is assessed in terms of part and component reliabilities. It becomes difficult to assess the reliability of any system or product with an increase in design complexity and number of components. It is possible to increase the product reliability by closely monitoring the interaction between system components. It is also important to include the critical role played by individual component in successful system performance.

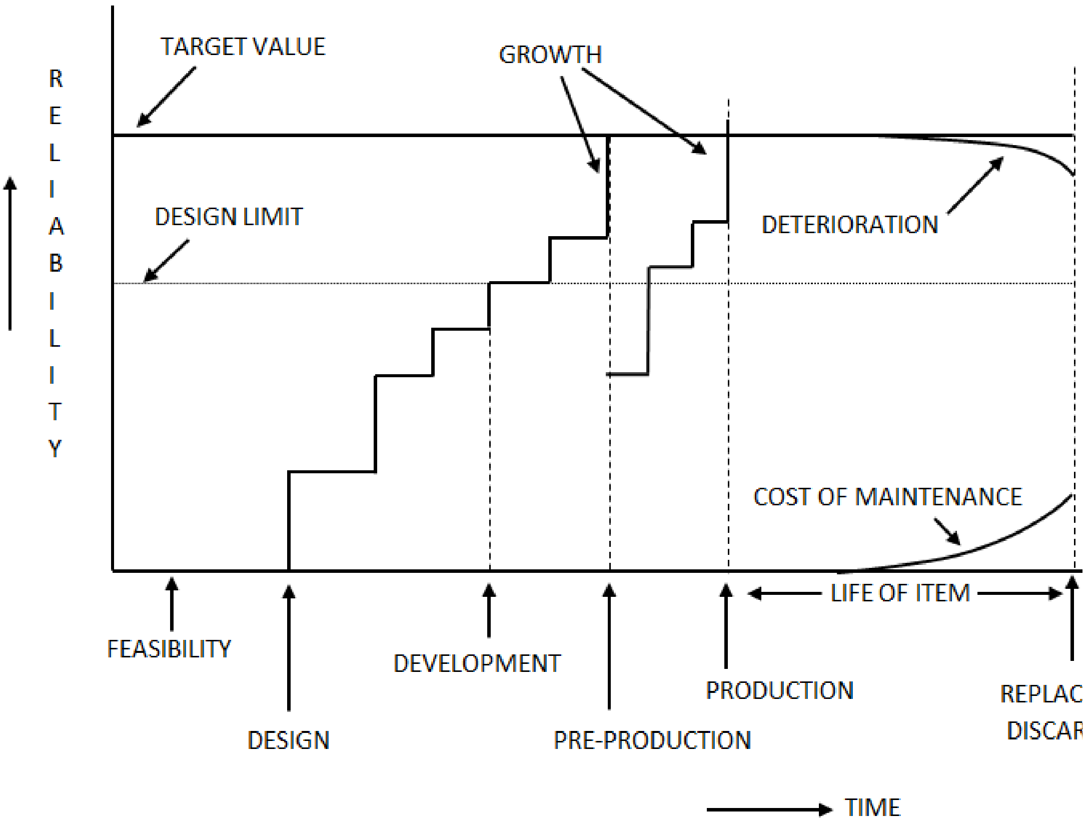

Figure 4.1: Reliability variation over the product life cycle (adopted from Murthy, 2006).

The reliability of an item in use decreases with age due to product deterioration. This deterioration is affected by several factors, including environment, operating conditions and maintenance. The rate of deterioration can be controlled through preventive maintenance.

Preventive maintenance (PM) over the warranty period has a greater impact on the warranty servicing cost. It is worthwhile to carryout maintenance as it affects the overall health of the product in future. Murthy and Jack (2003) have reviewed the literature pertaining to warranty and maintenance and suggested areas for future research. Park & Pham (2012) developed warranty cost models considering a periodic PM policy with both corrective maintenance and PM and also determines three decision variables including warranty period, repair time limit and periodic maintenance cycles. Huang & Yen (2009) have developed a two-dimensional warranty model in which the customer is expected to perform appropriate PM. The warranty policy that maximizes the manufacturers' profits is determined. Ben-Daya & Noman (2006) have developed an integrated model that simultaneously considers inventory production decisions, PM schedule and warranty policy for a deteriorating system that experiences shifts leading to an out of control state. Jung & Park (2003) have developed an approach for optimal periodic PM policies following the expiration of warranty by minimizing the expected long-run maintenance cost per unit time. Djamaludin et al. (2001) have developed a framework to study warranty and maintenance. Kim et al. (2004) have proposed a model to determine discrete time instants when PM actions are to be carried out over the warranty period. There were visible gaps in the aforementioned literature regarding an integrated approach for estimation of preventive maintenance time schedules of a multi-state multi-component system life cycle.

4.3. Implementation of MSMC system reliability assessment model in preventive maintenance

4.3.1. Nomenclature and Notations:

N	number of components in the system
n_s	number of subsystems in the system (Primary level)
R	threshold limit of system's acceptable working efficiency
M_x+1	discrete levels of functional performance for x^{th} component of the system
M_x	perfect functioning state for x^{th} component
$M_x - k$	k^{th} degraded functional level for x^{th} component

Λ_x	state transition probability matrix for x^{th} component.
${p_{ij}}^x$	probability of transition from state i to any lower state j for x^{th} component
G_{M_x}	sum of all transition probabilities when x^{th} component is in state M (perfect state)
$G_{M_{x-k}}$	sum of all transition probabilities when x^{th} component has degraded to k^{th} performance level
L	standard life of system
y_w	number of components under warranty claim
$\theta_{r_y}(t)$	repair cost of y^{th} component after time t
φ_{r_y}	replacement cost of y^{th} component

4.3.2. Assumptions

1. Each component at system level consists of M_x+1 state, where 0 is complete failure state and M_x is perfect functioning state.
2. The system only degrades with time and does not make transitions to a higher state.
3. Only a single transition occurs during one time instant ($t_{initial} = t_0$, next time instant (t_1) = $t_0+\tau$).
4. Every component follows a gradual and successive hierarchical pattern of degradation (i.e. states of degradation for any component x will be from $M_x \rightarrow M_x$ - 1; M_x -1 $\rightarrow M_x$ -2… M_x –k $\rightarrow$ 1 & 1 $\rightarrow$ 0). Here x =1, 2 …n.
5. The warranty time does not stop any moment.

4.3.3. Proposed Framework for system preventive maintenance schedule:

The following section presents a structured approach to decide the preventive maintenance schedule (PM) for various components of the system. This selection is done on the basis of transition state of every component which is estimated using the reliability assessment methodology as discussed in chapter 3. Figure 4.3 depicts the proposed framework for selecting optimal maintenance plan. The proposed approach also relates the financial aspect of warranty and preventive maintenance to service time selection decision.

