Dynamic Economic Load Dispatch with Renewable Energy Sources using an Improved Fireworks Algorithm

Master of Technology

(Power Systems)

by

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(2014PES5405)

Under the supervision of

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Assistant Professor



Department of Electrical Engineering Malaviya National Institute of Technology Jaipur June 2016

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This dissertation is submitted in partial fulfillment of the requirements for the award of degree of

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the Dissertation entitled **"Dynamic Economic Load Dispatch with Renewable Sources using an Improved Fireworks Algorithm"**, in partial fulfillment of the requirements for the award of the **Degree of Master of Technology** and submitted in the **Department of Electrical Engineering**, Malaviya National Institute of Technology Jaipur, is an authentic record of my own work under the supervision of Dr. Nikhil Gupta, Assistant Professor, Department of Electrical Engineering, Malaviya National Institute of Technology Jaipur.

The matter presented in this dissertation embodies the results of own work and studies carried out by me and have not been submitted for the award of any other degree of this institute or any other institute.

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Dr.Nikhil Gupta Supervisor

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ABSTRACT

Growing concerns for environmental regulation, scarcity of fossil fuel and steep rise in fuel price and on the contrary substantial generation from the renewable sources has emerged the hybrid (mix) generations in the modern power system. This is benefitting by reducing the demand of the conventional sources and creating ecofriendly environment. The solar and wind both are the most popular and inexhaustible source of renewable energy, but also subjected to uncertainty and variability. Thus, their integration in the power system affects the economic operation due to their unpredictable nature. Therefore, a Stochastic Dynamic Economic Load Dispatch (DLED) with the incorporation of wind and solar based power generation is addressed in this work. Further, the cost functions of the renewable sources are used to consider the economic factor and their variability is considered by the overestimation and underestimation.

DELD problem is highly non-linear, non-convex, multi-constraint optimization problem with continuous decision variables. Such problem can be efficiently solved using metaheuristic techniques. Thus, the present work utilizes Improved Fireworks Algorithm (IFWA) methodology for obtaining optimal schedule of generators in a stochastic dynamic economic load dispatch problem. The FWA is swarm based computational intelligent techniques. It is inspired by the explosion of the fireworks and extensively found to be powerful techniques in among all other metaheuristic techniques. However, the conventional FWA faces incompetence for the functions having optima far away from the origin as exists in DELD problem. Therefore, several modifications have been suggested in FWA and a new method is proposed named as Improved Fireworks Algorithm using Chaotic Sequence Operator (IFWA-CSO). The validation and effectiveness of the FWA and developed variants is first tested on the some well-known benchmark functions and then applied on the power system optimization problem of economic operation in static and dynamic states with variety of constraints. The obtained numerical results verify the efficacy of the proposed method and its variants and seem to be a promising optimizer than conventional FWA.

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Abbreviations

ED/ELD	Economic Load Dispatch
SELD	Static Economic Load Dispatch
DELD	Dynamic Economic Load Dispatch
EP	Evolutionary Programming
SQP	Sequential Quadratic Programming
FWA	Fireworks Algorithm
IFWA	Improved Fireworks Algorithm
PSO	Particle Swarm Optimization
GA	Genetic Algorithm
DE	Differential Evolution
CS	Cuckoo Search
GSO	Group Search Optimizer
CQGSO	Continuous Quick Group Search Optimizer
DG	Distributed Generation
DHSA	Differential Harmony Search Algorithm
TLBO	Teaching Learning Based Algorithm
POZ	Prohibited Operating Zones
DP	Dynamic Programming
ARMA	Auto-Regressive Moving Average
PSO-LRS	Particle Swarm Optimization Local Random Search
NPSO	New Particle Swarm Optimization
APSO	Adaptive Particle Swarm Optimization
SOH-PSO	Self-Organizing Hierarchical Particle Swarm Optimization
QPSO	Quantum-Behaved Particle Swarm Optimization
ABC	Artificial Bee Colony
AIS	Artificial Immune System
ICA-PSO	Improved Coordinated Aggregation-Based Particle Swarm
	Optimization
DE/BBO	Differential Evolution/Biogeography-Based Optimization;
HHS	Hybrid Swarm Intelligent-Based Harmony Search Algorithm;
CCPSO	Chaotic Sequences and Crossover Operation Particle Swarm

