EVALUATION OF CRITICAL COMBINATION OF GEOTECHNICAL PARAMETERS AND ASSESSMENT OF SEISMIC FRAGILITY CURVES FOR ROCK SLOPES

Submitted in partial fulfillment of the requirements for the award of degree of

Master of Technology

In

DISASTER ASSESSMENT AND MITIGATION

CIVIL ENGINEERING

Submitted by

Gaurav Fulwaria

(2014PCD5196)

Supervised by

Dr. Rajib Sarkar
&
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DEPARTMENT OF CIVIL ENGINEERING

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

JUNE 2016

A

DISSERTATION REPORT

ON

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DEPARTMENT OF CIVIL ENGINEERING

JAIPUR 302017

DECLARATION

I hereby certify that the work which is being presented in the dessertation report entitled **"EVALUATION OF CRITICAL COMBINATION OF GEOTECHNICAL PARAMETERS AND ASSESSMENT OF SEISMIC FRAGILITY CURVES FOR ROCK SLOPES"**, in partial fulfillment of the requirements for the award of the Degree of Master of Technology and submitted in the Department of Civil Engineering of the Malaviya National Institute of Technology Jaipur is an authentic record of my own work carried out during a period from August 2015 to June 2016 under the supervision of Dr. Rajib Sarkar, Assistant Professor and Dr. M. K. Jat, Associate Professor, Department of Civil Engineering, Malaviya National Institute of Technology Jaipur, India.

The matter presented in the report has not been submitted by me for the award of any degree of this or any other Institute.

(**GAURAV FULWARIA**)
Student ID: **2014PCD5196**

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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(Gaurav Fulwaria)

ABSTRACT

The uncertainties associated with geotechnical system in the form of material variability and seismic ground motion, pose great challenges to engineers. To account for these uncertainties in the planning of geotechnical infrastructure in mountainous regions, this study aims to highlight the applicability of reliability analysis and seismic fragility curves in assessment of slope performance.

In this study the First Order reliability Method (FORM) of slope under Pseudostatic condition is presented along with the development of fragility curves. Reliability analysis is done using Response surface Method (RSM) and FORM. Response surface method is used to generate the performance function *i.e.* the relation between the input variables and output response. Subsequently, fragility curves are generated using numerical analysis under dynamic condition.

Reliability analysis aims to quantify the system performance in terms of probability of failure. In addition, fragility curve defines the relation between the damage states corresponding to increasing intensity of ground motion. The study also presents significant computational advantage of reliability analysis over conventional methods.

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Chapter 1

INTRODUCTION

1.1 Introduction to Uncertainty

Uncertainties in geotechnical systems pose major challenges to geotechnical engineers. To ensure satisfactory performance of geotechnical facilities a due consideration of the associated uncertainties is required. Reliability based approach provides a robust and logical framework to incorporate these uncertainties in decision making process during preliminary design studies. The reliability based approach majorly aims to quantify the system performance in terms of probability of unsatisfactory performance or failure.

In the field of geotechnical engineering, the assessment of stability of slopes is perhaps a major field associated with risks arising due to the variable nature of the geological medium. Thus, probabilistic based approaches have found wide application in stability assessment in a number of past investigations. One such famous approach involves the assessment of reliability index along with the identification of design points.

In addition to the material variability, another form of uncertainty associated with safe design of slopes is due to the variable seismic motion which may occur to during the intended life cycle of the engineered slope especially in seismically active regions. In this regard, seismic fragility curve which describes the probability of exceeding particular performance level under variable ground shaking has been widely used for seismic vulnerability assessment. The purpose of this study is to demonstrate the use of reliability based approach and seismic fragility curves in assessing slope performance under variable geological settings and seismic scenarios.

1.2 Importance of Slope Stability Assessment

Cases of slope failure have been documented throughout history caused either by human activities or natural events driving the balanced slope towards instability. To ensure satisfactory slope performance, slope stability assessment considering the probable conditions assumes great significance. The primary purpose of slope stability analysis is to ensure safe and economic design. The slope stability analysis is basically conducted with an aim to achieve one or more of the following tasks:

- To assess the stability of slopes under short term and long term conditions.
- To assess the possibility of slope failure
- To understand failure mechanisms and influence of different factors.
- To study the effects of seismic loading on slopes and embankments.

1.3 Uncertainties and Variability in Soil

Physical properties of soils vary from place to place within resulting deposits thereby exhibiting inherent variability. The variability is generally categorized in terms of strength and deformation parameters such as elastic modulus (E), unit weight (γ), cohesion (c), friction angle (ϕ), Poisson's ratio (μ) etc. (Phoon and Kulhawy 1999).

The major challenges in geotechnical engineering arise from these uncertainties and hence they are advised to be incorporated in analysis and design phase. In addition, the geotechnical performance of a specific site, facility, system or regional geotechnical project may also be affected by other types of uncertainty such as the following (Bhattacharya *et al.* 2012):

1. Geological uncertainty (geological details).
2. Geotechnical parameter uncertainty (variability of shear strength parameters and of pore water pressure).
3. Hydrological uncertainty (aspects of groundwater flow).
4. Uncertainty related to historical data (frequency of slides, falls or flows).
5. Uncertainty related to natural or external events (magnitude, location and timing of rain/storm, flood, earthquake and tsunami).
6. Project uncertainty (construction quality, construction delays).
7. Uncertainties due to unknown factors (effects of climate change)

1.4 Reliability based Approach

Reliability methods are a class of probabilistic approach which allows systematic consideration of uncertainties and evaluation of system performance (Cherubini and Vessia 2009). Reliability approach incorporating random variables in computation of safety of slope have been widely used in the past (Dai and Wang 1992). It overcomes the limitations of conventional deterministic approaches providing better flexibility in making decisions with regard to alternate designs possible. This approach allows assessment of the likelihood that a particular slope section will have a higher fail0ure probability than the failure probability of the critical deterministic failure surface (Dodagoudar and Venkatachalam 2000).

1.5 Major Goals of the Study

Major goals set for the present dissertation work are as following.

(i) This thesis aims to understand the concept of reliability approach for evaluation of stability of slopes.

(ii) Brief review of literature dealing with reliability methods along and numerical solution of slope stability.

(iii) Understanding the basic modeling aspects of finite element method for pseudo-static and rigorous dynamic problems.

(iv) Utilization of Response Surface Methodology for framing performance function of slope considering pseudo-static seismic loading.

(v) Development of seismic fragility curves through rigorous dynamic analyses.

1.6 Organization of the Report

Keeping in view of the goals, the dissertation report is organized as following.

Chapter 2 discusses literature of slope stability analyses dealing with reliability based formulations.

Chapter 3 presents different methodology adopted for the study.

Chapter 4 presents probabilistic assessment of slope stability utilizing *First-Order Reliability Method* (FORM)

Chapter 5 presents development of seismic fragility curves for the most vulnerable slope configuration identified in *Chapter 4*.

Chapter 6 summarizes the report and concludes.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

Stability analysis of rock slopes has always been critical and challenging task to the geotechnical engineering professionals. The criticality of rock slopes is mainly attributed to the heterogeneous and anisotropic nature of the rock mass. Assessment of stability becomes more challenging when slopes are subjected to earthquake motions which also very random in nature. So uncertainties in both material and loading parameters need to be dealt with carefully for ensuring satisfactory performance of any slope. Reliability analysis is a tool to consider uncertainties in a very scientific way. This chapter mentions literature which is majorly being adopted for the present study for carrying out reliability analyses of a rock slope to come out with the critical design parameter and development fragility curves for assessment of seismic vulnerability of the critical slope.

2.2 Literature on Reliability Analyses

The rock mass is considered to have certain uncertainties in geometry and material properties. This poses challenge to engineers to consider these uncertainties in analyses and subsequent designs. Reliability analysis is a probabilistic approach which allows the systematic analysis of uncertainties and their inclusion in evaluation of performance of system.

Low and Tang (1984) discussed a method of calculating Hasofer-Lind second moment reliability index based on spreadsheet. Correlation is also accounted for setting quadratic form in spreadsheet. The method is based on perspective of an ellipsoid that is tangent to the failure surface in the original space of the variables. The reliability index reflects the effects of mean values and covariance of the random variables influencing the design.

Dodagoudar and Venkatachalam (2000) presented that a probabilistic approach using sets allowing for a logical and systematic analyses of uncertainties. They discussed reliability analysis in their study to process the fuzzy uncertainties in a slope. Their study shows that a slope have higher probability of failure than the critical failure probability estimate by deterministic approaches. They addresses the effects of correlation coefficient of the uncertain variables on the stability of slope.

Low (2007) discussed applicability of reliability analysis of slopes with correlated non normals. He carried out his study on *San Mau Ping* slope of Hong Kong. It is clear from the results that reliability method is faster than other conventional methods. The constrained optimization approach presented for reliability analysis can be used to approximate the limit state surface near the design points.

Low and Tang (2008) discusses the *First Order Reliability Method* which is very efficient in considering uncertainties in geotechnical system involving correlated non normal random variables. They demonstrated the method in spreadsheet by FORM analysis. They have focused on *Hasofer-Lind* reliability index and distinguish negative from positive reliability index. They performed a slope analysis of a clay slope in Norway with spatially auto-correlated soil properties. The new procedure gives same results as the classical Hasofer-Lind method but in faster manner.

Babu and Shrivastava (2008) presented the reliability analysis performed on an earth dam. They used *Response Surface Method* to formulate the performance function in combination with *First Order Reliability Method* and analyses results. Reliability Index is obtained and interpreted in terms of probability of failure. They compared their results with Monte Carlo Simulation and limit equilibrium methods. They found that consideration of variability in geotechnical parameters and seismic uncertainties play significant role in deciding the probability of failure. This approach is also helpful in identifying sensitive parameters involve in the analysis. The main advantage is that it takes less computational time as compared to other methods. Also the method is helpful in decision making for the quality control measures.

Song Ki-II et al. (2010) state that uncertainties in load, geometrical shape and material properties can cause unfavorable and unexpected response to the system in geotechnical design and analysis. The spatial variability of the ground affects the ground reaction and limits the amount of reliable information. Hence, the incorporation of uncertainties in design and analysis is important. They recommended that stochastic numerical analysis should be performed on design by considering the inherent spatial variability of geotechnical parameters.

Shukla and Hossain (2011) discussed about the discontinuities and variability's in rock mass and quoted that the there is not any slope that can be regarded as fully guaranteed for stability during their serviceable life. They explained types of rock slope failures such as plane failure, wedge failure, circular failure, toppling failure and buckling failure. Equations are presented for the factor of safety of multi directional anchorage rock slope against plane failure incorporating seismic variability.

Chowdhury et al.(2012) discusses about the variability of soil and rock mass. The major challenge for geotechnical engineers is to incorporate variability's (such as geological uncertainties, hydrological uncertainties, uncertainties in historic data etc.) in design procedure. The methods for the inclusion of such variability in design can be done by Reliability analysis with a probabilistic framework. They also discussed about the slope stability assessment and the use of *Geographic Information System* (GIS) in organization, processing, managing and updating of spatial and temporal information concerning geological, geotechnical, topographical and other parameters.

Myers et al. (2012) dealt with method of optimization of response surface. They explained the importance of RSM technique in design, development and formulation of new products as well improvement of existing products. The method of approximating the response function is explained in detail. The RSM plays important role in geotechnical system as variation in key parameter can result in worst situation for the system.

Arteaga and Soubra (2014) explained that reliability analysis method is most accurate method to incorporate the uncertainty in geotechnical system in a rational manner. They discussed the various methods of *First Order Reliability Method* (FORM) and the *Second Order Reliability Method* (SORM) which is also known as *Hasofer-Lind* reliability method. The failure probability and the reliability index used to quantify the risk. They explained that governing parameters are considered as random variables. The domain of random variables is divided in two parts such that one part represent safe zone while other represent the unsafe zone. The boundary between the two zones represents the limit state surface.

Cherubini and Vessia (2014) performed reliability analysis method on anchored rock slopes. A comparison between the partial factor approach and reliability based design is carried out by them.

2.3 Literature on Seismic Fragility Curves

The role of soil and site conditions in the vulnerability and risk assessment of structures is discussed by *Pitilakis et. al. (2010).* They discussed that due to the spatial extent of lifeline and other infrastructure they are subjected to non uniform and incoherent ground motions as a result of the variability of soil and geological conditions. They did vulnerability assessment i.e. quantification of risk by using fragility function. The vulnerability and loss assessment for infrastructure are evaluated on the basis of site specific seismic hazard using available inventory data. The vulnerability assessment helps in formulation of efficient mitigation strategies and policy.

Argyroudis and Pitilakis (2011) in their conference paper discussed the importance of seismic risk assessment in relation to transport facility. It was concluded that the vulnerability of transport facilities like road bridges and other structures can be accessed on the basis of fragility curves.

Argyroudis et al. (2013) performed their study on cantilever bridge abutments and the response of the abutment to increasing seismic intensity was evaluated by them using 2D nonlinear FE model. The effect of soil conditions and ground motion characteristics on the global soil and structural response was taken into account by considering different typical soil profiles and seismic input motions. The objective was to assess the vulnerability of the road network with regard to the performance of the bridge abutments. They discussed that in general the vulnerability assessment of geotechnical system is very limited as it is mainly based on empirical data but with this methodology the distinctive feature of the construction technology, input motion and the soil properties are considered in a more systematic may.

SYNER-G Reference Report-2013 by *Kaynia A.M*. on *"Guidelines for deriving seismic fragility functions of elements at risk: Buildings, lifelines, transportation networks and critical facilities",* proposed the utilization and guidelines for the fragility functions in references with the construction typology in Europe. They also validated the fragility curves against the observed damage. A software package was also developed for the storage, harmonization and estimation of the uncertainty of fragility functions.

Argyroudis and Kaynia (2015) discussed that fragility curve is fundamental tool in seismic risk assessment. They discussed that there may be a certain probability that a structure will reach a certain damage state for a given ground motion. A numerical approach was employed by them in development of fragility curves. The limitations of this method are also outlined. It is also noted in the study that reduction in uncertainties can be achieved by randomization of soil properties, geometry and use of large set of input motions.

Chapter 3

METHODOLOGY

3.1 Introduction

This chapter presents the methodology used in this study for the vulnerability assessment of slope under material and seismic uncertainty. Method of reliability analysis by *First Order Reliability Method* (FORM) and *Hasofer Lind* reliability index are discussed. Subsequently, performance function is developed by using *Response Surface Method* utilizing the results numerical analyses results.