Let $\boldsymbol{y_w}$ be the number of components under warranty claim. It should be noted that $\boldsymbol{y_w} \in \boldsymbol{n}$; where $\boldsymbol{n}$ is the total number of components in the system. The following are the cost attributes considered in the proposed framework:

$\boldsymbol{\theta_{r_y}(t)}$ = repair cost of $\boldsymbol{y^{th}}$ component; t is the time after which the component comes

for repair

$\boldsymbol{\varphi_{r_y}}$ = replacement cost of the $\boldsymbol{y^{th}}$ component

TCC = Total Cost Company = $C_{mf} + C_w + R_v + G_c$

where; $\boldsymbol{C_{mf}}$ **=** manufacturing cost of the system

$\boldsymbol{C_w}$ **=** Total warranty budget for the system {2-3% of TCC }

$\boldsymbol{R_v}$ = Reverse Logistics Cost

$\boldsymbol{G_c}$ = Goodwill cost

Table 4.1 represents the repair and replacement cost matrix for individual component of any MSMC system.

Table 4.1: Repair and Replacement Cost Matrix

Component	**Repair Cost** $\boldsymbol{\theta_{r_y}(t)}$	**Replacement Cost** $\boldsymbol{\varphi_{r_y}}$
y_1	$\theta_{r_1}(t)$	φ_{r_1}
y_2	$\theta_{r_2}(t)$	φ_{r_2}
. . .	. . .	. . .
y_w	$\theta_{r_w}(t)$	φ_{r_w}
TOTAL	$\sum \boldsymbol{\theta_{r_y}(t)}$	$\sum \boldsymbol{\varphi_{r_y}}$

The above cost data can be obtained from various part suppliers or production & manufacturing department of the case company.

Another important aspect of warranty decision is the time interval at which preventive maintenance is scheduled for various components of the system. The following time instants are taken under consideration for the proposed approach:

$\boldsymbol{t_{s_0}}$= system purchase time

$\boldsymbol{t_{s_1}}$= time of first service

$\boldsymbol{t_{s_2}}$= time of second service

.

.

.

$\boldsymbol{t_{s_w}}$= total warranty time

$\boldsymbol{PM_1}, \boldsymbol{PM_2}, \dots, \boldsymbol{PM_{s_w}}$ = preventive maintenance checklist for system's service, scheduled at

time instants t_{s_1}, t_{s_2},…, t_{s_w} respectively.

$\boldsymbol{t_{l_f}} = t_1 + t_2 + \cdots + t_l$

where; t_1 = time instant when system degrades to M_x *-1* state

t_2= time instant when system degrades to M_x *-2* state

t_l= time instant of final system transition before it fails

As stated above, $PM_1, PM_2, \dots, PM_{s_w}$ are the preventive maintenance checklist assigned for different service intervals. Further, each PM checklist includes a list of all the components to be serviced at different intervals.

The following table represents a generic form of PM checklists.

Table 4.2: Preventive Maintenance Checklist for System Components

PREVENTIVE MAINTENANCE SCHEDULE AND CHECKLIST	
Customer Name:	
Date of Purchase:	
Model Number:	
Unit #:	
$\mathbf{PM_1}$: $\mathbf{t_{s_1}}$ after system purchase.	
Task 1	$\boldsymbol{C_{p_{11}}}$: Component 1 of PM_1 to which task 1 is related
Task 2	$\boldsymbol{C_{p_{21}}}$: Component 2 of PM_1 to which task 2 is related
. . .	. . .
$\mathbf{PM_2}$: $\mathbf{t_{s_2}}$ after system purchase.	
Task 1	$\boldsymbol{C_{p_{21}}}$: Component 1 of PM_2 to which task 1 is related
Task 2	$\boldsymbol{C_{p_{22}}}$: Component 2 of PM_2 to which task 2 is related
. . .	. . .
$\boldsymbol{PM_{s_w}}$: $\boldsymbol{t_{s_w}}$ after system purchase.	
Task 1	$\boldsymbol{C_{p_{1s_w}}}$: Component 1 of PM_{s_w} to which task 1 is related
Task 2	$\boldsymbol{C_{p_{2s_w}}}$: Component 2 of PM_{s_w} to which task 2 is related

To reduce the replacement cost of various components, a comparative analysis is to be done between the actual service time interval provided by the manufacturer and the realistic time when the component needs repair. This realistic time is derived using the proposed MSMC reliability assessment model.

Let us assume a scenario where the warranty period given for a system is two years and the system is to be serviced after every six-month interval. If t_{s_0} indicates the time of product purchase then the first service will be provided at t_{s_1} and the last service at t_{s_4}. A parallel line above the vehicle service timeline shows the progressive cycle of vehicle life and t_0, t_1, t_2 and t_3 are the time of intermediate state transitions between *'perfect function'* to *'failed state'*. Figure 4.2 demonstrates the system service timeline along with its transition times from a higher state to a lower state. As the vehicle is designed to perform for a longer time period as compared to the warranty period, the product life cycle timeline exceeds the vehicle service timeline.

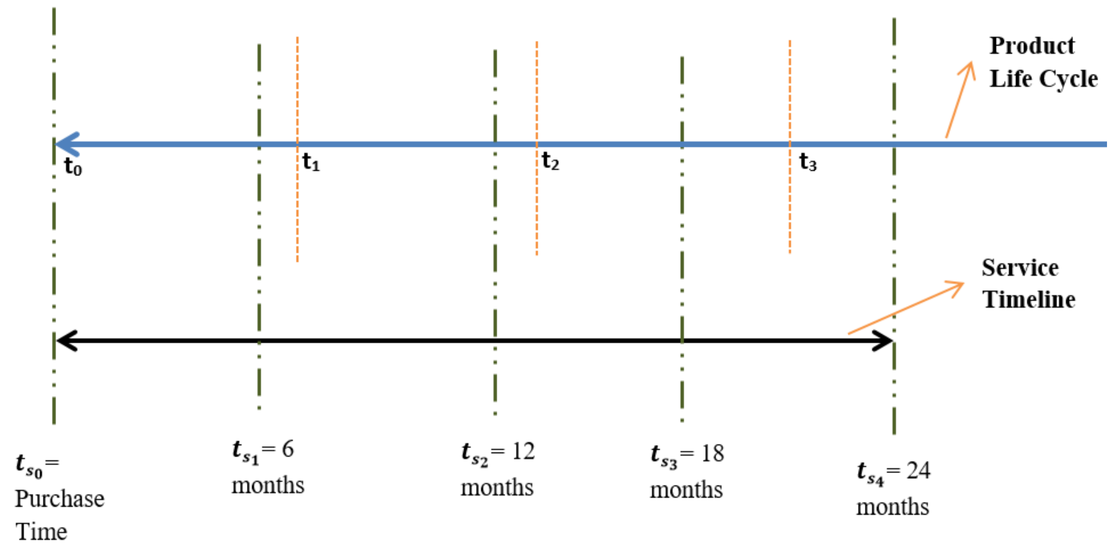

Figure 4.2: System Service Timeline

According to the existing maintenance plan, if the service of any component C_{p_3} is scheduled at t_{s_3} whereas the actual failure of the component is resulted (from MSMC system reliability assessment model) to happen somewhere between t_2 and t_3 then it would be suitable to get that component serviced at t_{s_2} instead of t_{s_3}. This would reduce the replacement cost of C_{p_3} to repair cost at time t_{s_2}. Also, prior service of C_{p_3} would save the future repair or replacement cost of other components which would degrade in future due to poor performance of C_{p_3}. A similar cost analysis estimation can be followed for each component of the MSMC system and an entire cost saving table can be generated to depict the overall replacement and repair savings. Figure 4.3 depicts the framework of the proposed preventive maintenance methodology.