	Optimization
IPSO	Improved Particle Swarm Optimization
IABC-LS	Incremental Artificial Bee Colony With Local Search
NAPSO	New Adaptive Particle Swarm Optimization
CSA	Cuckoo Search Algorithm
DCPSO	Dynamically Controlled Particle Swarm Optimization
RBF	Regularized Beta Function
UR_i	Up ramp
DR_i	Down ramp
PV	Photo-voltaic
WECS	Wind Energy Conversion System
EFWA	Enhance Fireworks Algorithm
CCPSO	PSO with the both chaotic sequence and crossover operation
EAPSO	Enhance Adaptive Particle Swarm Optimization
LP	Linear Programming
NLP	Nonlinear Programming
QP	Quadratic Programming
LR	Lagrangian Relaxation
ACO	Ant Colony Optimization
BF	Bacterial Foraging
pdf	Probability Distribution Function

CHAPTER 1 INTRODUCTION

In the nineteenth century, the beginning of the industrialization and later, the invention of electricity has changed the future of the whole world. This begins the exhaustive exploitation of energy resources and creates the multiple resources for electric power generation. However, the fossil fuels get hold the most of the common resource of the electricity power generation. Further, with the demographic expansion, energy requirement escalates exponentially, which leads to formation of large interconnected power system and need for better utilization of the energy rises. This developed one of the most versatile problems of power system known as economic load dispatch (ELD). This aims to obtain the minimal fuel cost of thermal generators by optimally allocating the several generators while satisfying generator and network constraints. Hence, it plays an essential role in economic operation of power system has significant reduction in the fuel cost.

However, the fossils fuels are becoming the serious threat of extinction in the near future due to their excessive consumption. This is creating energy crisis with everincreasing demand. In addition, emission of pollutants gas from these sources has also caused severity of global warming. Thus, this alarming bell has forced the human race to take necessary step to curb down the green emission gas as well as look for alternative resources with low or negligible pollutant potential. For this use of non-conventional sources and to control the emission level certain environmental regulation has been enforced. Similarly, to tackle energy demand crisis alternative approach such as energy efficient management system, renewable sources integration with conventional power system. The renewable technologies has been emerging potential of power generation source and becoming the researcher's interest. As it is perceived that the extraction of energy from renewable resources significantly increases in the recent years. Therefore it can be inducted into the conventional power systems.

The hybrid system consists of conventional and non-conventional resources. The non-conventional resources include wind, solar, hydro, bio-mass etc. However, in this context the wind and solar has drawn significant attention in the recent years. It is due to its wide availability, plentiful, clean resource of energy and free from fuel cost. There substantial generation can reduce the fuel cost of thermal generators as well as pollutant emission from burnt fuels. However, being of environment friendly and of substantial potential, the renewable sources suffer the drawback of being uncertain and unpredictable nature. Thus, this limits the accountability of the renewable sources. This uncertainty and vagueness of these resources also reflects in the power generation with disturbance. This raises the issue of reliability, security and stability in the power system, where these concerns are paramount of power system operation. Therefore, power extracted from renewable energy conversions device are highly non-linear and dis-continuous in nature. The wind and solar both are intermittent source of nature but, the wind is considered to be more unpredictable than solar, which is partial predictable due to zero solar insolation availability at night and sun movement. Therefore, renewable resources are sustainable source of energy but, they also are unreliable source of electricity generation in power system operation.

Wind and solar based power generations are highly variable and non-deterministic in nature. Both sources depend upon the weather and geographical condition which are partially predictable. Though, there are various forecasting methods available which forecast the future scenario on the basis of the past information. But, as far as electrical demand is concern the demand variation can be predicated on the basis of load duration curve and it's forecasting reasonably fit according to past information due to electrical demand does not vary suddenly with time while weather prediction is not very accurate and causes sudden variation with time. Therefore, renewable sources based power generation cannot be employed independently in the large power system. Thus, mix generating system is employed. However, to avoid the large changes in the system rescheduling of conventional generator require in minutes to hour.