3.2 Finite Element Method & Strength Reduction Factor Method

Finite Element Method (FEM) is a numerical tool for solving a physical problem. FEM subdivides a large problem domain into smaller and simpler parts called finite elements. The equations of each finite element are then assembled to simulate the entire system. FEM uses various methods to approximate a solution by minimizing an associated error function. One of these methods which have been widely applied in slope stability assessment is *Shear Strength Reduction* (SSR) method.

The principle behind the SSR technique is to reduce cohesion (c) and angle of internal friction (ϕ) until slope failure occurs where instability of slope is defined as the condition where the equilibrium of the system considered is lost.

Some of the advantages of the SSR method include (*Rocscience Inc. 2015*):

- No prior requirement to define a failure surface
- Equations of equilibrium are all satisfied.
- Strains and displacements in the soil and/or rock can be evaluated,
- Strains, displacements, axial force and moment distributions in support can be evaluated.
- Progressive failure can be modeled.

The disadvantages SSR method include:

- Not as widely known as the limit-equilibrium methods.

- Requires more data such as material modulus, stiffness, plasticity parameters, in-situ stress, boundary conditions etc. which may not be easily available
- Mesh generation and model setup can be difficult and may require a high level of modeling expertise.
- Finite-element is prone to convergence, tolerance, and numerical instability issues.
- It is much slower and time intensive. The computational requirement may be very high depending on the complexity involved with the system.

In shear strength reduction method, soil's shear strength is gradually decreased by applying finite element and finite difference programs as long as till indication of failure appear.

Fig. 3.1 shows the forces acting on the slope. Out of these forces cohesion and friction acts as resisting forces while self weight of the slope, pore water pressure acts as driving agents. Safety factor is defined as the ratio of real shear strength of soil to reduced shear strength. Mathematically, it may be represented by Eq. 3.1.

$$FOS = F_r / F_d \tag{3.1}$$

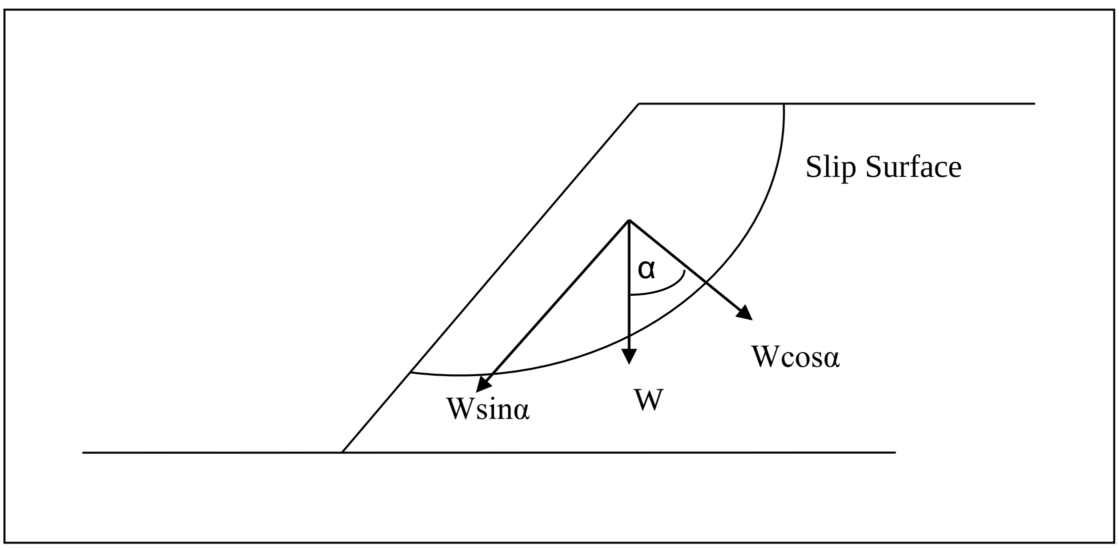


Fig. 3.1. Forces acting in a slope (Matsui and San 1992)

3.3 Reliability Analysis

All engineering designs are aimed to provide minimum level of serviceability during the lifetime. However, this is a challenging task as there are various sources of uncertainties associated with geotechnical systems as discussed in *Chapter 2*. In this context, reliability analysis methods may be used to evaluate the ability of systems or components to remain safe and operational during their lifecycle (Arteaga and Soubra 2014).

Reliability analysis is a probabilistic approach which allows the systematic inclusion of uncertainties in evaluating performance of the system. Important geotechnical parameters such as shear strength parameters are treated as random variables, each with an appropriate probability distribution, rather than deterministic values.

The importance of reliability method through which uncertainties are incorporated in design can be understood by example that if one is going to compute *Factor of Safety* (FOS) with absolute precision, a value of FOS = 1.1 or even 1.01 would be acceptable. However, there are certain uncertainties present in the geotechnical system, so the computed values of factor of safety are never absolutely precise. The reliability of a slope is the computed probability that a slope will not fail and is given by Eq. 3.2.

$$R = 1 - P_f \tag{3.2}$$

where P_f is the probability of failure and R is the reliability of the system. A brief description of the background of the probabilistic approach adopted in the present study is provided below.

In this approach the governing parameters are considered as random variables and grouped in vector X. f(X) is assumed to be the response function. In reliability analysis, the domain of X is divided in two regions; safe and failure regions respectively. Safe region is represented by f(X) > 0 while failure region corresponds to f(X) < 0 for the domain of vector X. f(X) = 0 is the boundary between failure region and safety region and it is known as limit state surface (Low and Tang 2009).

The performance function g(x) is defined as the difference between capacity R(x) and demand S(x).

$$g(x) = R(x) - S(x) \tag{3.3}$$

3.3.1 Methods of Reliability Analysis

First Order Reliability Methods (FORM): This method uses second moments of the random variables characterized by mean and standard deviation. The term First Order Reliability Method (FORM) comes from the fact that the performance function g(X) is approximated by the first order Taylor expansion (linearization) (Cornell 1967). This method includes two approaches which are *First Order Second Moment* (FOSM) and *Advanced First Order Second Moment* (AFOSM) approaches. In FOSM, the information on the distribution of random variables is ignored. However, in AFOSM, the distributional information is appropriately incorporated. A brief description of the two approaches has been provided below.

(a) *First Order Second Moment* (FOSM): This method uses mean and standard deviation, which are known as second moment statistics of the random variables (Sanchez-Silva 2010). The failure probability P_f is related to the failure event represented by R < S and calculated as-

$$P_f = P(R - S) = P(R - S < 0)$$

$$\text{Or} \qquad P_f = P(Z < 0) \tag{3.4}$$

Where Z is called as performance function denoted as Z = (R – S) as the random variables representing the capacity and demand following Gaussian distribution so that Z also follows a Gaussian distribution. The performance function Z is characterized by mean $\mu_Z = \mu_R - \mu_S$ and a standard deviation $\sigma_Z^2 = \sigma_R^2 - \sigma_S^2$. The failure of probability is related to the event P(Z < 0).

Hence P_f can be calculated as :

$$P_f = \varphi\left[-\frac{\mu_Z}{\sigma_Z}\right] = \varphi = [-\beta] \tag{3.5}$$

Where φ represent the standard normal cumulative distribution function (CDF) and $\beta = \mu_z / \sigma_z$ is the 'reliability index' which is used to quantify risks of failure. Fig. 3.2 represents the normally distributed probability distribution function with mean value μ_z, and standard deviation σ_z. β is the distance from the mean value indicating the limiting surface for the performance function and hence the distance β form mean is the reliability index.

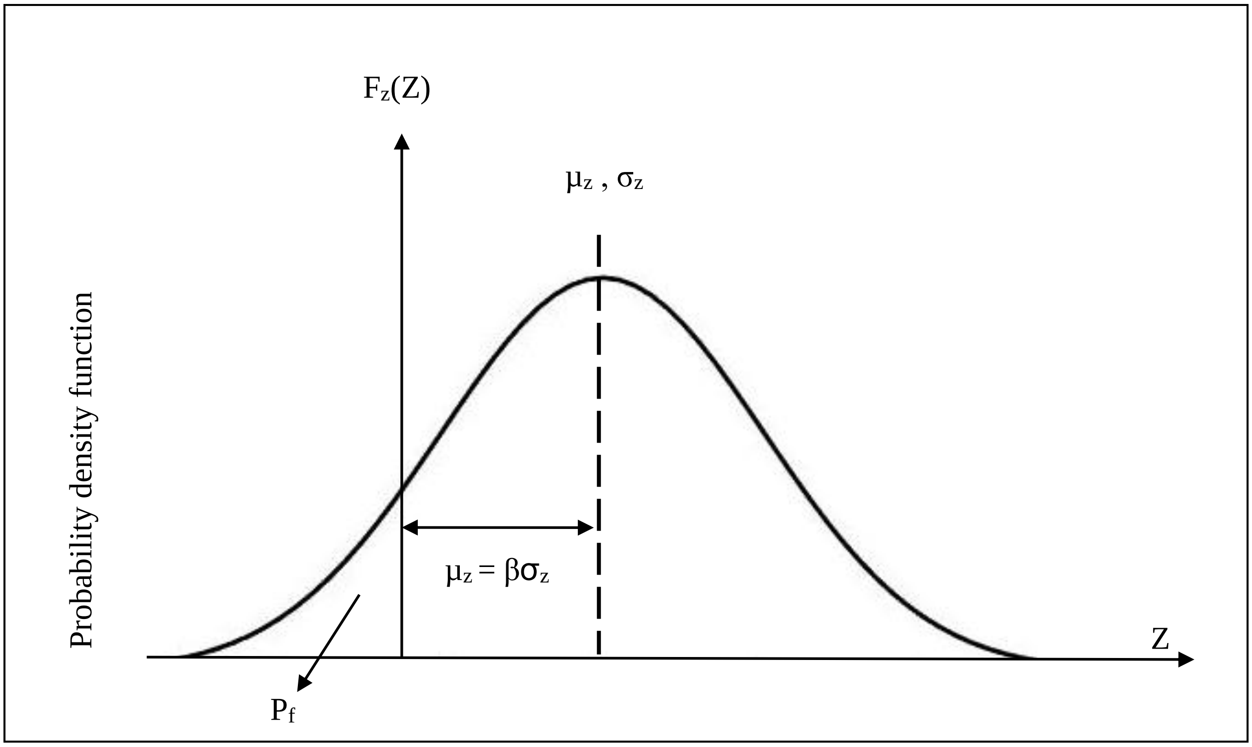


Fig. 3.2. Probability density function of Z

(b) *Advanced First Order Second Moment* (AFOSM): Reliability index with correlated normal variables and first order reliability method (FORM) is known as Advanced First Order Second Moment method (Low and Tang 2007; Hasofer and Lind 1974). AFOSM is also called ***'Hasofer-Lind'*** method. The assessment of the reliability index is mainly based on the reduction of the problem to a standardized coordinate system. Thus, a random variable X_i is reduced as:

$$X_i^{'} = \left(X_i - \mu_{xi}\right) / \sigma_{xi} \qquad (i = 1,2,3,\ldots\ldots,n) \qquad (3.6)$$

Where $X_i^{'}$ is a random variable with zero mean and unit standard deviation. X denotes the random variable in original coordinate system and $X^{'}$ denotes random variable in reduced coordinate system. In reduced system, the Hasofer-Lind reliability index β_{HL} is the minimum distance from the origin of the axes to the limit state surface-

$$\beta_{HL} = \sqrt{\left(X^{'*}\right)^T * \left(X^{'*}\right)} \qquad (3.7)$$

The minimum distance point on the limit state surface is called '*Design Point*'. It is denoted by X^* in original coordinate system and by X'^* in reduced coordinate system. As shown in Fig. 3.3a straight line represents the capacity minus demand curve. The region above line is failure region as demand is more than the capacity and region below the straight line refers to safe region. The point on the straight line denotes the determined design point and the minimum distance between the mean value point and the design point is reliability index. In reduced coordinate system mean value point is mathematically transferred to origin point (Fig. 3.3b) hence the distance from origin to design point is the require reliability index which is denote by $X^{'*}$ or $\beta_{HL.}$

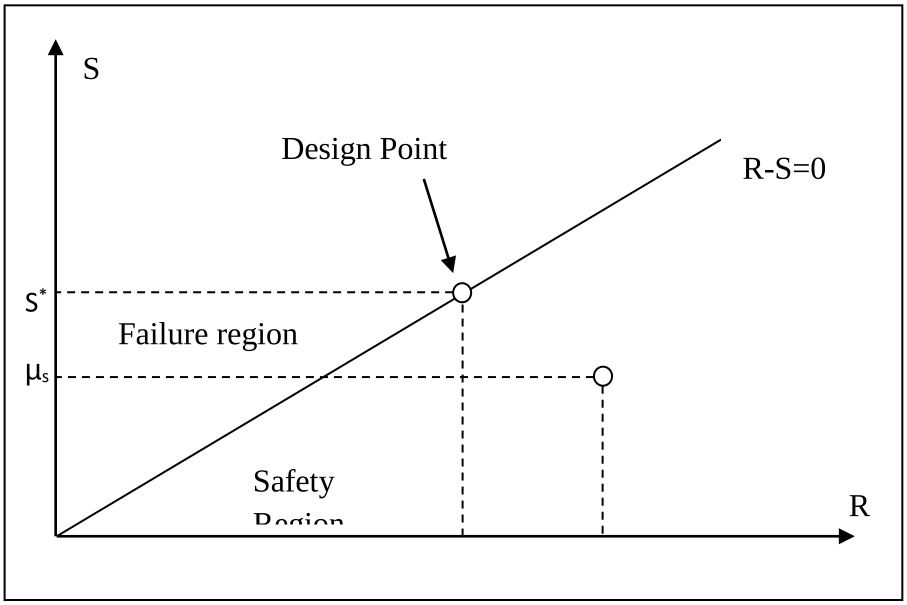
The reliability index corresponds to the minimum distance between the limit state surface and the origin (in the reduced coordinates system). By using simple trigonometry, this distance (reliability index) can be estimated as:

$$\beta_{HL} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3.8}$$

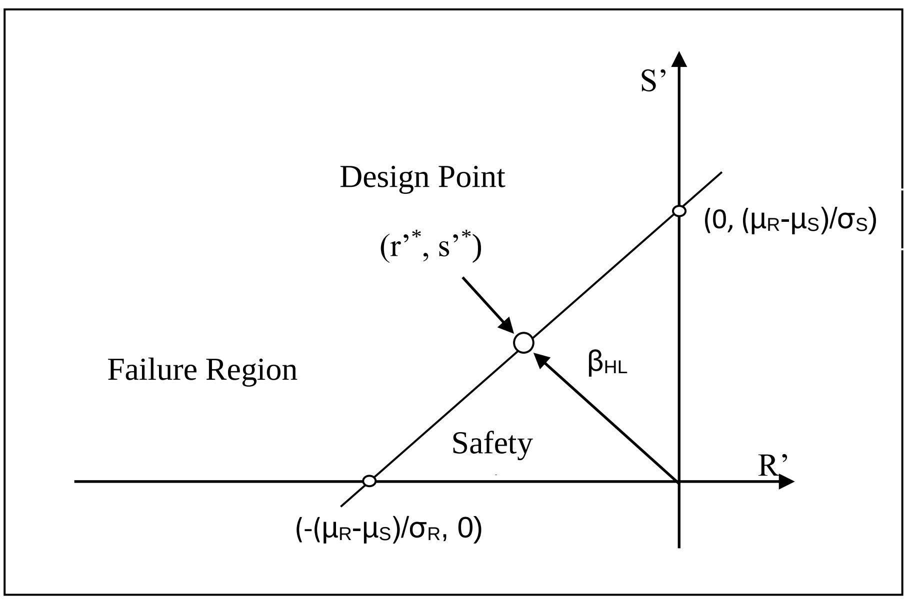
The index reflects not only the effect of mean values but also the covariances of the random variables influencing the design (Low B.K. 1997). The matrix formulation of Hasofer-Lind index may be given following Ditlevsen (1981) and Veneziano (1974) as

$$\beta_{HL} = \min_{x \in F} \sqrt{(x-m)^T * C^{-1} * (x-m)} \tag{3.9}$$

Where x is a vector representing the set of random variables, m is their mean values, C is covariance matrix. The procedure for computing the β is to transform the failure surface into the space of reduced variable. Shortest distance from the transformed failure surface to the origin of the reduced variables is reliability index.