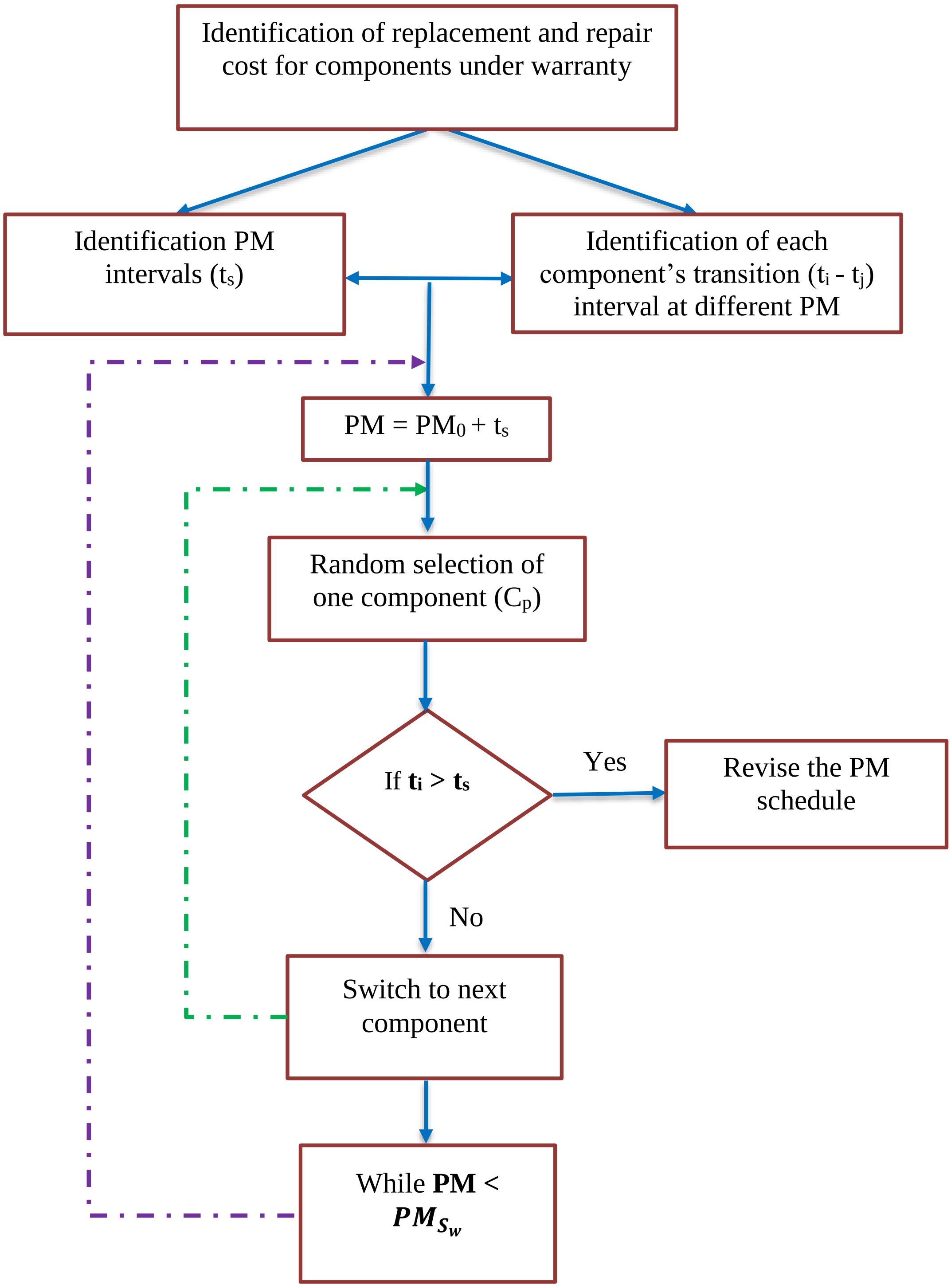

Figure 4.3: Preventive Maintenance Framework

The proposed framework shall reduce the overall replacement costs associated with different components. It will further lead to better warranty decisions based on the proposed preventive maintenance plan.

4.4. Chapter Summary:

In this chapter, the managerial application of proposed MSMC system reliability assessment is demonstrated in the area of preventive maintenance and repair. A general system repair and replacement cost matrix is developed using past degradation and warranty data for an existing product. A system service timeline is designed by estimating the reliability of the existing system using the proposed model. The main contribution of this chapter is that the framework proposed here gives a better and cost effective maintenance schedule for repair and replacement of system components. This is done by identifying the transition states of individual components at various time instants and further comparing them with the original service time instants. The application of proposed preventive maintenance (PM) framework (Figure 4.3) is demonstrated in chapter 5 using a case study. The impact of better PM service timeline on warranty decisions of the case company is also discussed.

Chapter 5
Case Study

5.1 Introduction

In this chapter, all the steps of the proposed MSMC system reliability assessment mode and preventive maintenance framework are illustrated in detail with the help of a case study. The idea of this case study is adopted from the work done by González Díaz et al., (2010). The company is a large manufacturer in the automotive industry that operates worldwide. It designs and manufactures and purchases a wide range of heavy and light duty vehicles for commercial use.

5.2 Model Implementation:

The given example demonstrates reliability assessment of a light duty pick truck and further proposes and improved maintenance plan for the same. The company has delivered a specific number of vehicles to its customers with a standard warranty of one year on the vehicle. There is an additional warranty of one year on few top vehicle parts as listed in Table 5.1.

A classification tree is used to explain different levels of vehicle assemblies, sub-assemblies and components as shown in Figure 5.1. The *primary level* constitutes of four major systems (electrical system, auxiliary system, hydraulic system and mechanical system) which are identified on the basis of their functional outcomes. The *secondary level* further comprises of all possible s*ub-assemblies* that enables their respective primary systems to perform its expected function. The *component level* is the last level in the vehicle design at which it is possible to perform the maintenance and repair. Usually, the number of levels stated above depends on the design and modularity of vehicle.

In order to have a better clarity and understanding of the model application, we have limited this study to a single, primary level system of mechanical assembly. However, the approach can be repeated to identify the time at which repair or replacement of other vehicle components is required.