In power system ELD is defined in two ways, first called static economic load dispatch (SELD) as being independent of time and second is termed as dynamic economic load dispatch (DELD) being dependent on the time. In SELD generators are schedule for one operating period with optimized fuel cost. However, in the real time

of operation, generators are scheduled continuously. Thus, the SELD is not all form of practical ED operation. Furthermore, in SELD, a ramp rate constraint is usually not considered, which incurred due to physical limitation of the generator that results in the restricted operating range of the generator instead of the entire operating range in the successive schedule. Therefore, to consider it ED operation must be extended into DELD operation.

DELD is process of the economic operation in the power system for a day-ahead scheduling with forecasted load demand. It is the process of obtaining the optimum schedule of the generator units with minimizing the total fuel cost over continuous time periods while satisfying the generator and network constraints. It is noted that DELD is extended version of conventional ELD problem with incorporation temporal constraints (e.g. ramp rate limits) of generation units. This is used to avoid the physical limitation of the generator, which results in change in generation limits of the generator in each time period. In DELD, the generator are schedule in discretized manner over continuous periods and this time periods can be range from minute to hours. It is noted that DELD is similar to unit-commitment in terms of scheduling of the generators for day-ahead demand. However, it is different in terms of operation, where unit-commitment is problem of possible combinations of units to meet the particular load demand, and then select the optimal combination, while DELD is directly applied for optimal scheduling of generators on the available units.

The renewable sources embankment in modern power system has transformed the power system into the (hybrid) system and this shifting the focus toward the development in the renewable technologies. This is becoming the investigating context in the modern power system. However, the integration of renewable source will reflects on the economic as well as security concerns of the power system. Therefore, in this work, a stochastic DELD with renewable sources is proposed to incorporate the wind and solar based power generation. The cost model of wind and solar is employed to consider the economic factor. The stochastic nature of wind and solar is modeled by suitable probabilistic distribution functions. Further, to solve the problem a recently developed technique Firework Algorithm is used. However, due to its severe limitation on shifted function, an improved version of FWA, Improved Fireworks Algorithm using Chaotic Sequence operator is proposed. The proposed

algorithm is tested and analyzed on the standard system and applied in the stochastic DELD.

The thesis has been organized in the seven chapters.

Chapter 1: In this chapter a brief introduction about the ED and DELD and its importance are described. The needs of hybrid generating systems and its challenge in operation in modern power system are discussed.

Chapter 2: The literature review about the different methodology and solution techniques of ED and DELD problems techniques are discussed. Further, the various approaches for the inclusion of wind and solar sources are discussed.

Chapter 3: The modeling of renewable sources (wind and solar) are carried-out with suitable probabilistic distributions using probabilistic model and their probabilistic cost functions are formulated to consider the effect of underestimation and overestimation due to unpredictability of these sources.

Chapter 4: The objective function of stochastic DELD problem with renewable sources is formulated using the various generator and network constraints in this chapter.

Chapter 5: The problem solution technique FWA is discussed in this chapter and its fabrication in computational intelligence is explained. Further, the inadequacy of FWA is asserted with the reason and to overcome this certain modifications have been proposed in IFWA-CSO algorithm. The pseudo code of proposed IFWA-CSO techniques is developed.

Chapter 6: The simulation results are presented in this chapter. The standard FWA and proposed IFWA-CSO algorithm is programmed in MATLAB. The effectiveness and validation of proposed techniques is established using standard ED and DELD system. Finally, the proposed techniques implemented on the formulated function objectives.

Chapter 7: Conclusions and future scope are discussed

In the next chapter, a brief history of ELD development with different methodologies and their solution techniques are discussed.

CHAPTER 2 LITERATURE REVIEW

This chapter discusses the brief history of development of ED problem with different methodological approaches and then extended to the DELD system. Further, the various approaches for the inclusion of the renewable source have been discussed. The various techniques used for the optimization are explained progressively. Finally, the methods and solution technique aspects are critically reviewed and drawn conclusions are framed in the objective and scope of the dissertation.

Economic load dispatch is primarily deals with minimization of operating fuel cost of the generators to serve the power demand while satisfying the generator and network constraints. Generally, the fuel cost function of thermal generators is approximated by the quadratic function. However, for multi-fuel fired thermal generators exhibit non-convexity in the fuel cost function in the account of valve-point loading effect. Further, to incur practical operating conditions of generators, gives rise to constraints of ramp rate limits and prohibited operating zones. Similarly, transmission loss can also be included to consider the power loss. In general, the economic load dispatch problem is highly non-linear complex combinatorial optimization problem. It is one of the old classical problems of power system. However, the structure of the ELD problem is evolved with time. A brief literature review about the development of different methodologies, solution techniques, etc. is presented in this chapter.