(a) Original coordinates



(b) Reduced Coordinate

Fig. 3.3. Schematic representation of terminologies in reliability analysis

As suggested by Low and Tang the concept of expanding ellipse that is presented in Fig. 3.4 led to a simple method for computing the Hasofer-Lind reliability index in the space of random variables using an optimization tool. Fig. 3.4 shows that there are two uncorrelated non-normal random variables with an one-sigma distance ellipse with center at mean value. After doing iterative process of optimizing beta, final β-ellipse is obtained which is the smallest ellipse just touching the limit state surface.

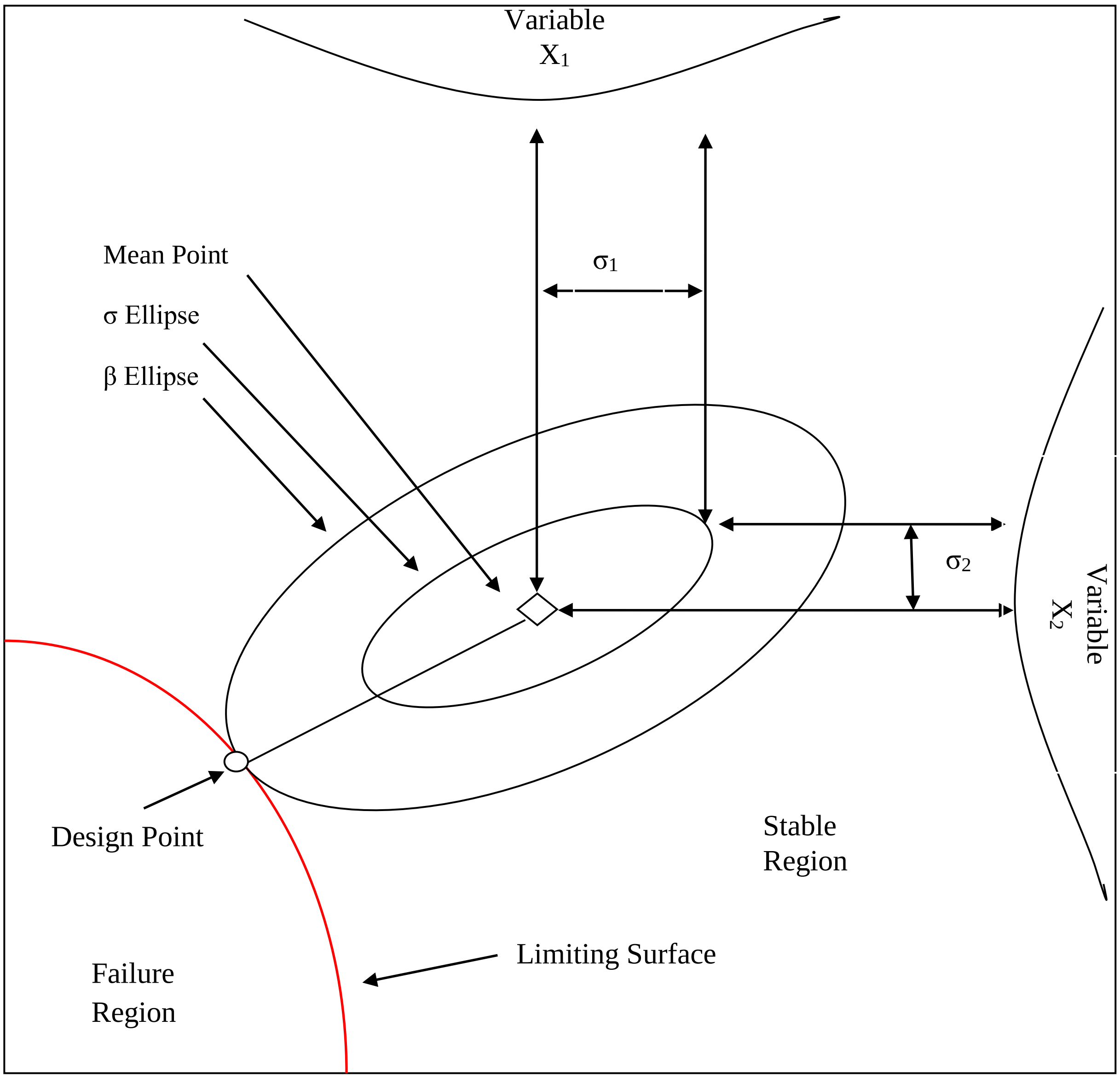


Fig. 3.4: FORM analysis showing design point, mean point and reliability index (Source: Low and Tang 1997)

3.3.2 Selection of Sampling Point

Selection of correct sampling point is very important in conducting the reliability based analyses through the use of the methodology discussed above. Starting from mean values, estimation of □ is carried out. The first estimate of the approximate limit state may not be close enough to the actual critical limit surface at design point. To ensure sufficient accuracy the approximation is refined through an iterative solution process. After the reliability index and design point are determined in any iteration, a new center point (X_m) is chosen on a line from mean vector to design point (X_D) by linear interpolation as following (Langford 2013).

$$X_m = X + \left(X_D - X\right)\frac{G^{'}(X)}{\left(G^{'}(X) - G^{'}(X_D)\right)} \tag{3.10}$$

Using X_D as new center and σ_n as their deviation from mean are selected as new sample and next iteration is performed till the convergence of Hasofer-Lind reliability index. The use of corner points, as represented in Fig. 3.5, allows the user to assess compound impacts of multiple variables and determine which combinations have greater impact on the performance of the system. The general representation of selection of sampling points is shown in Fig. 3.4 representing mean point, axial points and corner points at one sigma distance. (2n+1) sample points are required for the evaluation. This is for a single iteration giving critical limit surface.

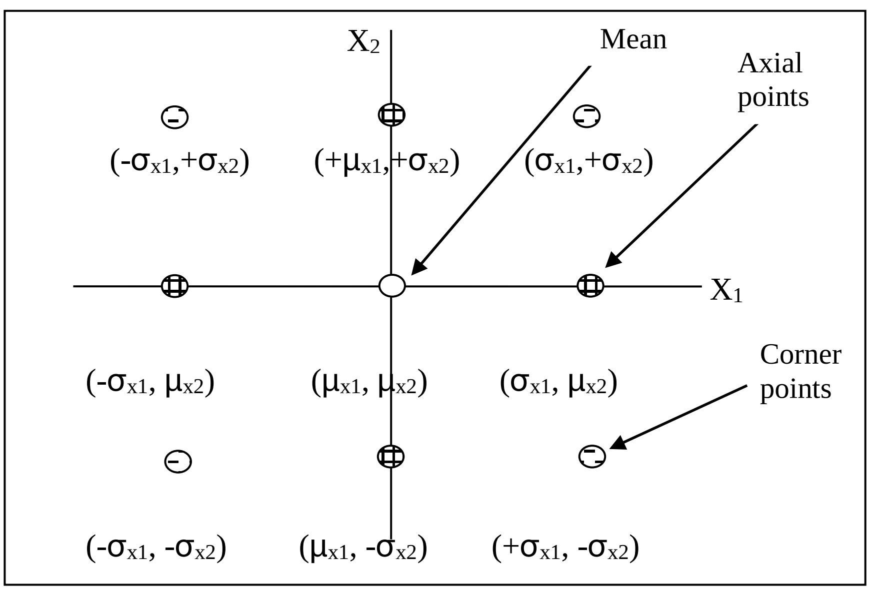


Fig 3.5. Sampling points and their coordinates for two variable systems.

The combined FORM and RSM technique is shown in Fig. 3.6 for two variables system. In first iteration, estimated limit surface may not be closer to the critical surface at the design points. To ensure accuracy, approximation of limit surface is repeated. Each iteration has new design point that is chosen on straight line by interpolation as previously discussed in selection of sampling points. This iterative process continues till change in reliability index between iterations is less than the set tolerance. Once the final reliability index is determined the probability of failure can be calculated.

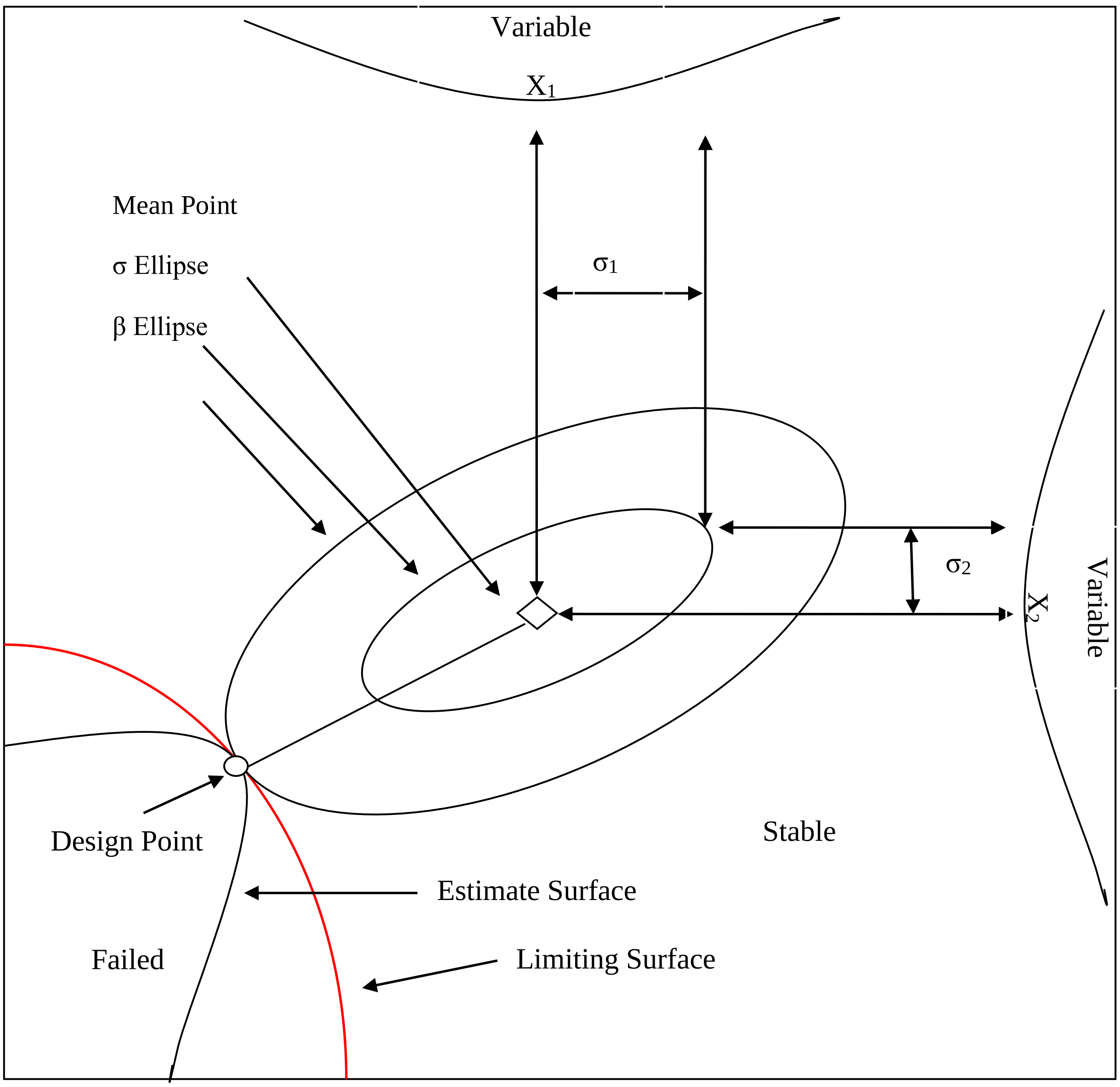


Fig 3.6. Showing combined process of RSM and reliability method.

3.4 Response Surface Method

Response surface methodology (RSM) is a collection of statistical and mathematical techniques. It is the process of identifying and fitting an approximate response surface model from input and output data obtained from experimental studies or from the numerical analyses where each run can be regarded as an experiment (Myers *et al.* 2009). It has important applications in the design, development, and formulation of new products, as well as in the improvement of existing product designs. The most extensive applications of RSM are in the industrial world, particularly in situations where several input variables potentially influence some performance measure or quality characteristic of the product or process. This performance measure or quality characteristic is called the response. It is typically measured on a continuous scale, although attribute responses, ranks, and sensory responses are not unusual. Most real world applications of RSM will involve more than one response. The input variables are called independent variables or random variables. The RSM, which has been used extensively in civil engineering to approximate the mechanical response of a structure, allows an approximate limit state to be developed by relating the input and output parameters for a system by a simple mathematical expression (Langford 2013).