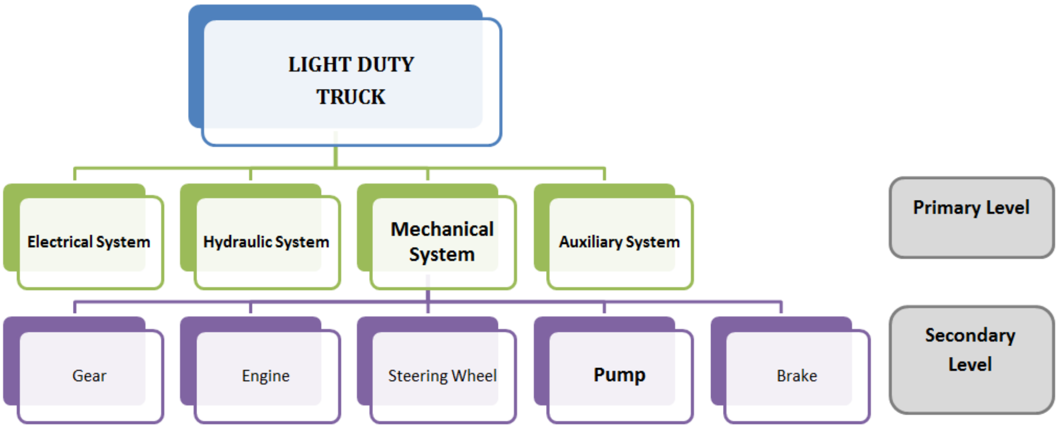

Figure 5.1: Classification of vehicle levels.

As the study takes place for an existing product; therefore, historical data regarding costs, failed items etc. are available for the research. A statistical survey is carried to gather data for a defined population of consumers and the number of claims done by them during a *standard vehicle life* **(L)**. In case of a new product with no past data, a set of probabilistic data can be generated using expert's opinion and test run data based on degradation behaviour of product and its components.

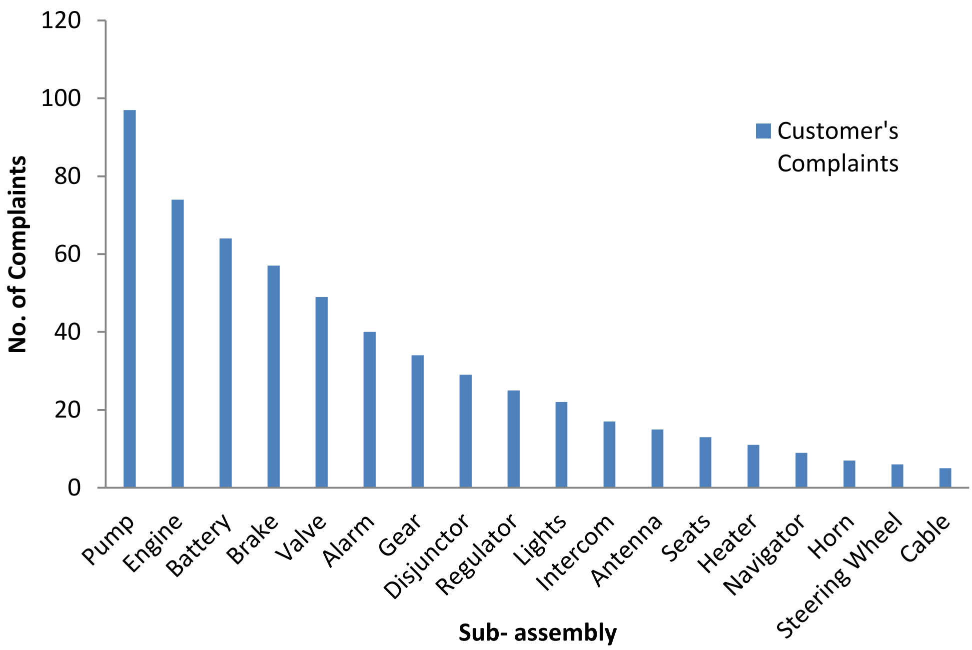

Figure 5.2: Statistical data for consumer complaints

Referring to the results of the statistical data collected for the case study, it is observed that highest claims have been made by the customers for hydraulic pump (Figure 5.2), followed by engine and other sub-assemblies of the vehicle and hence pump needs an immediate manufacturer's attention. Therefore, in order to simplify user understanding, the reliability and warranty assessment in this study is restricted to the component assembly with highest consumer claim i.e. pump.

Figure 5.3 shows component level of vehicle which is the root level of pump assembly. This classification is done using the knowledge of vehicle design and is limited to be the maximum level at which repair and replacement team can work for error rectification.

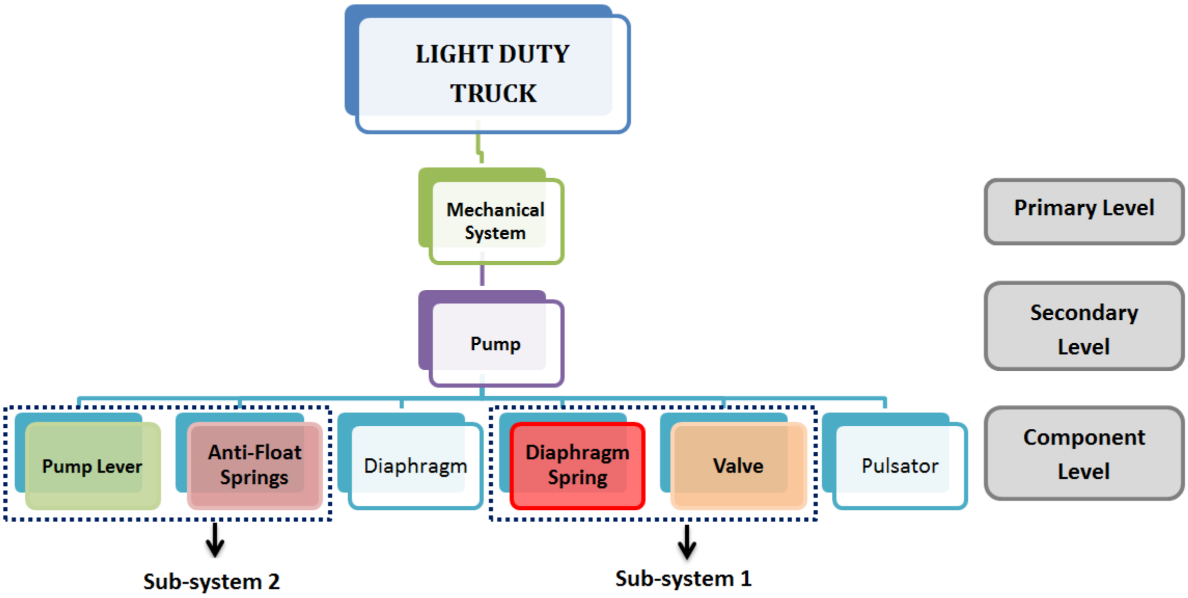

Figure 5.3: Pump categorization based on vehicle design modules*

***Source:** *Durability of fuel pumps in neat and aggressive E15, 2013.*

Component C_1 with critical order 1
Component C_2 with critical order 2
Component C_3 with critical order 3
Component C_4 with critical order 4

5.3. Stage I: Reliability assessment of the system:

In order to further perform a reliability assessment for the pump, a priority order is assigned to each of its parts. As shown in figure 5, the priority order for pump parts is diaphragm spring (C_1) > anti-float spring (C_2) > valve (C_3) > pump lever (C_4). This priority order is assigned using the knowledge of design expert's and the past data of repair and replacement. It has been observed that there were negligible cases of repair or replacement of diaphragm and pulsator. Hence, these parts can be ignored while the system reliability assessment. These parts are further grouped in two sub-systems on the basis of their function, design and criticality. In the figure above, sub-system 1 holds a high order of criticality as compared to sub-system 2.