In Ref [1], the author discussed the various aspects of ED problem and also insight about the different methodology in accordance with the progressive development in twentieth century. Liang *et al.* [2] proposed a dynamic programming method using a zoom feature to solve the ED problem with consideration of transmission loss. In [3] the author discussed the concept of ramping costs in ED problem. As the unit ramping process to cost of fatigue effect in the generator scheduling of thermal systems. Fan *et al.* [4] attempted more realistic approach to solve the ED problem with generating units' sub-region to account the prohibited operating zones (POZs) and developed a novel strategy to find feasible search space by utilizing the lambda-iteration method to avoid the multiple decision search space due to POZs. Yoshikawa *et al.* [5] addressed the fuel cost dynamics on online ED with auto-regressive moving average (ARMA) model. This proposed supplemented ARMA quadratic model to counter the dynamic variation in the fuel cost with generation. Irisarri *et al.* [6] proposed the ED problem with the network flow and generator's ramping constraints by employing Interior Point Methods to measure the economic and security issues simultaneously.

However, DELD problem is more realistic in operation than ED which continuously schedules the generator units. Moreover, it is also suffice to consider pre-long scheduling of generators optimally to maintain economy, stability and security. Thus, ED problem must be extended into long term i.e., DELD. Some of the adapted methodologies of DELD are discussed as follows: Wood *et al.* [7] attempted ED problem with the spinning reserve in static and dynamic state to avoid high load pick-up with reasonable spinning reserves using an efficient use of computer resources. Han *et al.* [8] put forwarded the factors that affect the feasibility and optimal solution of DELD problem and solved it by two methods. Attaviriyanupap *et al.* [9] proposed a hybrid evolutionary programming and sequential quadratic programming (EP-SQP) for DELD with non-convex fuel cost function to account the multi fired-fuel system.

Victoire *et al.* [10] solved the DELD problem with valve-point loading effects taking the reserve constraint into account using deterministically guided Particle Swarm optimization (PSO) and test the feasibility on three different load patterns. In [11] author presented the Enhance adaptive PSO for DELD with valve-points effects and ramp rates. T. Niknam *et al.* [12] proposed a new modified TLBO for reserve constrained DELD with more practical formation of DELD problem. Arul *et al.* [13] addressed the DELD problem using Harmony search algorithm with a chaotic self-adaptive differential operator (DHSA).

As mentioned in the previous chapter the mix generating system becoming the part of the modern power systems. Thus, some of the approaches for the inclusion of renewable sources in ED and DELD operation are discussed here. In Ref [14] the author addressed DELD problem with integration of renewable sources by setting the penetration level as fraction of load demand. But, it does not consider cost function and any uncertainty related to the renewables. Villanueva *et al.* [15] addressed the issues of wind uncertainty by using correlated wind speed data for economic dispatch

formulation. This limits wind uncertainty to some extent. However, correlated data is generated using the past information. Thus, it cannot handle sudden variation. Wang *et al.* [16] solved the ED problem with the integration of wind power by accessing the risk and cost both using the fuzzy logic approach. Since the wind variability limits the penetration level of wind generators. Thus, fuzzy membership based function to balance the risk and cost of wind power. Karaki et al. [17] presented a solar and wind probabilistic model to measure their performance in autonomous solar-wind energy conversion systems. Bilil et al. [18] used the probabilistic approach for economic/emission optimization in multi-source system. In Ref [19], the author presented generation adjustment methodology using base point and participation factor approach to account the fluctuations in load and renewable sources. The problem is solved in real-time economic dispatch to take measure against the fluctuations. Cheng-Chien Kuo [20] suggested combine economic and emission dispatch by considering the large penetration of wind power. In this wind energy consider as dispatch able unit. Khan et al. [21] suggested solar power generation in combine economic/ emission power dispatch.