Advantages of RSM are as following (Babu and Shrivastava 2008).

- The RSM approach reduces the number of runs required in the reliability analysis compared to other computationally more demanding approach like Monte Carlo simulation.
- The response surface model is a simplified relationship that can be used for practical engineering purposes, where spending high costs for performing advanced numerical analysis is not desired.
- The response surface developed in the process replaces the original model in an uncertainty analysis and with fewer numbers of runs it reduces the computational time drastically.
- It also helps in identifying the sensitive parameters influencing the system response.

3.4.1 Response Surface Modeling

Let us consider a problem in which 'Y' is a response which depends on random variables $\xi_1, \xi_2, \xi_3 \ldots$ i.e.

$$Y = f(\xi_1, \xi_2, \xi_{3.......}) + \varepsilon \tag{3.11}$$

Where ξ_1, ξ_2, ξ_3 are natural variables as they are expressed in natural measurement units and ε is a term that represents other sources of variability not accounted for in *f*. Thus ε includes effects such as measurement error on the response, other sources of variation that are inherent in the process or system, the effect of other variables etc. In RSM it is easy to work on reduced variables so we transform the natural variables to coded variables x_1, x_2, . . . , x_k, which are usually defined to be dimensionless with mean zero and the same spread or standard deviation. In terms of the coded variables, the true response function is written as-

$$Y = f(x_1, x_2, ..., x_k) + \varepsilon \tag{3.12}$$

For example if two variables are there then first order model in terms of reduced variables is

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon \tag{3.13}$$

The above equation is main model equation as it contains main effect of the variables. While if there is interaction between variables then it can be written as-

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon \tag{3.14}$$

In case of curvature in the response surface then second order model equation is used. For the case of two variables, the second-order model is

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1^2 + \beta_4 X_1 X_2 + \varepsilon \tag{3.15}$$

Here the values of unknown regression coefficients β can be determined by data of the system under consideration. These data can be obtained by numerical analysis or field data collection.

Natural variable should be transformed to reduced variable as

$$X_1 = \frac{\xi_{i1} - [\max(\xi_{i1}) + \min(\xi_{i1})]/2}{[\max(\xi_{i1}) - \min(\xi_{i1})]/2}$$

$$X_2 = \frac{\xi_{i2} - [\max(\xi_{i2}) + \min(\xi_{i2})]/2}{[\max(\xi_{i2}) - \min(\xi 2)]/2} \tag{3.16}$$

This coding scheme is widely used in fitting linear regression models, and it results in all the values of X_1 and X_2 falling between -1 and +1.

The solution to the normal equations will be the least square estimators of the regression coefficients. It is simpler to solve the normal equations if they are expressed in matrix notation. So,

$$Y = X\beta + \varepsilon \tag{3.17}$$

Where,

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & . & . & . & x_{1k} \\ 1 & x_{21} & x_{22} & . & . & . & x_{2k} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & x_{n1} & x_{n2} & . & . & . & x_{nk} \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_n \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ . \\ \varepsilon_n \end{bmatrix}$$

When the above indicated multiplication is performed we get the values of β and the scalar form of the normal equations will result. The fitted regression model gives us fitted response as.

$$\hat{Y} = X\beta \tag{3.18}$$

The difference between the observation Y and the fitted value $\hat{Y}$ is a residual, say $\varepsilon_i = Y - \hat{Y}$. The (n x 1) vector of residuals is given by following.

$$\varepsilon = Y - \hat{Y} \tag{3.19}$$

3.5 Regression Analysis

Regression analysis is a statistical process for estimating the relationships among variables. Regression analysis helps one understand how the typical value of the dependent variable (or Response variable) changes when any one of the independent variables (or random variable) is varied, while the other independent variables are held fixed.

$$Y = f(X_1, X_2, X_3......) \tag{3.20}$$

Here Y is a response function whereas X_1, X_2, X_3 are the random variables. For example in case of slope stability, Factor of Safety (FOS) is a response function while cohesion (c), angle of friction (ϕ), Young's modulus (E) and unit weight (γ) are the random variables.

3.5.1 Linear Regression Analysis

Linear least squares regression is the most widely used modeling method. It is also known as "regression", "linear regression" or "least squares" that is used to fit a model to their data. This simple linear regression model for two variables is as shown in figure.

$$Y = \beta_0 + \beta_1 X + \varepsilon \tag{3.21}$$

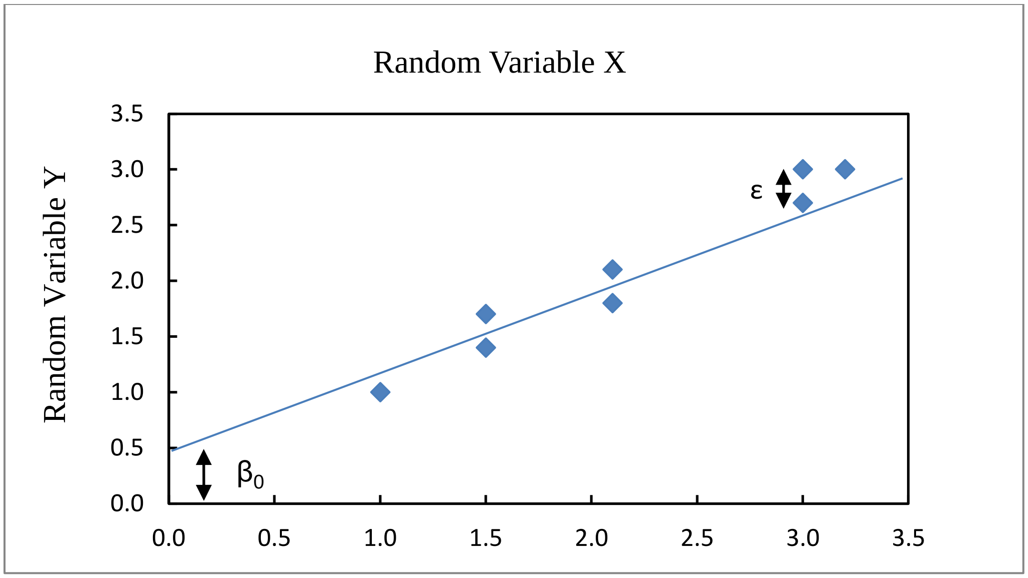


Fig. 3.7. Relation between X and Y variables.

Where the intercept β_0 and the slope β_1 are unknown regression coefficients and ε is a random error component. The errors are assumed to have mean zero and unknown variance σ^2. Also it is assumed that the errors are uncorrelated. This means that the value of one error does not depend on the value of any other error. There is a probability distribution for 'Y' at each possible value for 'X'. The mean of this distribution is

$$E(Y|X) = \beta_0 + \beta_1 X \tag{3.22}$$

and the standard deviation is

$$\sigma(Y|X) = \varepsilon \tag{3.23}$$

3.5.2 Nonlinear Regression Analysis

Nonlinear regression is a regression in which the dependent or criterion variables are modeled as a non-linear function of model parameters and one or more independent variables. There are several common models, such as Asymptotic Regression/Growth Model etc. The reason that these models are called nonlinear regression is because the relationships between the dependent and independent parameters are not linear. A nonlinear regression model can be written as

$$Y_n = f\left(x_n ; \beta\right) + \varepsilon \qquad (3.24)$$

Where *f* is the expectation function and x_n is a vector of associated independent variables for the n^{th} case and β is the disturbance in the data. For nonlinear models, at least one of the derivatives of the expectation function with respect to the parameters depends on at least one of the parameters.

Linear and non linear regression analysis can be performed by using various software’s like MINITAB, SPSS etc. In software, we need to define response and random variables and then appropriate model can be set to define interaction between random variables and also order of variables can be defined as quadratic, cubic etc. Software generally generates various graphs as normal probability plot of residuals, residuals versus fits, residuals versus order and histogram of residuals.

Chapter 4

PROBABILISTIC ASSESSMENT OF SLOPE FAILURE

4.1 Numerical Modeling of a Slope

Continuum modeling is best suited for the analysis of slopes that are comprised of massive, intact rock, weak rocks, and soil-like or heavily fractured rock masses. In the present study, a rock slope has been modeled numerically and analyzed in a finite element based software package using continuum approach. A 50m high slope with inclination angle of 45^0 has been modelled. Plane strain elements have been employed for modeling of the slope. In the model, rock mass has been divided into zones with each zone being assigned a material model and properties. Fig. 4.1 shows the discretized view of slope by continuum based PHASE^2 software.

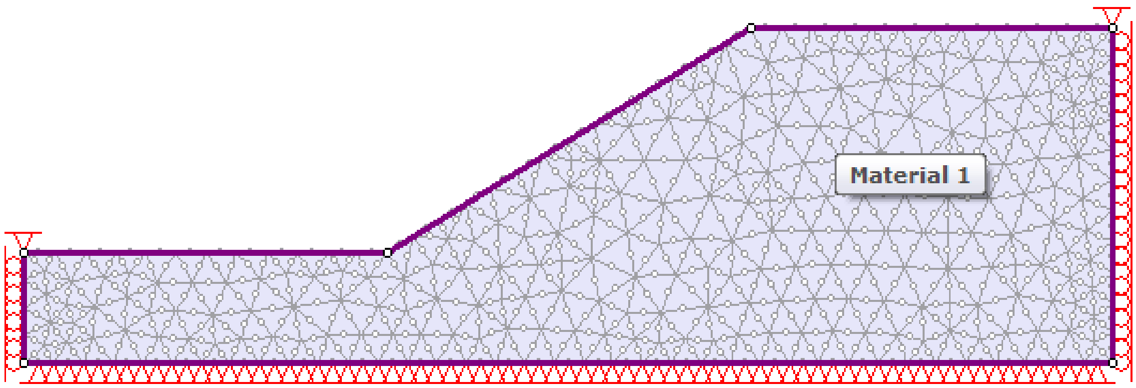


Fig. 4.1. Finite element model showing discretized slope

4.1.1 Boundary Condition

Boundaries are either real or artificial. Real boundaries in slope stability problems correspond to the natural or excavated ground surface that is usually stress free. All problems in geomechanics including slope stability problems require that the infinite extent of a real problem domain be artificially truncated to include only the immediate area of interest. The far-field boundary location and its condition must be specified in any numerical model for slope stability analyses.

The base of model is set fixed boundary while lateral boundaries are made to roller supports. The extents of model boundary are so fixed that it maintains the recommendations of as shown in Fig. 4.2.

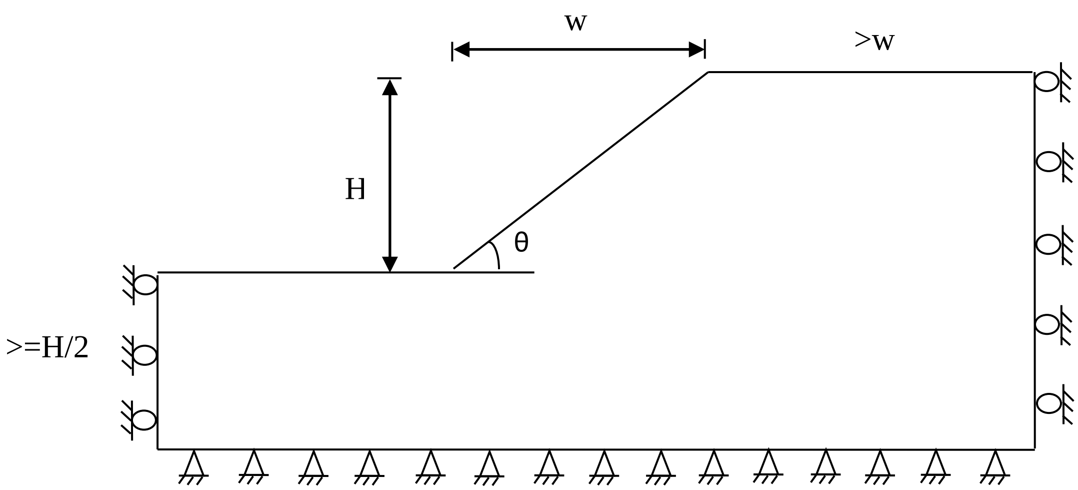


Fig. 4.2. Typical recommendations for locations of artificial boundaries in slope stability analyses

4.1.2 Slope Parameters

Variability in properties of slope material (such as cohesion, angle of friction, unit weight of soil etc.) influences the stability of slope in a major way. The anisotropy and heterogeneity of geotechnical parameters are the main issue of uncertainty in the stability analysis. The uncertainty limits the reliability of the information obtained from various tests and that makes quantification of uncertainties a necessity. This variability can be quantified by statistical parameters such as mean, coefficient of variance etc. The standard deviation (σ) normalized by the local mean geotechnical property (μ) provides a useful dimensionless ratio known as the *Coefficient Of Variation* (COV) (Song *et al.* 2011) as shown in Eq. 4.1 and the COV of various properties of rock material adopted in the present study is furnished in Table 4.1.

$$COV = \frac{\sigma}{\mu} \quad (4.1)$$

4.2 Static Slope Stability Analysis

In static equilibrium the sum of all the forces on each element of system is zero *i.e.* at equilibrium total resisting force is equal to total driving force. The resisting and driving forces are calculated by using equilibrium equations. The factor of safety is defined ratio of resisting force to driving force, given as following.

$$FS = \frac{Resisting\ \ Force}{Driving\ Force} \tag{4.2}$$

Table 4.1. Adopted COV of rock properties

Parameter	COV (%)
Elastic modulus	16-42
Friction angle	10-20
Cohesion	20-40
Uniaxial compressive strength	18-43
Tensile strength	10-12

When FS < 1 then the system is considered as unstable; while FS > 1 shows a stable condition. At equilibrium state factor of safety is 1.0 (Hoek and Bray 1981). Factor of safety of a rock slope as shown in Fig. 4.3 against plain failure is given by

$$FS = \frac{F_R}{F_D} = \frac{cA + W\cos\psi_P + \tan\phi}{W\sin\psi_P} \tag{4.3}$$

Where F_r is resisting force, F_d is driving force, c is cohesion, W is self weight of slope, Ψ_p is angle of failure plane and ϕ is the angle of friction.