Using the MSMC reliability assessment model proposed in chapter 3, the following state transition probability matrices are generated for individual component using past performance data and their expected degradation path function as inputs for matrix generation.

Diaphragm Springs (C_1)	M_1	M_1 - 1	M_1 - 2	1_1
M_1	0.2482	0.6898	0.0475	0.0145
$M_1 - 1$	0	0.1019	0.7971	0.1010
$M_1 - 2$	0	0	0.9347	0.0653
1_1	0	0	0	F
Time	t_0	t_1	t_2	t_{max}

Anti-float Springs (C_2)	M_2	M_2 - 1	M_2 - 2	1_2
M_2	0.4420	0.0784	0.2266	0.2531
$M_2 - 1$	0	0.6907	0.1534	0.1559
$M_2 - 2$	0	0	0.4690	0.5310
1_2	0	0	0	F
Time	t_0	t_1	t_2	t_{max}

Valve (C_3)	M_3	M_3 - 1	M_3 - 2	1_3
M_3	0.0825	0.1106	0.6479	0.1590
$M_3 - 1$	0	0.1132	0.5735	0.3133
$M_3 - 2$	0	0	0.7588	0.2412
1_3	0	0	0	F
Time	t_0	t_1	t_2	t_{max}

Pump Lever (C_4)	M_4	M_4 - 1	M_4 - 2	1_4
M_4	0.4053	0.0249	0.5172	0.0526
$M_4 - 1$	0	0.0012	0.7569	0.2420
M_4 - 2	0	0	0.9547	0.0453
1_4	0	0	0	F
Time	t_0	t_1	t_2	t_{max}

Further implementation of the proposed MSMC reliability assessment model discussed in chapter 3, the following degradation and interaction probabilities are calculated at both component as well as system level. The following values are obtained for the reliability assessment of the pump.

Probabilities of $\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$ and $\mathbf{C_4}$ degrading from state M to state M-1:

$\boldsymbol{P(D_{11})} = 0.4486$ $\boldsymbol{P(D_{21})} = 0.0714$

$\boldsymbol{P(D_{31})} = 0.5764$ $\boldsymbol{P(D_{41})} = 0.0427$

Interaction probabilities at component level:

$\boldsymbol{P(D_{3,1}|D_{1,1})} = 0.0257$ $\boldsymbol{P(D_{4,1}|D_{2,1})} = 0.0120$

Cumulative Sub-system Level Probabilities

$P(S_{1,1})$= 0.4743 $P(S_{2,1})$= 0.0594

Interaction Probabilities at Sub-system Level: $P(S_{2,1}|S_{1,1})$= 0.0013

Cumulative System Level Probabilities: $P(I)$= 0.3945 & $P(II)$ = 0.0582

Total System Transition Probability $(TSTP)_1$: **0.4527** **(From time t_0 to t_1)**

The above results are generated for a single transition between initial time t_0 to the next time interval t_1. A repetitive simulation of the for next state transitions, the following total system transition probabilities **(TSTP)**$_2$: 0.7035 **and (TSTP)$_3$:** 0.9647 are generated for time period between t_1 to t_2 & t_2 to t_3 respectively.

The expected efficiency of pump should not be less than 85% for successful vehicle performance and in a case otherwise, the pump will be considered to be in failed condition. Therefore, the threshold value R taken for comparison with TSTP comes out to be 0.85. Clearly, the value of $(TSTP)_3$ obtained from last transition is greater than the 0.85 and hence the pump is likely to fail somewhere t_2 and t_3 time.

The following section presents a comparative analysis of the warranty cost of the vehicle before and after the implementation of the proposed approach. A better preventive maintenance plan is proposed using the results obtained after estimation of vehicle reliability.

5.4. Stage II: Preventive Maintenance Schedule for the Vehicle

Table 5.1 below shows the list of all components that comes under the warranty claim along with their respective repair and replacement costs. It should be noted that the repair cost includes both labour as well as part cost. The repair cost varies over a given range for each component depending on the time of repair and condition of the component at the time of service.

Table 5.1: Repair and replacement cost of components with additional manufacturer warranty.

S. No.	Component	Claims	Repair Cost (θ_r)*	Replacement Cost (φ_r)*
1	Pump	97	$193 - $245	$ 475

S. No.	Component	Claims	Repair Cost (θ_r)*	Replacement Cost (φ_r)*
2	Engine	74	$950 - $1500	$ 6500
3	Brake	57	$200 - $750	$ 1200
4	Gear	34	$1400 - $1700	$ 3500
5	Steering Wheel	6	$350 - $500	$ 1300
	TOTAL			**$\sum \varphi_r$ = 12,975**

***Source:** *The Bureau of Automotive Repair (BAR) 2015*

The following data presents the manufacturing and the warranty cost of the vehicle along with other specifications indicated by the manufacturer prior to vehicle sale.

1.	Vehicle manufacturing cost	$5,54,875
2.	Total warranty budget (3.5 % of the manufacturing cost)	$20, 125
3.	Total cost of the vehicle	$5,75,000
4.	Vehicle Warranty	12 months or 36,000km (whichever comes first)

Few other costs such as reverse logistics cost and goodwill cost are not considered for calculation in this study for simplification purposes. However, these costs can be directly added to the warranty cost if data is available for the same.

Figure 5.4 shows the timeline of current service plan proposed by the manufacturer.