Hetzer et al. [22] attempted an approach for ED modeling with the integration of wind power. This takes economic factor of wind power by considering a cost model related by proportional to scheduled power and variability by overestimation and underestimation terms. In [23] the author addressed the issues of economic/ emission by considering the factor of overestimation and underestimation of wind power. As, wind power penetration will certainly affects the conventional unit generation. Similarly, it will also affect the pollution from the conventional sources. Thus emission function is modeled to similar wind cost model as in [22]. In similar way overestimation and underestimation factor can be considered for other renewable sources with uncertain nature. Therefore, same approach is put forwarded the inclusion of solar cost model as wind source in [24]. Peng et al. [25] proposed DELD problem in the conventional thermal system including the wind power using a novel differential evolution with bi-population chaotic sequence. In this, the author has modeled the wind speed in the time sequence using probability distribution model. Reddy et al. [26] attempted to solve the ED problem in the real time by considering the variability of wind, solar and load demand in the small time frame (minute) for a

given time. It employed participation factor approach to handle the uncertainty related to renewable sources.

The solution technique of ED can be classified into two categories as classical and heuristics. In classical method mathematical based techniques are gradient, lambda iteration [27-28] and computational optimization based method are Dynamic Programming (DP) [2], Linear Programming (LP) [29], Nonlinear Programming (NLP) [30], quadratic programming (QP) [31], and Lagrangian relaxation (LR) [32] Similarly, heuristics based algorithms can be defined into various types based on their mechanism such as swarm, evolutionary etc. Some of evolutionary techniques applications on the ED problems are: In Ref [33] the author solved ED problem with non-smooth fuel cost function using Genetic Algorithm (GA). Amjady et al. [34] suggested the non-convex economic dispatch with ac constraints using GA. In Ref. [35] author presented the economic-emission using binary successive approximationbased Evolutionary Programming (EP). Coelho et al. [36] tested ED problem using Differential Evolution (DE). On the contrary, swarm intelligence based algorithms also proven be substantial solver for ED problem. It includes Particle Swarm Optimization (PSO) [37-38]. Ant Colony Optimization (ACO) [39] is a swarm intelligent based meta-heuristic technique inspired by ant behavior, which search good path through graphs. In [40] Bacterial Foraging (BF) is employed for economic and emission load dispatch problem. In [41] a non-convex ED problem is solved by Cuckoo Search problem. This technique is stimulated from the particular cuckoo species behavior, which laid their eggs in the others nests of other species for the survival. Dalyand et.al [42] addressed a non-convex ED problem using Continuous Quick Group Search Optimizer (CQGSO). It is an enhanced version of quick group search optimizer (QGSO) algorithm. It employed the tactics of current and global best updating process of PSO for updating the scrounger locations up to current iterations etc. Similarly, various metaheuristics techniques employed for DELD problem. It include hybrid EP and SQP [10], Enhance adaptive PSO [11], chaotic self-adaptive differential Harmony search algorithm (DHSA) [13], Artificial Bee colony (ABC) [43], Artificial Immune system (AIS) [44] and TLBO [12] etc.

2.1Critical Review

As, mentioned that ELD is most integral part of power system for the economic operation and their significance in power system is perpetual. However, with mix generations their importance is perplexed as security and reliability issues arise. Therefore, there is need to revise the approach to consider the effect of renewable sources. Thus, it is critically reviewed in the methodology aspects. Similarly, the techniques analysis is investigated in solution techniques aspects.

2.1.1 Methodology Aspects

The ED is optimally schedule the thermal generators for the forecasted load demand. However, with the integration of renewable sources, economic operation changes in various ways. In DELD problem [14] generation from the renewable sources is subtracted from load demand and rest demand share by conventional sources. Some authors [15, 22-26] have investigated renewable (wind or solar or both) sources as utility to consider the economic factor. This employs a cost model consists of proportional scheduled power. Moreover, variability is related by overestimation and underestimation factor. Their cost coefficients is defined as penalty and reserve cost, where penalty on the underestimation and reserve (purchase) on overestimation of available power. Thus, this cost model justified economics and security of system. In [22] analysis reveals that the cost parameters and availability of power has significant affect in the scheduling of wind powered generator. Thus, the cost model changes dynamically. Therefore, a reasonable cost coefficient value should be selected based on the source availability. Here, wind and solar power is being incorporated in conventional power system. Since, the wind and solar both are variable source of nature and their availability varies over time of period. Thus, problem is formulated in DELD form. Therefore, wind and solar has been modeled over time interval to measure their potential with conventional power system.