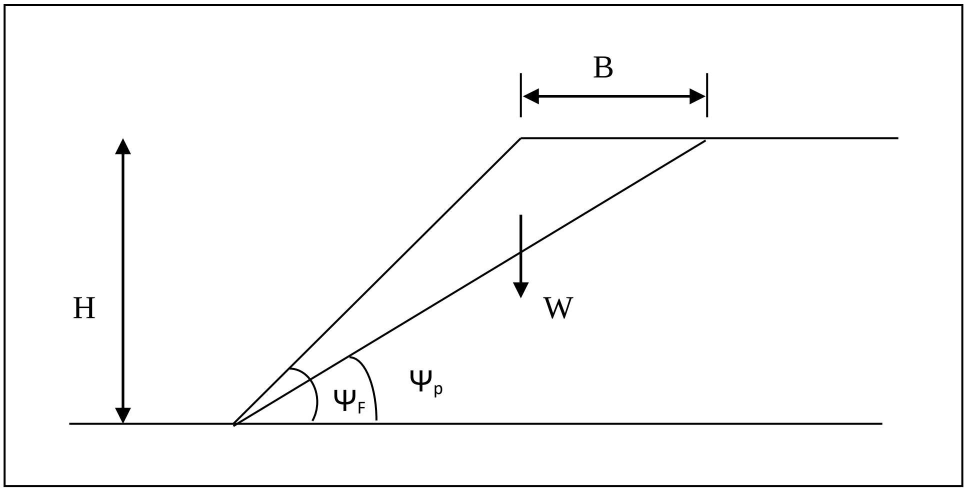


Fig. 4.3. Mechanism of slope failure under self weight

4.3 Pseudo-Static Slope Stability Analysis

Pseudo-static analysis is commonly used to analyze the seismic response of soil embankments and slopes. In pseudo-static slope stability analysis the inertial forces generated by earthquakes are simulated by the inclusion of static horizontal and vertical forces in limit equilibrium analysis (Melo and Sharma 2004).

The cyclic earthquake motion is replaced with a constant horizontal acceleration equal to $k_c\, g$, where 'k_c' is the seismic coefficient and 'g' is the acceleration due to gravity. A force is applied to the soil mass equal to the product of the acceleration and the weight of the soil mass. Fig. 4.4 shows that static force is acting on the slope and the curve surface represent the failure surface. The 'k_h' shown in figure is the seismic horizontal coefficient and 'k_v' is seismic vertical coefficient. It is assumed that the cyclic motion is idealized by equivalent static force applied on the slope due to earthquake. A lateral force acting through centroid of sliding mass, is applied which acts out of slope direction. This pseudo-static lateral force F_h is calculated as follows:

$$F_h = ma = \frac{Wa}{g} = \frac{Wa_{\max}}{g} = k_h W \qquad (4.4)$$

Where F_h = horizontal pseudo-static force acting through centroid of sliding mass out of slope direction; m = total mass of sliding material; W = total weight of sliding mass; a= acceleration; a_{max} = peak ground acceleration; a_{max}/g = seismic coefficient

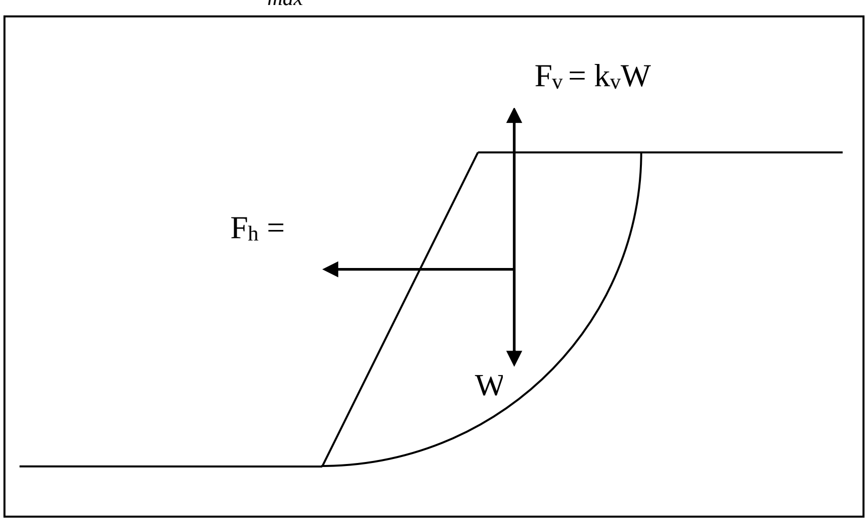


Fig. 4.4. Pseudo-static analysis approach

The values of the seismic coefficient are considered from literature as presented in Table 4.2. There are no specific rules for selection of an appropriate seismic coefficient for design. However, the different selection criteria suggest that the seismic coefficient should be based on the anticipated level of acceleration within the failure mass and should correspond to some fraction of the anticipated peak acceleration (Kramer 1997).

Table 4.2. Recommended Horizontal Seismic Coefficient (Melo and Sharma 2004)

Horizontal seismic coefficient, k_h	**Description**	
0.05-0.15	In the US	
0.12-0.25	In Japan	
0.1	"severe" earthquake	Terzaghi
0.2	"violent, destructive" earthquakes	
0.5	"catastrophic" earthquakes	
0.1-0.2	FOS >= 1.15	Seed
0.1	Major Earthquake, FOS>1	Corps of Engineers
0.15	Great Earthquake, FOS>1	
½ to 1/3 of PHA	FOS> 1	Marcuson
½ of PHA	FOS>1	Hynes-Griffin
FOS = Factor of safety. PHA = Peak Horizontal Acceleration.		

4.4 Validation Study

The validation study is based on the slope problem considered by Babu and Shrivastav (2008). In the study, a typical slope as shown in Fig. 4.5 is considered. For the validation purpose, identical procedure and data are adopted and results have compared with the results given in the paper.

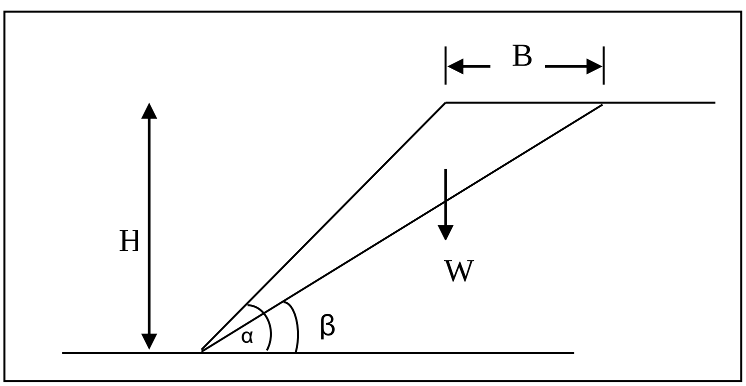


Fig. 4.5. Slope problem considered for validation

The properties of the slope are provided in Table 4.3.

Table 4.3. Properties of slope

Geometric and Soil Properties	Values
Height of the slope (H)	2.5 m
Slope angle (α)	60˚
Angle of planer failure surface (β)	20˚
Unit weight of soil (γ)	17 kN/m^3
Cohesion (c)	5 kPa
Angle of internal friction (ϕ)	30˚

It is assumed that input parameter c, ϕ and γ are uncorrelated normally distributed with coefficient of variation 10%. The statistical properties of parameters with the upper limit and lower limit values are as presented in Table 4.4. k_h=0.15 is assumed.

Table 4.4. Statistical properties of parameters

Input Parameter	Mean	COV (%)	Std. Dev.	x_{max} (μ+1.65σ)	x_{min} (μ-1.65σ)
c	5.0	10	0.5	5.83	4.18
ϕ	30	10	3.0	34.95	25.05
γ	17	10	1.7	19.81	14.20

Response surface generation is carried by computation of pseudo-static factor of safety for 8 different combinations of input parameters. For computation of *Factor of Safety* (FOS) PHASE^2 software is used in which 8 models are made with different combinations of input parameters. The input parameters are provided in Table 4.5 and also the computed FOS from numerical analysis are presented.

Regression analysis is performed by using these data to get a representative equation between dependent and independent variables. The generated response model is given below:

$$FS = 0.46 + 0.16c + 0.04\phi - 0.04\gamma \tag{4.5}$$

Now the reliability index has been calculated by using methodology of reliability analysis as discussed in *Chapter 3*. Fig. 4.6 shows a typical spreadsheet for obtaining reliability index by minimizing the performance function.

Table 4.5. Factor of safety for various combinations of input parameters

Sl. No.	c	ϕ	γ	FOS
1	5.83	34.95	19.81	1.94
2	5.83	34.95	14.11	2.2
3	5.83	25.05	19.81	1.51
4	5.83	25.05	14.20	1.77
5	4.16	34.95	19.81	1.76
6	4.16	34.95	14.20	1.94
7	4.16	25.05	19.81	1.33
8	4.16	25.05	14.20	1.52

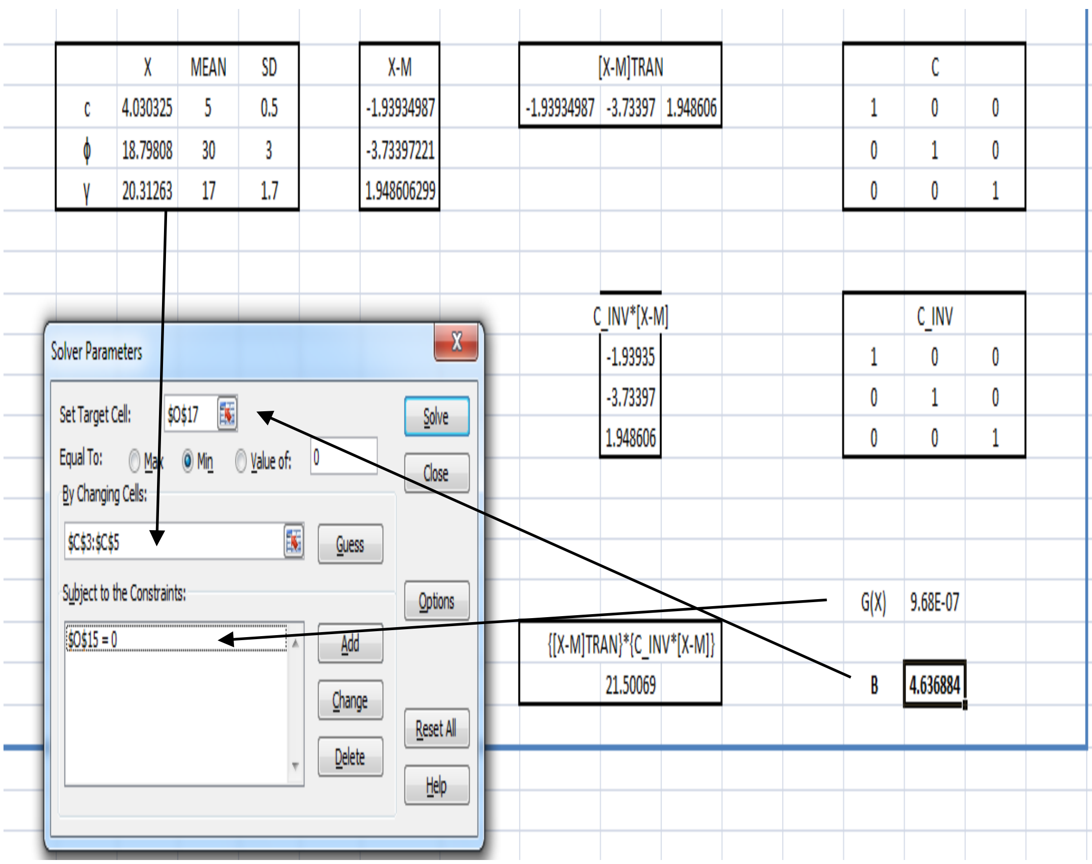


Fig. 4.6. Typical spreadsheet calculation of reliability index

With the values adopted for the study, reliability index obtained in the present study is 4.63 while the value calculated by Babu and Shrivastav (2008) was 4.61 which are approximately equal. The little difference in reliability indices is due to the reason that Babu and Shrivastav (2008) calculated the factor of safety using analytical method while in this study factor of safety is calculated by numerical analyses in $PHASE^2$ Software.

4.5 Present Problem Statement

The present study aims to evaluate the reliability of an engineered slope under pseudo-static condition using response surface method. A 50m high rock slope with angle of slope θ, cohesion c, angle of friction ϕ, unit weight γ, Young's modulus E has been considered. Out of these variables cohesion, angle of friction, Young's modulus are assumed to be normally distributed while horizontal seismic coefficient and angle of slope are considered to be linearly distributed. Geometric and material properties are summarized in Table 4.6.

Table 4.6. Properties of slope

Parameter	**Mean**
Cohesion	0.2 MPa
Angle of friction	30 Degree
Angle of slope	45 Degree
Young's modulus	1200 MPa
Horizontal seismic coefficient	0.15
Unit weight	18.50 MN/m^3

4.5.1 Steps Involved

In the present study, a reliability analysis of a slope with single layer has been considered. The steps considered in the study are below.

a. Calculation of FOS using numerical analysis ($PHASE^2$ software)
b. Derive performance function using RSM technique

In reliability analysis of finite slope, variables cohesion, angle of friction, Young's modulus, slope angle and horizontal seismic coefficient are considered as random variables.

4.5.2 Analysis of Single Layered Slope

In single layered slope angles of 60^0 and $30^0$are considered. For which geometry constructed in PHASE^2 with the criteria as shown in Fig. 4.2. The statistical properties of the parameters used in the analysis are provided in Table 4.7.

Table 4.7. Statistical properties of parameters

Parameters	Mean	COV (%)	Std. Dev.	X_{max} (μ+1σ)	X_{min} (μ-1σ)
c (MPa)	0.2	40	0.08	0.28	0.12
ϕ (Degree)	30	20	6.00	36	24
E (MPa)	1200	40	480	1680	720
θ (Degree)	45	-	8.66	60	30
k_h	0.15	-	0.57	0.25	0.05

Pseudo-static FOS has been calculated for 2^n combination of input parameters (i.e. 32 analyses) using above data in PHASE^2. The FOS has been tabulated in Table 4.8.