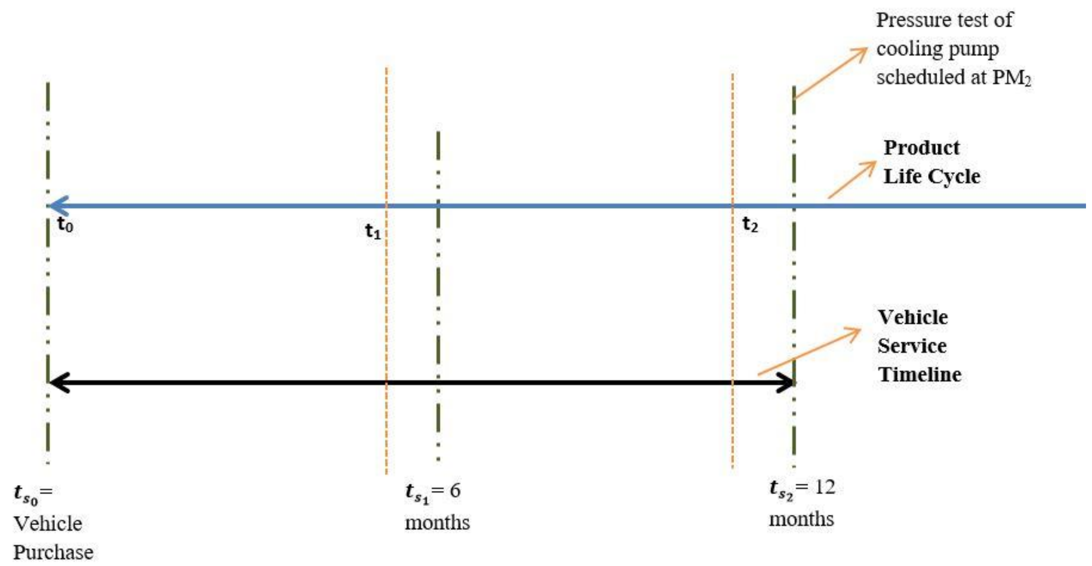

Figure 5.4: Vehicle service timeline

In above figure, the vehicle service is due after every six months, i.e. the company shall provide the service twice within the warranty period. The following preventive maintenance (PM) plan is suggested by the manufacturer for a warranty period of one year.

PM$_1$: 6 months after vehicle purchase or 18000 km (whichever comes first)

- Change engine oil and filter
- Change air filter (as needed)
- Check and fill all fluids
- Grease chassis
- Visual safety inspection

PM$_2$: 6 months after vehicle purchase or 18000 km (whichever comes first)

- Change engine oil and filter
- Change air filter (as needed)
- Check and fill all fluids
- Grease chassis
- Visual safety inspection
- Inspect all brakes
- Rotate tires
- Pressure test cooling system
- Load test battery and inspect terminals
- Complete 36000 km maintenance check list

It is observed from the given schedule that the first service (PM_1) is scheduled after six months of vehicle purchase and includes elementary services engine check, air filter check etc. The second service (PM_2) is provided after six months of the first service and includes advanced services like inspection of brakes, pressure testing of cooling system.

According to the existing maintenance plan, the pressure test of cooling system is scheduled at t_{s_2}, whereas the proposed reliability assessment model estimates the likelihood of pump failure between $\boldsymbol{t_2}$ and $\boldsymbol{t_3}$. Therefore, there is a possibility that the pump fails before the vehicle comes for second service at t_{s_2} and the manufacturer have to pay cost for replacing the pump along with a repair cost of other associated parts damaged due to poor pump performance. In order to avoid these high costs, it becomes important for the manufacturer to provide a timely check on the cooling system. This will save the manufacturer from unnecessary replacement cost along with retaining company's goodwill and customer satisfaction.

5.5. Results and Discussions

The following section provides an estimation of repair and replacement costs before and after the reliability assessment of the vehicle sub-assemblies.

It is clear from the present PM plan that the pump shall need a replacement when the vehicle comes for service at t_{s_2}. From Table 5.1 it is observed that the replacement cost of pump is $475. On the other hand, if the pump is checked at t_{s_1} it will only require a repair job instead of complete replacement. Therefore, maximum cost of repair for pump will be $245. This amounts to a total savings of $230.Thus it is suggested to perform 'pressure test cooling' at PM_1 i.e. time t_{s_1} instead of performing it at PM_2 i.e. time t_{s_2} (Figure 5.5).

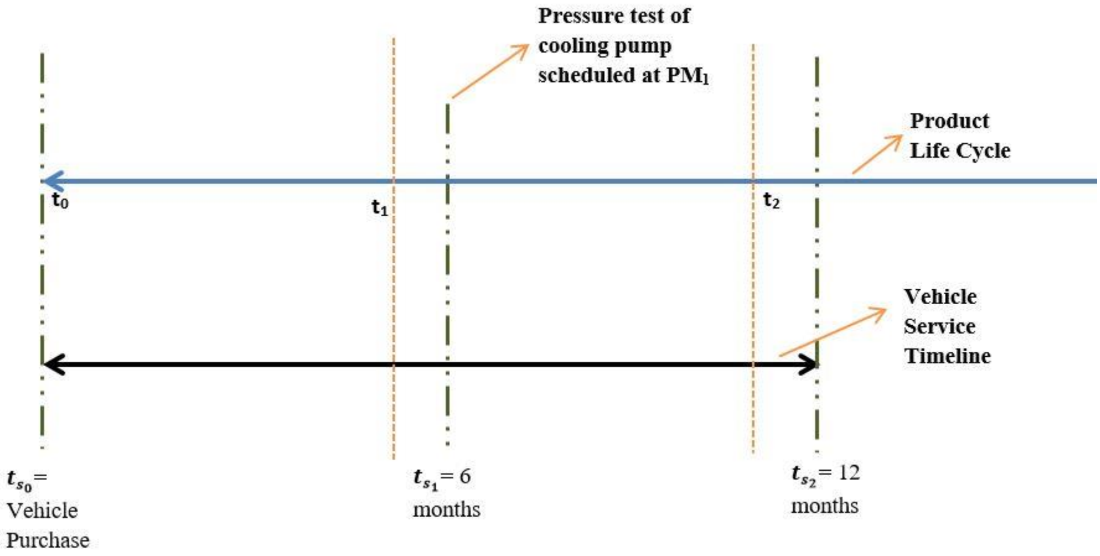

Figure 5.5: Proposed preventive maintenance schedule

A similar cost analysis can be done for other components that are listed in table 1 after performing individual reliability assessment for each component. There can be a subsequent improvement in the vehicle performance as the components shall be repaired well in advance before they reach to a failed state. It shall also save the associated components from degrading rapidly due the adverse effect of a particular degrading component. Thus, an increase in vehicle performance can give the manufacturer an edge over the existing competitors while making warranty decisions. In this case, the manufacturer can give an extended warranty of 6 months to get a stronger market hold.

Chapter 6
Conclusion

6.1 Introduction:

The final chapter presents a summary of the thesis. The emphasis is on how this work has contributed to the body of research in multi-state system reliability considering different intermediate states of performance at both system and component level.

Most of the conventional approaches reported in literature have estimated system reliability from a binary perspective of either *working* or *complete failure* (Kapur and Lamberson, 1977; Hoyland and Rausand, 1994). This assumption contradicts the real life scenario of functioning of any system and its components. An extant review 319 articles from the system reliability domain suggests that most of the physical systems have a tendency to perform at more than single state of working efficiency (Liu & Kapur, 2008; Summers, A. E., Ford, K., & Raney, 2007; C. Wang et al., 2014; Liudong Xing & Amari, 2008). These systems with more than single level of performance are known as multi-state systems and the intermediate states between perfect function and complete failure are known as '*transition states*'.