2.1.2 Solution Technique Aspects

Since, practical ED problem is bound to various constraints such as valve-point loading effect and POZs etc. Therefore, ED is a non-convex, non-linear, discontinuous complex combinatorial optimization problem and cannot be solved by classical mathematical method [27-28]. However, classical computational optimization methods [2, 29-32] are suitable to solve such problem but, suffer from curse of dimensionality when dealing with the large-scale problems [12]. This led the population based heuristics techniques, which are not restricted to shape of the objective function and known to be most affirmative methods. However, these techniques cannot guarantee optimum solution due to their stochastic nature. Therefore, there is constant evolution on these algorithms as discussed. GA [33-34] is based on the genetic operation. However, GA suffers from the slow convergence and its encoding and decoding which, is essential part of GA causes more computation time; the methods based on the evolution are Evolutionary Programming (EP) [35] and Differential Evolution (DE). EP is similar to genetic programming, with fixed the programming structure. It shows slow convergence; Differential Evolution (DE) [36] forms the solution by utilizing one or more existing solutions. It shows better exploration, but premature convergence causes the solution to trap in local optima [45]. However, PSO a swarm based meta-heuristics technique exhibits faster convergence, [37-38, 46], but suffers from premature convergence on large scale problem dealing with complex constraints, resulting a local optimum likely [38]. Moreover, these algorithms suffer heavily in DELD optimization problem due to more complex constraints than ED, and huge and irregular search space due to ramp rate limits.

The last decade has witnessed the potential of several newly established metaheuristic techniques. Fireworks Algorithm (FWA) [47] is one of the recently developed powerful meta-heuristic techniques based on the explosion of fireworks. In FWA, every individual mimics the explosion process of fireworks and thus enables the algorithm to perform local and global search simultaneously. Hence, this unique features of FWA, adaptively performs the exploration and exploitation. The algorithm effectiveness is found to be remarkably well on the benchmarks functions having optima at the origin of the search space. However, its performance deteriorates severely when being applied to functions with optimum resides away from the origin [48]. Moreover, FWA is computationally demanding than other meta-heuristic techniques. Zheng, *et al.* [48] proposed Enhance Fireworks Algorithm (EFWA) to overcome these limitations of FWA. Thereafter several other improved variants [49-5 1] of the algorithm have been proposed by performing different experiments on the selection method and operators of the algorithm.

2.2The objectives and scope of the dissertation

The objectives are

- 1. To model the uncertainty related to renewable sources using the probabilistic method.
- 2. The formulation of stochastic DELD system's objective considering the conventional thermal generator system as well as to consider economic factor of renewable sources with operational and network constraints.
- 3. Implementation of FWA.
- 4. Development of proposed IFWA-CSO to overcome the drawbacks of standard FWA
- 5. The validation and efficacy of proposed techniques on standard benchmark function and to implement on the power system optimization of economic operation with standard ED and DELD system in detailed analysis.
- 6. Detail analysis of renewable sources with conventional system to carried-out the impact of hybrid system on conventional system.

In the next chapter, mathematical modeling of wind speed and solar irradiance are framed. For this suitable probabilistic distribution function is used and their corresponding power potential is obtained using transformation through their output characteristics. The cost model is formed based on proportional power and their unsettledness is considered by overestimation and underestimation terms.

CHAPTER 3 PROBABILISTIC MODELING OF RENEWABLE SOURCES

In this chapter, a probabilistic distribution model has been employed to model the variability of wind speed and solar irradiance. The wind speed has been modeled by using Weibull distribution and solar irradiance through Beta distribution based on analysis. Further, their corresponding power probabilistic distribution functions are derived using their power performance characteristics at each time interval. These functions are further employed for the probabilistic cost calculation under the condition of overestimation and underestimation.

The probabilistic model depends upon the fitting of the data. Therefore, it requires thorough research on the various probability distribution models. Several probabilistic models [52-53] have been employed for the modeling of the renewable sources. The prior research has established that Weibull probability distribution is most frequently used for the modeling of wind speed. Similarly, solar irradiance is modeled by Beta distribution. Thus, here these two distributions are considered for modeling of wind speed and solar irradiance, and their respective transform power variables are investigated through their output performance characteristics of each source.