Table 4.8. Pseudo-static FOS for iteration 1

Sl. No.	c (MPa)	ϕ (Degree)	E (MPa)	θ (Degree)	k_h	FOS
1	0.28	36	720	30	0.05	3.66
2	0.28	36	1680	30	0.05	3.66
3	0.28	24	720	30	0.05	2.95
4	0.28	24	1680	30	0.05	2.95
5	0.12	36	720	30	0.05	2.49
6	0.12	36	1680	30	0.05	2.49
7	0.12	24	720	30	0.05	1.87
8	0.12	24	1680	30	0.05	1.87
9	0.12	36	1680	30	0.25	1.73
10	0.12	36	720	30	0.25	1.73
11	0.28	24	720	30	0.25	2.00
12	0.28	36	1680	30	0.25	2.52
13	0.28	36	720	30	0.25	2.52
14	0.28	24	1680	30	0.25	2.00
15	0.12	24	1680	30	0.25	1.30

Sl. No.	c (MPa)	ϕ (Degree)	E (MPa)	θ (Degree)	k_h	FOS
16	0.12	24	720	30	0.25	1.30
17	0.12	24	720	60	0.25	0.86
18	0.12	36	720	60	0.25	1.08
19	0.12	24	1680	60	0.25	0.86
20	0.28	24	720	60	0.25	1.46
21	0.28	36	720	60	0.25	1.74
22	0.28	36	1680	60	0.25	1.74
23	0.28	24	1680	60	0.25	1.46
24	0.12	36	1680	60	0.25	1.08
25	0.28	24	1680	60	0.05	1.95
26	0.28	24	720	60	0.05	1.95
27	0.12	36	720	60	0.05	1.44
28	0.12	36	1680	60	0.05	1.44
29	0.12	24	1680	60	0.05	1.14
30	0.12	24	720	60	0.05	1.14
31	0.28	36	1680	60	0.05	2.31
32	0.28	36	720	60	0.05	2.31

For the generation of response surface model, regression is performed. An equation is fitted in between the input and output values. Linear response surface model obtained for above data is given in Eq. 4.6 with R^2 adjusted value 0.94.

$$(FOS)_1 = 1.49 + 5.22c + 0.04\phi - 0.03\theta - 3.20k_h \tag{4.6}$$

As the R^2 adjusted value is close to 1.0 and normal probability plot is approximately a straight line, this validates the adequacy of the fitted model. The Fig. 4.7 shows the normal probability curve of the residuals.

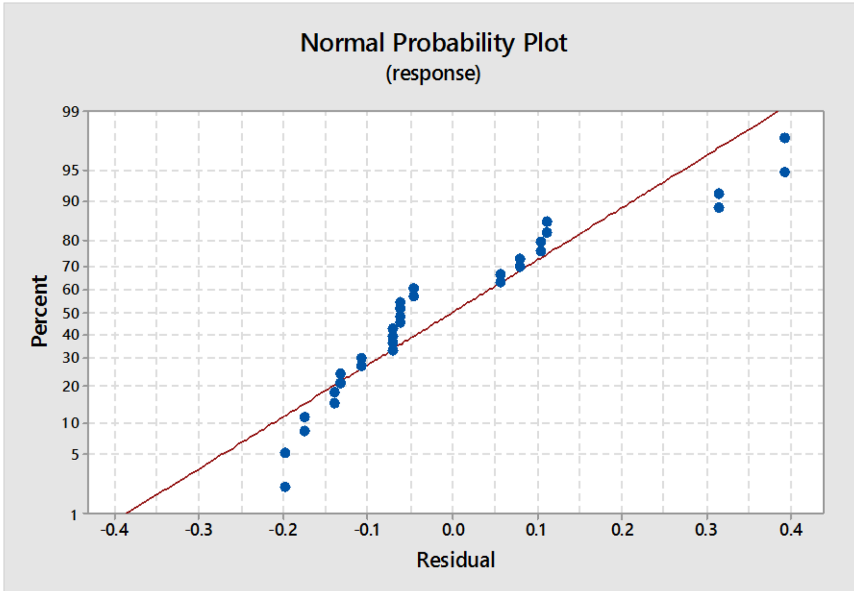


Fig. 4.7. Normal probability plot of the linear regression model

	X	MEAN	ST.DEV.	(X-M)
C	0.10218	0.2	0.08	-1.22275122
ϕ	26.22838	30	6	-0.62860317
E	0.485042	1200	480	-2.4989895
θ	50.98401	45	8.66	0.690994509
αh	0.180478	0.15	0.057	0.534696582

X-M(T)

-1.22275	-0.6286	-2.4989895	0.690995	0.534697

Transpose matrix

C

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Covariance matrics

	-1.22275
	-0.6286
C_INV*(X-M)	-2.49899
	0.690995
	0.534697

C_INV

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Inverse of Cov.

n =	8.898585
BETA =	2.98305
G(X) =	-5.6E-08

Fig. 4.8. Spreadsheet showing calculation of reliability index

Now after the formulation of response surface model reliability analysis is performed. The *Hasofer-Lind reliability index* can be calculated using spreadsheet based calculation. It is assumed that input parameters are uncorrelated and it is linearly related to the output response. MS Excel's optimization tool "Solver" is used to calculate reliability index and is shown in Fig. 4.8. Here following has been considered.

$$G(x) = FOS - 1 \tag{4.7}$$

To minimize the $\beta_{HL,}$ G(x) is constrained to zero with random variables c, ϕ, E, θ and α_h. When we run the SOLVER tool it minimizes G(x) giving β = 1.77 fulfilling all conditions. The process of optimizing β is an iterative process and it ends when β gets converged i.e. β having same value in two conjugative iterations. In the first iteration, design point is obtained at β=1.77. The values of different variables in this iteration are provided in Table 4.9.

Table 4.9. Values of variables at design point in iteration 1

Variables	**Values**
c (MPa)	0.08
ϕ (Degree)	26.99
θ (Degree)	48.37
k_h	0.18

Young's modulus *'E'* is not considered in optimizing beta because Young's modulus is a deformation parameter and it does not affect the FOS. Response surface remains unchanged with any change in Young's modulus.

Now using these design points we again select the new center point and repeat the process again. So the new design points for iteration second with their statistical parameters are given as in Table 4.10.

Further Pseudo-static analyses have been carried out for new 32 combinations of cohesion, friction angle, Young's modulus, slope angle and seismic coefficient as shown in Table 4.11. Corresponding FOS has been noted and shown in the same Table 4.11.

Table 4.10. Statistical parameters for iteration 2

	X_D	St. dev.	X_{max}	X_{min}
c (MPa)	0.06	0.08	0.09	0.03
ϕ (degree)	20.31	6	26.31	14.31
E (MPa)	1199.99	480	1679.99	719.99
β (degree)	60.34	8.66	60	30
k_h	0.23	0.06	0.25	0.13

Table 4.11. Pseudo-static FOS for iteration 2

Sl. No.	c (MPa)	ϕ (Degree)	E (MPa)	θ (Degree)	k_h	FOS
1	0.09	26.31	719.99	30	0.13	1.51
2	0.09	26.31	1679.99	30	0.13	1.51
3	0.09	14.31	719.99	30	0.13	1.04
4	0.09	14.31	1679.99	30	0.13	1.04
5	0.03	26.31	719.99	30	0.13	1.01
6	0.03	26.31	1679.99	30	0.13	1.01
7	0.03	14.31	719.99	30	0.13	0.76
8	0.03	14.31	1679.99	30	0.13	0.76
9	0.03	26.31	1679.99	30	0.25	0.82
10	0.03	26.31	719.99	30	0.25	0.82
11	0.09	14.31	719.99	30	0.25	0.84
12	0.09	26.31	1679.99	30	0.25	1.22
13	0.09	26.31	719.99	30	0.25	1.22
14	0.09	14.31	1679.99	30	0.25	0.84
15	0.03	14.31	1679.99	30	0.25	0.57
16	0.03	14.31	719.99	30	0.25	0.57

Sl. No.	c (MPa)	ϕ (Degree)	E (MPa)	θ (Degree)	k_h	FOS
17	0.03	14.31	719.99	60	0.25	0.55
18	0.03	26.31	719.99	60	0.25	0.68
19	0.03	14.31	1679.99	60	0.25	0.55
20	0.09	14.31	719.99	60	0.25	0.67
21	0.09	26.31	719.99	60	0.25	0.76
22	0.09	26.31	1679.99	60	0.25	0.76
23	0.09	14.31	1679.99	60	0.25	0.67
24	0.03	26.31	1679.99	60	0.25	0.68
25	0.09	14.31	1679.99	60	0.13	0.81
26	0.089	14.31	719.99	60	0.13	0.81
27	0.025	26.31	719.99	60	0.13	0.92
28	0.025	26.31	1679.99	60	0.13	0.92
29	0.025	14.31	1679.99	60	0.13	0.74
30	0.025	14.31	719.99	60	0.13	0.74
31	0.09	26.31	1679.99	60	0.13	0.91
32	0.09	26.31	719.99	60	0.13	0.91

Response surface model is generated using regression analysis with the above data. New fitted equation is given by Eq. 4.8.

$$(FOS)_2 = 0.91 + 3.36c + 0.02\phi - 0.01\theta - 1.63k_h \tag{4.8}$$

The *Hasofer-Lind reliability index* is calculated using spreadsheet based calculation. It is assumed that input parameters are uncorrelated and it is linearly related to the output response. MS Excel's optimization tool "Solver" is again invoked to calculate reliability index. SOLVER tool minimizes beta giving β = 1.89 fulfilling all conditions. At β=1.89, the values various variables are shown in Table 4.12.

Table 4.12. Values of variables at design point in iteration 2

Variables	Values
c (MPa)	0.07
ϕ (Degree)	25.79
θ (Degree)	48.28
k_h	0.18

Now using these design points we again select the new center point and repeat the process again. New design points for iteration second and statistical parameters are provided in Table 4.13.

Table 4.13. Statistical parameters for iteration 3

	X_D	St.dev.	X_{max}	X_{min}
c (MPa)	0.07	0.08	0.13	0.01
ϕ (degree)	18.66	6	24.66	12.66
E (MPa)	1199.99	480	1679.99	719.99
β (degree)	53.84	8.66	60	30
k_h	0.24	0.06	0.25	0.14

Table 4.14. Pseudo-static FOS for iteration 3

Sl. No.	c (MPa)	ϕ (Degree)	E (MPa)	θ (Degree)	k_h	FOS
1	0.13	24.65	719.99	30	0.14	1.7
2	0.13	24.65	1679.99	30	0.14	1.7
3	0.13	12.65	719.99	30	0.14	1.21
4	0.13	12.65	1679.99	30	0.14	1.21
5	0.01	24.65	719.99	30	0.14	0.75
6	0.01	24.65	1679.99	30	0.14	0.75
7	0.01	12.65	719.99	30	0.14	0.61

Sl. No.	c (MPa)	ϕ (Degree)	E (MPa)	θ (Degree)	k_h	FOS
8	0.01	12.65	1679.99	30	0.14	0.59
9	0.01	24.65	1679.99	30	0.25	0.61
10	0.01	24.65	719.99	30	0.25	0.61
11	0.13	12.65	719.99	30	0.25	0.98
12	0.13	24.65	1679.99	30	0.25	1.39
13	0.13	24.65	719.99	30	0.25	1.39
14	0.13	12.65	1679.99	30	0.25	0.98
15	0.01	12.65	1679.99	30	0.25	0.44
16	0.01	12.65	719.99	30	0.25	0.44
17	0.01	12.65	719.99	60	0.25	0.28
18	0.01	24.65	719.99	60	0.25	0.32
19	0.01	12.65	1679.99	60	0.25	0.28
20	0.13	12.65	719.99	60	0.25	0.71
21	0.14	24.65	719.99	60	0.25	0.93
22	0.14	24.65	1679.99	60	0.25	0.93
23	0.13	12.65	1679.99	60	0.25	0.71
24	0.01	24.65	1679.99	60	0.25	0.33
25	0.13	12.65	1679.99	60	0.13	0.84
26	0.13	12.65	719.99	60	0.13	0.84
27	0.01	24.65	719.99	60	0.13	0.8
28	0.01	24.65	1679.99	60	0.13	0.78
29	0.01	12.65	1679.99	60	0.13	0.59
30	0.01	12.65	719.99	60	0.13	0.57
31	0.13	24.65	1679.99	60	0.13	1.09
32	0.13	24.65	719.99	60	0.13	1.09

Response surface model is again generated for iteration 3 using regression analyses with the above data. The equation is

$$(FOS)_3 = 0.95 + 4.37c + 0.02\phi + 0 * E - 0.09\theta - 2.09k_h \tag{4.9}$$

The *Hasofer-Lind reliability index* is calculated and β = 1.80 has been obtained fulfilling all conditions. It may be observed that the β value is almost identical to that of iteration 2. Therefore it is assumed that the β has converged at iteration 3.

4.5.3 Critical Design Points from Reliability Analysis

The iteration, at which reliability index gets converged, the obtained design points are considered to be the critical design points. So, for given problem the critical design points are presented in Table 4.14 with reliability index β=1.80.

Table 4.15. Critical design points obtained at β=1.80

Variables	**Values**
c (MPa)	0.07
ϕ (Degree)	26.68
E (MPa)	1200
θ (Degree)	48.04
k_h	0.18

Chapter 5

DEVELOPMENT OF SEISMIC FRAGILITY CURVES

5.1 Introduction

To meet the increasing demand of infrastructure facilities in mountainous regions, there has been a significant increase in construction of facilities such as rail and road networks on sloping ground. However, as these areas fall within seismically prone region, high risks are associated with such infrastructure facilities (HAZUS-MH 2004) (Pitilakis *et al*. 2002). Hence, to construct and maintain sustainable infrastructure in such regions and avert any catastrophic disaster, assessment of seismic vulnerability of such facilities is necessary.

In this context, the seismic fragility curves have proven to be a robust tool for the prediction of seismic vulnerability of a number of geotechnical systems (Rossetto and Elnashai 2013). The fragility curve relates the probability of unsatisfactory performance of any infrastructural facility with increasing intensity of earthquake motions. Thus, in a broad sense, the uncertainty associated with probable seismic scenarios of a particular site is effectively considered in the assessment of the vulnerability (Andersen *et al.* 2008).

However, another important form of uncertainty in geotechnical systems is associated with the inherent variability of strength and deformation parameters characterizing the geological medium as discussed in *Chapter 4*. For effective planning of lifeline facilities in mountainous regions, a due consideration of both the forms of uncertainties *i.e.*, variable geological setting and seismic motion, needs to be considered. In view of this objective, the present chapter demonstrates the application of seismic fragility curves considering the results obtained in *Chapter 4*.