Reliability assessment of such multi-state systems have been addressed in two ways in the existing literature. First by *modeling system reliability based on the cause of failure* (Fiondella, 2010; Fiondella & Gokhale, 2010; Mitra et al., 2000; Modarres, 2011; Summers, A. E., Ford, K., & Raney, 2007; C. Wang et al., 2014) and *second by evaluation of multiple competing failure processes acting on the system* (Keedy & Feng, 2012; Lei Jiang et al., 2012; Rafiee et al., 2014; Song et al., 2014; Y. Wang & Pham, 2011a). Although, these studies have made significant contributions to multi-state system reliability, it was observed that certain issues pertaining to reliability assessment of multi-state systems with multiple components have not been addressed. Based on these, issues, the following research gaps were identified:

- Majority of the existing studies have assumed the system and its components to perform at same states. This assumption means that any system and all its components will be in same state at any time instant '*t*'. However, in reality, different components in a system architecture are exposed to varying environmental conditions, and hence the individual behaviour of each component cumulative constitute the system state.

- Another important issue in estimating multi-state multi-component (MSMC) system reliability is the consideration of the fact that deterioration of individual component is affected or sometimes even accelerated due to the correlative effect of other associated components. Most of the existing model have not evaluated system reliability from the perspective of component dependency.
- Additionally, each component in the system is intended to perform a distinct function which have a different importance in successful operation of the system and other relative components. Hence, it becomes necessary to take under consideration the critical nature of individual components while estimating multi-state multi-component system reliability.
- A good reliability model should not only be capable of estimating the overall system reliability but should simultaneously identify the cause of system failure. Therefore, it becomes very important from MSMC reliability assessment perspective to identify the components which may accelerate the system degradation process.

In this dissertation, a new reliability assessment model for multi-state system with multiple components is presented. The novelty of the proposed work lies in considering the dependency of individual component along with their criticality order in system functionality.

Section 6.2 provides a detailed discussion on the research contribution and findings. Section 6.3 illustrates the limitations of the present study and the ends with highlighting the future research opportunities.

6.2. Research Contribution:

The proposed approach provides reliability assessment for multi-state multi-component (MSMC) systems by considering dependency among components along with critical behaviour of individual component. Each component follows a discrete-time Markov process as it degrades over a given period of time. The component transition probability is captured in transition probability matrix for each component.

Each probability transition matrix considers various functional states for the component where the final state of the system is derived from the cumulative combination of individual component state. The proposed approach investigates the state dependence among components and their effect on the overall all system reliability.

The developed model also helps in identifying the performance states of critical components using transition probabilities which further helps to develop an improved and more accurate maintenance & repair measure prior to system failure. This improved preventive maintenance plan can reduce the overall warranty cost associated with the product. A comparative evaluation of the present and proposed maintenance plan can be used by the manufacturer to propose a better and more competitive warranty plan by giving repetitive repairs (preventive maintenance schedule) at component level thus avoiding complete part replacement or failure. This will help to reduce the overall warranty cost hauled by the company along with providing it competitive edge in the market.

The major contribution of this research are:

- There are a number of studies on multi-state system reliability assessment but not much work has been reported on modeling system reliability at different hierarchy levels of the system architecture. This study proposes a model that considers system transition at both component as well as assembly level.
- Majority of the reported work has addressed multi-state system reliability by modeling the independent and dependent nature of failure processes acting on it. This limits the reliability assessment only at broad system level. The proposed MSMC reliability assessment model considers the dependent nature of system assemblies by taking the physical as well as functional association of the components and sub-assemblies.
- The model considers the transition of individual component from a higher state of performance to a lower state of performance in two dimensions:
    i. deterioration of each component due its age effect where various external and internal factors cause exponential degradation of the component;
    ii. correlative effect on each component due degradation of other functionally associated components in the system.

- In any multi-component system, there are some critical components whose degradation may have a high impact on the overall system reliability. None of the existing research has considered this aspect in system reliability estimation. The proposed model successfully incorporates the priority orders of each component and assembly in the system.
- Another important contribution of the proposed model is that it provides a holistic approach for reliability assessment of MSMC systems by not only assessing the degradation behaviour of system and its components but also by identifying the components with very low states of performance which may lead to system breakdown or permanent failure.
- This study is further extended to integrate managerial aspects of preventive maintenance and warranty planning by developing a more cost effective repair and replacement schedule for a given system. A case study for an existing product is used to demonstrate the proposed preventive maintenance framework.

6.3. Limitations and Future Scope:

This study contains many unique mathematical approaches, concepts and procedures for MSMC system reliability assessment for application in modern industry. Any such research aimed at meeting the academic requirements is bound to suffer from certain limitations. This study is not an exception as well. While deliberating various issues related to the study reported in this thesis, a few points were noticed which could be identified as the limitations of the present work, some of which are as follows.

- The proposed model has assumed that a single kind of failure process is acting on individual components. The same model can be extended for multiple competing failure process acting on each component.
- Existing study has estimated system reliability with the assumption that the degradation of each component follows an exponential distribution. However, the degradation of individual component can be modeled more precisely based on its environmental and operational conditions.
- The transition of system and its components is modeled using a discrete-time Markov Chain process and every transition has a discrete probability value. Although, this assumption holds good for systems designed for medium life

cycle range, a continuous-time Markov Chain process is reported to give better results for wide range of product life cycle.

- The proposed reliability model can also be extended to consider both external and internal causes of failure for both system and its components.
- The proposed preventive maintenance framework can be extended to develop inventory plan for various system components and their parts.