In the present chapter, a brief description about the seismic fragility curve is provided. The method to evaluate the fragility curve is also discussed. To demonstrate the applicability of the method, dynamic modeling of the slope considered in *Chapter 4* has been explained. This is followed by a brief description of the earthquake time histories considered. Finally, the construction of fragility curves for the slope considered in the present study has been presented.

5.2 Seismic Fragility Curves

Seismic Fragility Curves provides an effective way of predicting the seismic vulnerability of an infrastructural facility giving a relation between the probabilities of unsatisfactory performance as a function of probable seismic ground motion. It provides a robust and flexible tool to designers in evaluating the relative performance of alternate designs by considering variable failure criteria and probable ground motion thereby striking a balance between safety and economy.

A brief discussion of various components required for the generation of seismic fragility curve is presented in subsequent sections. This is followed by the application of the discussed methodology for the generation of seismic fragility curve for slope problem discussed in *Chapter 4*.

5.2.1 Methodology of Development of Fragility Curves

The generation of fragility curves is based on defined damage state and input ground motion. By defining the damage level the fragility curves could be generated as a function of the level of seismic excitation.

Damage state

Damage states define the severity of damage sustained by the infrastructure facility. It may be different for various kinds of facilities depending on their importance and associated repair cost. The selection of threshold values for damage is either based on observed performance or expert judgment. In this study damage state is based on displacement criteria which are taken as 50mm of horizontal displacement. This damage state has been selected on the basis of that the exceedance of 50mm horizontal displacement will compromise the integrity of any infrastructure facility planned on the considered slope.

Input Motion

To characterize the probable seismic motion, ten acceleration time histories have been adopted for the dynamic analyses of considered slope. The earthquake time histories considered include the records of the following earthquake shown in Table 5.1.

These records have been selected to include a wide range of possible seismic scenarios possible in reality. One such acceleration time history of Kobe earthquake is shown in Fig. 5.1.

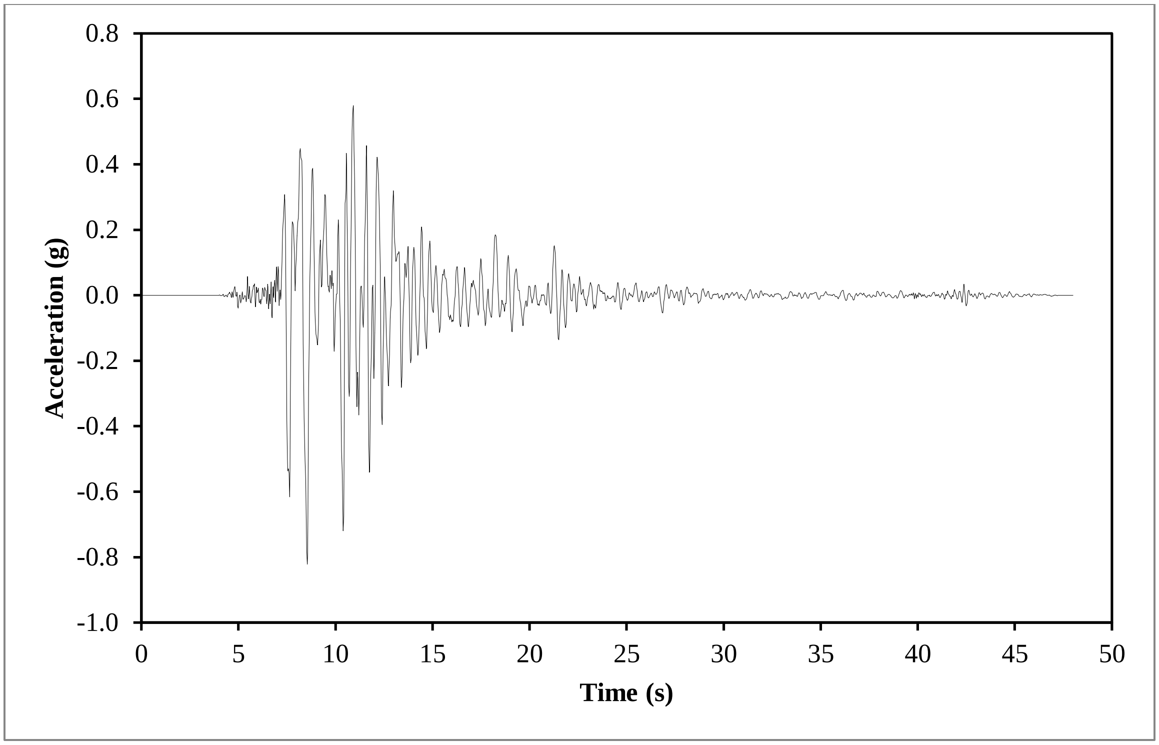


Fig. 5.1. Acceleration time history of Kobe earthquake (1995)

In order to evaluate the vulnerability with the intensity of seismic motion, all earthquake time histories have been normalized using the respective peak ground acceleration (PGA). All the earthquake motions have been scaled in steps of 0.1g to include a range between 0.1g to 1.0g of PGA in order to calculate the response of the slope considered in study with increasing intensity. The properties of the rock mass adopted for the assessment of fragility curve corresponds to the critical design points identified in *Chapter 4*.

Table 5.1. Earthquake time histories considered in the study

Earthquake Name	Year	PGA (g)	Pred. Time Period (s)
Chi Chi	1999	0.18	0.005
Coyote	1979	0.12	0.005
Imperial Valley	1940	0.17	0.01
Kobe	1995	0.82	0.02
Kacaeli	1999	0.22	0.005
Loma Gilroy	1989	0.17	0.005

Earthquake Name	Year	PGA (g)	Pred. Time Period (s)
Loma Gilroy2	2002	0.36	0.005
Mammoth Lake	1980	0.43	0.005
Nahanni	1985	0.15	0.005
Northridge	1994	0.22	0.02

Development of Seismic Fragility Curves

The construction of fragility curves is based on the permanent horizontal displacement and the seismic intensity in terms of PGA at ground surface. The vulnerability assessment is carried out by these curves and evaluation of damage with increasing earthquake intensity. The median threshold and total standard deviation must be calculated for each damage state.

The Fragility Curves are usually described by a lognormal probability distribution function as (Argyroudis *et al*. 2013):

$$P_f\left(ds \geq ds_i \middle| S\right) = \phi\left[\frac{1}{\beta_{tot}}.\ln\left(\frac{IM}{IM_{mi}}\right)\right] \tag{5.1}$$

Where P_f is the probability of exceedance of a particular damage state, *ds*, IM is intensity measure (ex. PGA, PGV etc.), ϕ is the standard cumulative function, IM_{mi} is the median threshold value and β_{tot} is the total lognormal standard deviation. A lognormal standard deviation (β_{tot}) that describes the total variability associated with each fragility curve needs to be estimated. Because of the lack of a rigorous estimation, the uncertainty associated with the definition of damage states (β_{DS}) is set equal to 0.4 following HAZUS for buildings. The uncertainty due to the capacity (β_C) is assigned equal to 0.3 based on engineering judgment. The last source of uncertainty, associated with the seismic demand, is described by the variability in response due to the variability of ground motion.

A log normal standard deviation (β_{tot}) describes the total variability associated with each curve. Total variability is a combination the three contributors as:

$$\beta_{tot} = \sqrt{\beta_{DS}^2 + \beta_C^2 + \beta_D^2} \tag{5.2}$$

Here β_D is obtained from different input motions at each PGA level as shown in Fig. 5.2. General steps of generation of seismic fragility curves are provided in flow chart Fig. 5.3.

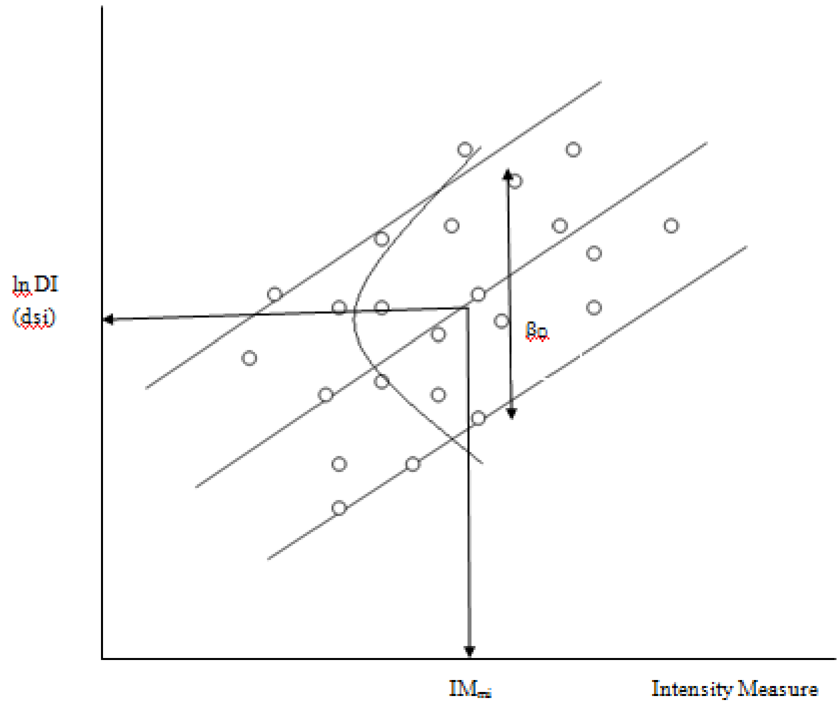


Fig. 5.2. Evaluation of intensity measure and standard deviation

5.3 Slope Considered

The slope considered for the analysis is the same as adopted in Chapter 4 along with the critical design parameters obtained from the reliability analysis. The details of the slope are shown in Table 5.2.

Table 5.2. Geometric and material properties of slope considered

Slope angle	45˚
Height	50 m
Unit weight of soil	0.0185 MN/m^3
Cohesion	0.07275 MPa
Angle of friction	26.6817
Young's modulus	1200 MPa
Poisson's Ratio	0.3

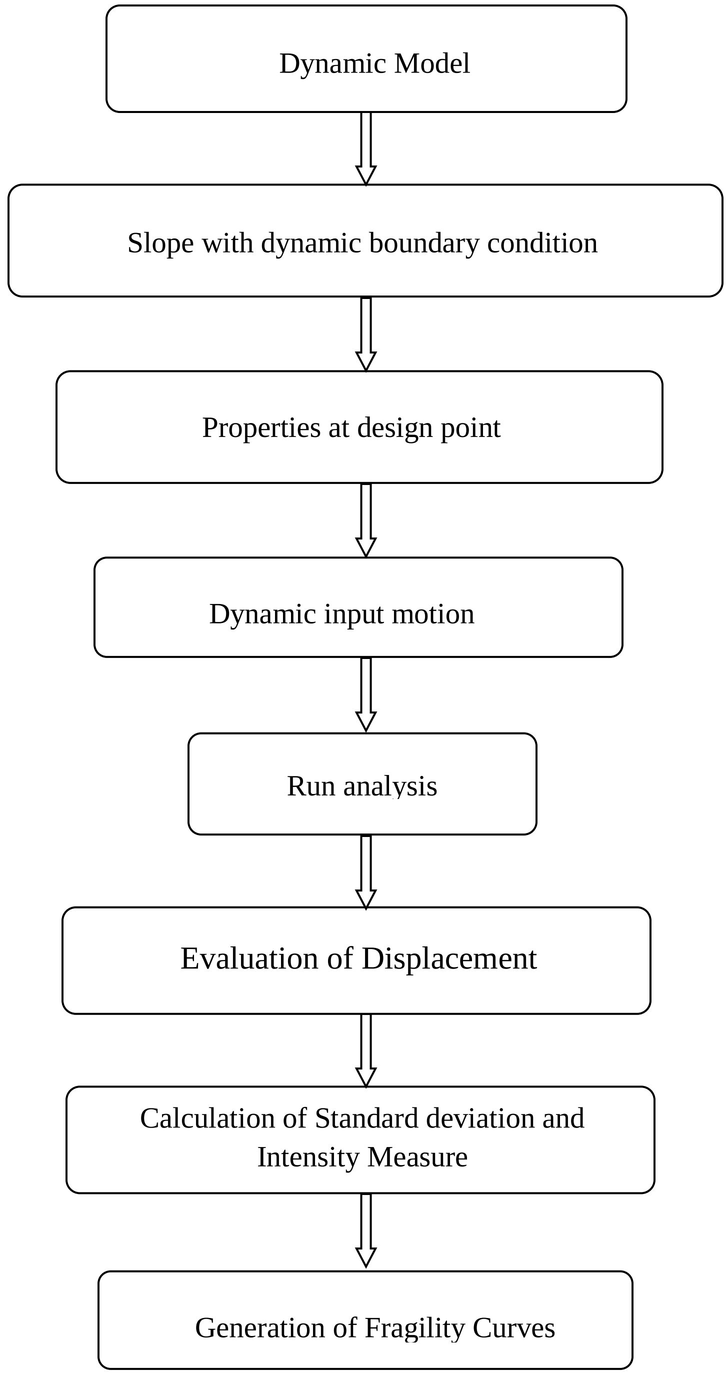


Fig. 5.3. Flow chart showing procedure for generation of fragility curves.

5.4 Dynamic Analyses of Slope

The analyses are performed with 2D finite element software package PHASE^2. All the analyses were performed using the elastoplastic soil behavior with Mohr–Coulomb criterion.