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Appendix-I: MATLAB Code

```
% STEP I : CALCULATING DEGRADATION PROBABILITY OF INDIVIDUAL COMPONENT:
4-142 %


% INPUT A SQUARE MATRIX/ELEMENTS ARE EXPONENTIALLY DISTRIBUTED Fn
% row=i(1 to 4), column =j(1 to 4)
clc
clear all
x = 4;
P = randi([1,8],x,x);
%P = randi(x,x)
    for i=1:x
        for j=1:x
            if (i>j)
            P(i,j)=0;
            elseif (i==x)&(j==x)
                P(i,j)=1;
            else
                %P(i,j)= 1/exp(P(i,j))
                P(i,j)= P(i,j);
            end
        end
       i=i+1;
    end

  P;

C1=P

%row_sum is matrix containing the sum of each row of C1
 row_sum1= sum(C1,2);
 C1_norm = bsxfun(@rdivide, C1, row_sum1); %to generate a normalized
matrix
 C1_norm

%calculating column sum for C1_norm
column_sum1 = sum (C1_norm, 1);  %to calculate the sum of each column
for matrix S_norm

%calculating degradation probability for one lower level: M to M-1
P11_first= C1_norm(1,2)/((column_sum1(1,1))-(column_sum1(1,2)))
P11_second = (exp(-column_sum1(1,2)))* (exp(-column_sum1(1,1)))
P11= P11_first * P11_second


% MODULE 2 FOR PROBABILITY CALCULATION OF C2 FROM M TO M-1

%component C2 matrix
C2 = randi([1,8],x,x);
%P = randi(x,x)
    for i=1:x
        for j=1:x
            if (i>j)
            C2(i,j)=0;
            elseif (i==x)&(j==x)
                C2(i,j)=1;
```

```
            else
                %Q(i,j)= 1/exp(Q(i,j));
                C2(i,j)= C2(i,j);
            end
        end
       i=i+1;
    end
    C2

 %row_sum is matrix containing the sum of each row of C2
 row_sum2= sum(C2,2);
 C2_norm = bsxfun(@rdivide, C2, row_sum2); %to generate a normalized
matrix
 C2_norm

%calculating column sum for C2_norm
column_sum2 = sum (C2_norm, 1);  %to calculate the sum of each column
for matrix C2_norm

%calculating degradation probability for one lower level: M to M-1
P21_first= C2_norm(1,2)/((column_sum2(1,1))-(column_sum2(1,2)))
P21_second = (exp(-column_sum2(1,2)))* (exp(-column_sum2(1,1)))
P21= P21_first * P21_second


% MODULE 3 FOR PROBABILITY CALCULATION OF C3 FROM M TO M-1

%component C3 matrix
C3 = randi([1,8],x,x);
%P = randi(x,x)
    for i=1:x
        for j=1:x
            if (i>j)
            C3(i,j)=0;
            elseif (i==x)&(j==x)
                C3(i,j)=1;
            else
                %Q(i,j)= 1/exp(Q(i,j));
                C3(i,j)= C3(i,j);
            end
        end
       i=i+1;
    end
    C3

 %row_sum is matrix containing the sum of each row of C3
 row_sum3= sum(C3,2);
 C3_norm = bsxfun(@rdivide, C3, row_sum3); %to generate a normalized
matrix
 C3_norm

%calculating column sum for C3_norm
column_sum3 = sum (C3_norm, 1);  %to calculate the sum of each column
for matrix C3_norm

%calculating degradation probability for one lower level: M to M-1
P31_first= C3_norm(1,2)/((column_sum3(1,1))-(column_sum3(1,2)))
P31_second = (exp(-column_sum3(1,2)))* (exp(-column_sum3(1,1)))
P31= -1 * P31_first * P31_second  % special introduction of -1 component
to avoid negative value of P31
```

```
% MODULE 4 FOR PROBABILITY CALCULATION OF C4 FROM M TO M-1

%component C4 matrix
C4 = randi([1,8],x,x);
%P = randi(x,x)
    for i=1:x
        for j=1:x
            if (i>j)
            C4(i,j)=0;
            elseif (i==x)&(j==x)
                C4(i,j)=1;
            else
                %Q(i,j)= 1/exp(Q(i,j));
                C4(i,j)= C4(i,j);
            end
        end
       i=i+1;
    end
    C4

 %row_sum is matrix containing the sum of each row of C4
 row_sum4= sum(C4,2);
 C4_norm = bsxfun(@rdivide, C4, row_sum4); %to generate a normalized
matrix
 C4_norm

%calculating column sum for C4_norm
column_sum4 = sum (C4_norm, 1);  %to calculate the sum of each column
for matrix C4_norm

%calculating degradation probability for one lower level: M to M-1
P41_first= C4_norm(1,2)/((column_sum4(1,1))-(column_sum4(1,2)))
P41_second = (exp(-column_sum4(1,2)))* (exp(-column_sum4(1,1)))
P41= -1 * P41_first * P41_second  % special introduction of -1 component
to avoid negative value of P41


% STEP 2: CALCULATING INTERACTION PROBABILITIES AT COMPONENT LEVEL:
147-150 %
P11_P31= 0.02;   % Impact of degradation of C3 on C1
P31_P11 = ((P31)*(P11_P31))/(P11)  % Impact of degradation of C1 on C3

P21_P41= 0.02;    % Impact of degradation of C4 on C2
P41_P21 = ((P41)*(P21_P41))/(P21)  % Impact of degradation of C2 on C4


% STEP 3: CALCULATING CUMULATIVE SUBSYSTEM LEVEL PROBABILTIES: 155-156
%

P_S11 = P11 + P31_P11   % Probability that sub-system 1 will move to a
lower state M_S1-1
P_S21 = P21 + P41_P21   % Probability that sub-system 1 will move to a
lower state M_S2-1


% STEP 4: CALCULATING INTERACTION PROBABILITIES AT COMPONENT LEVEL:
160-165 %
```

```
P_S11__P_S21_B = 0.01;  %Effect of sub-system 2 on sub-system 1: Baye's
Value
P_S21__P_S11 = ((P_S21) * (P_S11__P_S21_B)) / P_S11  % Effect of sub-
system 1 on sub-system 2

P_S21__P_S11_B = 0.01;  %Effect of sub-system 1 on sub-system 2: Baye's
Value
P_S11__P_S21 = ((P_S11) * (P_S21__P_S11_B)) / P_S21  % Effect of sub-
system 2 on sub-system 1



% STEP 5: CALCULATING CUMULATIVE SYSTEM LEVEL FAILURE PROBABILTIES:
171-172 %

I= P_S11 + P_S11__P_S21   %Failure probability due to self-degradation
of sub-system 1 and effect of sub-system 2 on sub-system 1
II = P_S21 + P_S21__P_S11  %Failure probability due to self-degradation
of sub-system 2 and effect of sub-system 1 on sub-system 2

%STEP 6: CALCULATING TOTAL FAILURE PROBABILITY (TFP): 175-176 %
TFP = I + II
```

Appendix-II: List of Publications

- Jain N, Yadav O.P., Rathore A. & Jain R., (2017), “Reliability Assessment Framework for a Multi-State Multi-Component System”, *Journal of Industrial and Production Engineering (Taylor and Francis),* 1-10.

- Jain N, Yadav O.P., Rathore A. & Jain R., “A reliability and warranty framework for a multi-state multi-component system with prioritized functionality order”, *Journal of Industrial Engineering and Management (Elsevier),* [Under Review].

- Jain N, Yadav O.P., Rathore A. & Jain R., (2017), “An Approach for Multi-State Multi-Component System Reliability Considering Dependency Behavior*”, Proceedings of the 63rd Annual Reliability and Maintainability Symposium (IEEE),* Florida, USA.

- Jain N, Yadav O.P., Rathore A. & Jain R., (2016), “Multi-State Reliability Framework for a Multi-Component System”, *In proceedings of the 2nd International Conference on Mathematical Techniques in Engineering Applications*, Dehradun (India).

- Jain N, Yadav O.P., Rathore A. & Jain R., “A comprehensive review on multi-state system reliability models”. (IEEM) [Communicated]

Appendix-III: Biographical Profile of Researcher

Niketa Jain is born in Ajmer, Rajasthan (India). She did her B.E. in Mechanical Engineering from Government Engineering College Ajmer, Rajasthan (India) and M.Tech. in Manufacturing Systems Engineering from Malaviya National Institute of Technology Jaipur, Rajasthan (India). She is presently working as Assistant Professor for Automobile Engineering Department, School of Automobile, Mechanical and Mechatronics Engineering, Manipal University, Jaipur and pursuing Ph.D. from Malaviya National Institute of Technology Jaipur, India. She has over one year of industrial experience as a product design engineer at Cummins Research and Technology, Pune, India. Her research interests are focused around reliability and quality engineering, product development, operations management and preventive maintenance and warranty management.