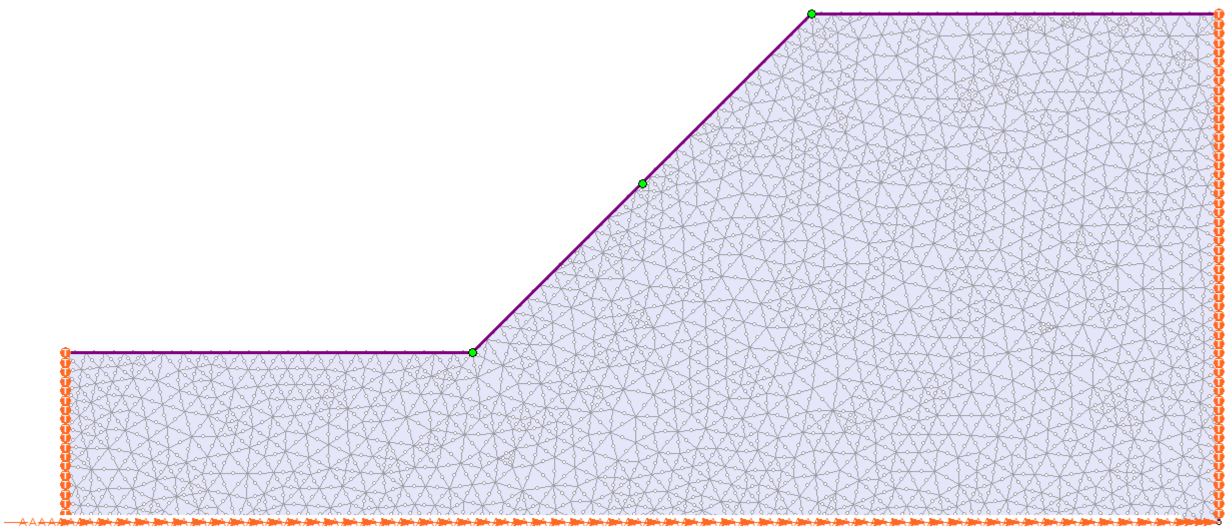


Fig. 5.4. Representative Slope for the dynamic analysis

Fig. 5.4 represents the dynamic model used in the study. The mesh is created by using 6-node triangles and having uniform mesh type. Finer mesh size has been considered based on preliminary sensitivity analyses. Finite element domain is discretized using 4701 nodes and 2268 elements.

5.4.1 Boundary Condition

In dynamic analyses, external boundaries representing the ground surface should be free to move in any direction. For this study lateral boundaries has been allowed to move vertically hence displacement in X-direction is restrained.

Absorbing boundaries has been applied at the base boundary of the model in order to absorb incoming waves travelling in the soil and lateral boundaries are set as transmitting boundary. Applying the earthquake time histories to the model and performing the dynamic analyses, displacement is obtained for the each model run. To get the displacement values time query has been incorporated in the model. Time query is applied at the required points where the dynamic data will be recorded for all time steps of the simulation.

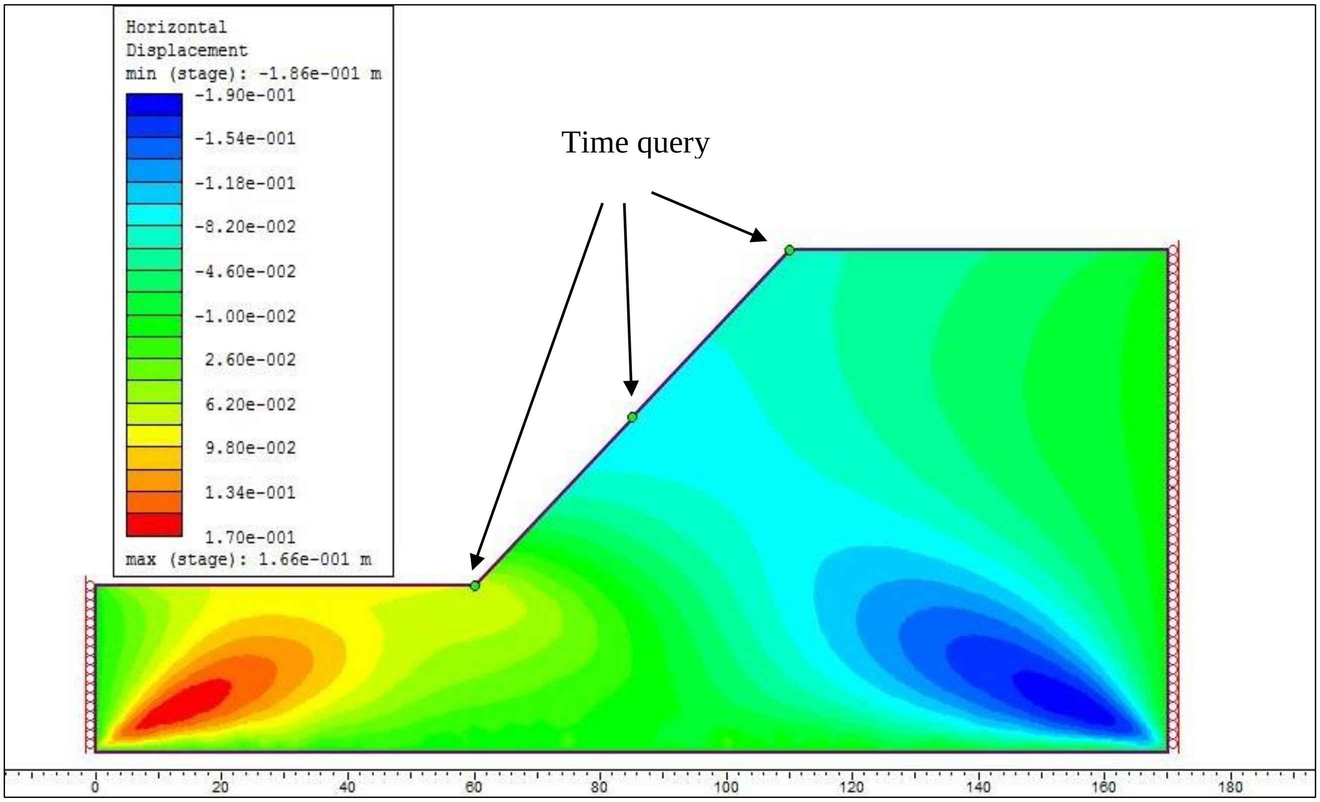


Fig. 5.5. Displacement Contour for Kobe earthquake time history and Location of displacement time queries

Fig. 5.5 represent the displacement contour from the analysis with Kobe earthquake motion. The three green dots in the figure show the point where the time query is noted.

5.5 Evaluation of Damage in Slope

The deformations corresponding to each scaled PGA is presented in Fig. 5.6. The standard deviation (β_D) is calculated from results of displacements. Intensity measure is determined from linear equation fitted for displacement vs. PGA.

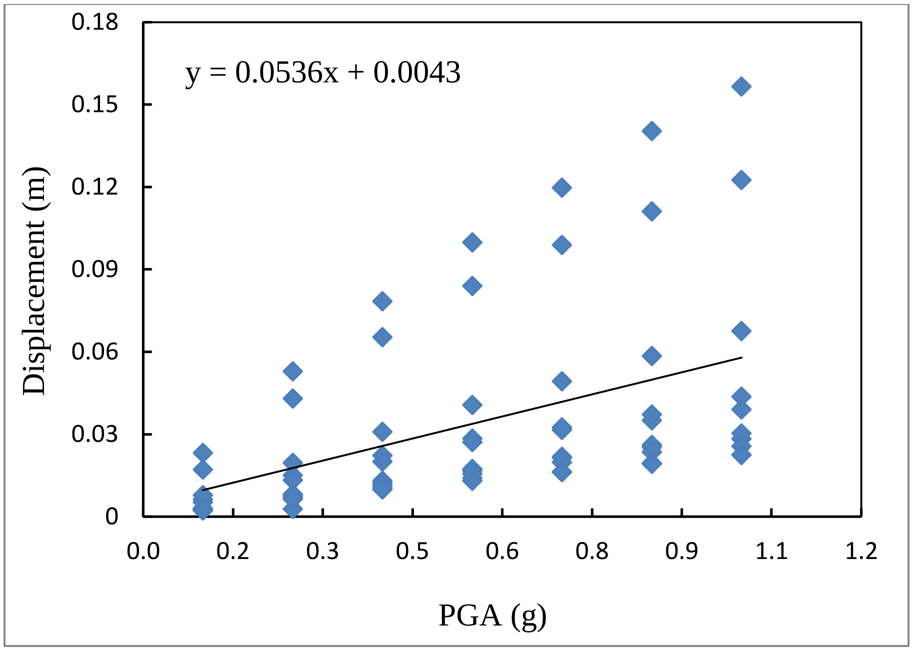


Fig. 5.6. Plot of maximum displacement vs PGA from dynamic analyses

Further for the evaluation of damage in slope, determination of total standard deviation and intensity measure are required. So as discussed in procedure to generate fragility curve the total standard deviation is calculated as:

$$\beta_{tot} = \sqrt{0.3^2 + 0.4^2 + (0.001)^2} = 0.5$$

and intensity measure for each damage state is determined from the equation established between displacement and PGA. The intensity measure determined for each damage state considered is presented in Table 5.3.

Table 5.3. Intensity Measure corresponding to each damage state

Damage State (m)	**Intensity measure**
0.01	1.78
0.02	0.29
0.03	0.47
0.04	0.66
0.05	0.85
0.75	1.78
0.10	1.29

5.6 Final Seismic Fragility Curves

Following the procedure finally the fragility curves are obtained and presented in Fig. 5.7.

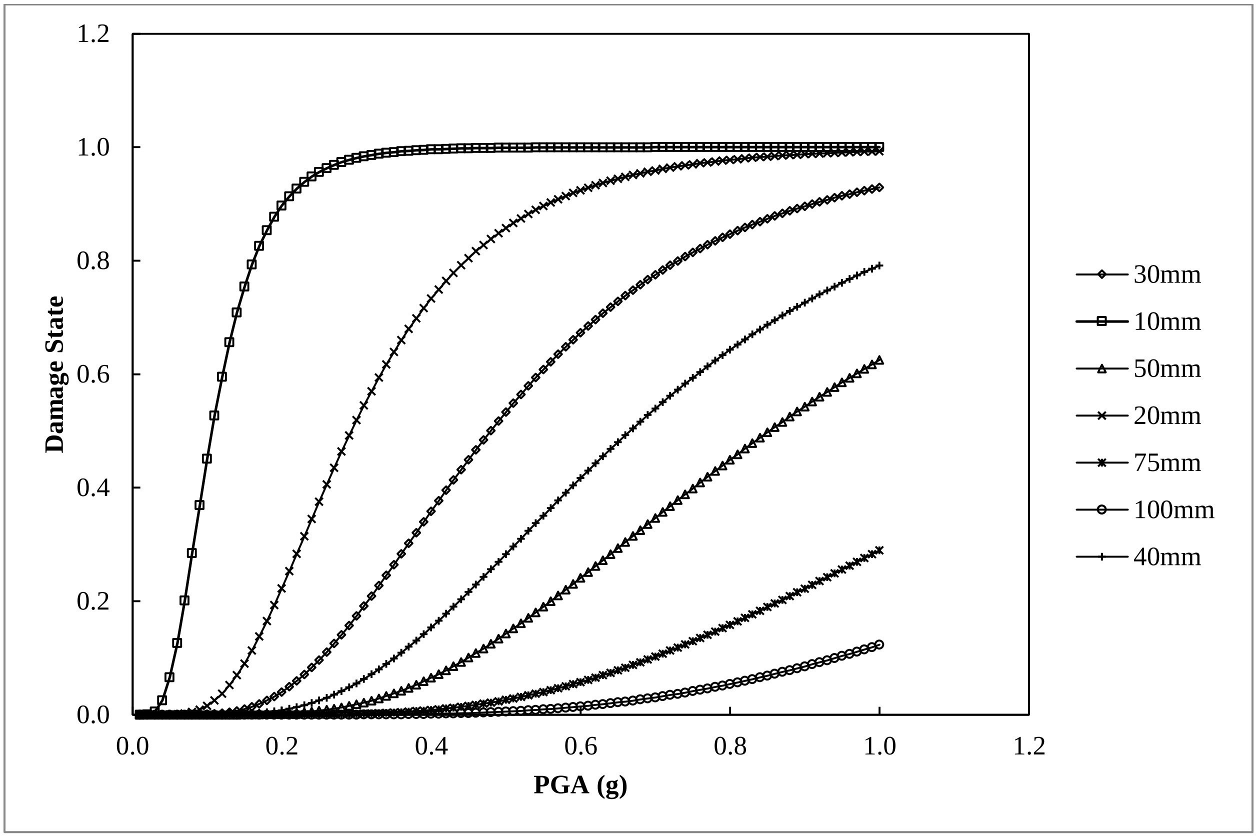


Fig. 5.7. Final seismic fragility curves for the slope

5.7 Discussion on Seismic Fragility Curve

A methodology is described to develop the seismic fragility curves for geotechnical systems for various seismic inputs with different frequency contents, scaled to different levels of seismic loading. The slope system response is evaluated by 2D dynamic analysis in PHASE^2. Then the seismic fragility curve is generated. Results denote the significance of soil conditions on the seismic vulnerability of slope system.

The vulnerability and loss estimates for geotechnical infrastructures can be evaluated on the basis of generated fragility curves. The importance of site-specific dynamic analysis is explored for the vulnerability assessment and the efficient mitigation strategies and policies can be prepared for pre and post earthquake actions. The actual vulnerability and the associated risk of any element at risk may be reduced with appropriate mitigation counter measures. The accurate evaluation of the input motion in terms of ground shaking characteristics, for a given probability of occurrence of a specific magnitude seismic event, always plays the decisive role in the risk assessment and use of the fragility curves becomes most accurate in vulnerability assessment of infrastructure susceptible to seismic ground motion.

Chapter 6

Summary and Conclusion

6.1 Summary

Vulnerability assessment of geotechnical system is a difficult task for geotechnical professionals. It gets more complicated because of variability in geologic material and the expected ground motion. Reliability based approaches are very popular in evaluation slope stability with better confidence level. The reliability based approaches majorly aim to quantify the system performance in terms of probability of unsatisfactory performance or failure. These approaches are very useful in evaluating the critical parameters for a particular slope.

In the present work, *First Order Reliability Method* (FORM) has been adopted for reliability analysis of a rock slope utilizing the analysis results obtained from software package PHASE^2. *Response Surface Method* (RSM) has been used for the process of identifying and fitting an approximate response surface framing the performance function.

Seismic vulnerability of the slope system has been assessed by development of seismic fragility curves. Relation between the probabilities of unsatisfactory performance as a function of probable seismic ground motion (PGA) has been established by the seismic fragility curves of the slope using the critical design parameters obtained from the reliability analyses.

6.2 Conclusion

The major conclusions resulting from this present study may be summarized as below.

- Influence of material uncertainties on the stability of slopes are very significant and hence material uncertainties need to be considered.
- Reliability based approaches have significant computational advantage over conventional methods like Monte Carlo approach. These reduce computation burden in a significant way.
- Critical design parameters has been evaluated for the considered slope.

- Seismic fragility curves have been developed for the slope with the identified critical design parameters considering Peak Ground Acceleration (PGA) of the earthquake ground motion.

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