3.1Probabilistic Characterization of Wind Speed and Power

The two parameters Weibull distribution [54] function is employed to model the wind speed over the time sequence. The probability density function fv, $t(v_t)$ is defined below.

$$f_{v,t}(v_t) = \frac{k_t}{c_t} \left(\frac{v_t}{c_t}\right)^{k_t - 1} \exp\left[-\left(\frac{v_t}{c_t}\right)^{k_t}\right]$$
(3.1)

Where k_t and c_t are the shape and scale parameters of Weibull distribution at time interval *t*. The k_t and c_t are defined as dimensionless unit and unit of speed respectively. The parameters k_t and c_t can be approximately estimated by using different methods [55-56]. The parameters values of k_t and c_t can be evaluated approximately, using the average wind speed $v_{m,t}$ and the standard deviation σ_t at time interval *t* as calculated.

$$k_t = \left(\frac{\sigma_t}{v_{m,t}}\right)^{-1.086} \tag{3.2}$$

$$c_{t} = \frac{v_{m,t}}{\Gamma \left[1 + \left(\frac{1}{k_{t}} \right) \right]}$$
(3.3)

The shape and scale parameters of Weibull distribution at time interval *t* measure the potential of wind speed over time horizon. The Weibull cumulative probability distribution can be expressed as:

$$F(v_t) = 1 - \exp\left[-\left(\frac{v_t}{c_t}\right)^{k_t}\right]$$
(3.4)

The power from wind speed is extracted through electromechanical action. It converts the mechanical energy (wind speed) into electrical energy through electrical generator. The output power of wind generation is associated with the wind speed at the unit hub height. As being the function of wind speed, wind power exist highly non-linear relationship. The calculated output power at height hub is defined as [57].

$$P_w = \frac{1}{2} K \rho A v^3 \tag{3.5}$$

where P_w is the output power of wind generation; K is the constant power coefficient, defined as nonlinear function of tip speed ratio and pitch angle. ρ is air density; A is rotor swept area and v is wind speed respectively. Further, in order to sort a simplified approach, non-linearity of wind power characteristics is dropped and a simplified piecewise linear function is employed. The wind power for given wind speed v_t at time interval t can be stated as [Deshmukh M, Deshmukh S, 2008, 50]:

$$P_{W,t}(v_t) = \begin{cases} 0 & v_t < v_{in}, v_t > v_f \\ P_{Wr} \frac{v_t - v_{in}}{v_r - v_{in}} & v_{in} \le v_t \le v_r \\ P_{Wr} & v_r \le v_t \le v_f \end{cases}$$
(3.6)

where P_{Wr} is the rated electrical output power of wind generation unit, v_{in} is the cut-in wind speed [m/s], below which wind generation unit remains shut off, v_f is the cut-off wind speed [m/s], above this speed unit is shut down to avoid the damage to the rotor by using breaking system and v_r is the rated wind speed [m/s] at this speed wind generator operate at the rated power P_{Wr} . It is shown from the equation (3.6) that the below the cut-in and above the cut-off wind speed, the wind power generation

becomes zero. The power between the cut-in and rated speed varies linearly and remain constant from rated to cut-off speed. Therefore, the output of wind generation unit is mixed random variable, which remains continuous from cut-in to rated and discrete below cut-in and above the cut-off speed.

As Weibull distribution takes into account to models the wind speed with its scaling and shape parameters. Therefore, a distribution model is also required to model wind power. Hence transformation of random variable [58] is employed to transform the wind speed to wind power variable. Since, the wind power function is expressed in discrete according to (3.6). Therefore wind power *pdf* will also exhibit discrete probability. The probability of wind power being zero is i.e.; $P_{w,t}=0$.

$$\Pr(P_{W,t} = 0) = \Pr(v_t < v_{in}) + \Pr(v_t > v_f) = 1 - \exp\left[-\left(\frac{v_{in}}{c_t}\right)^{k_t}\right] + \exp\left[-\left(\frac{v_f}{c_t}\right)^{k_t}\right]$$
(3.7)

And the probability of event $P_{Wt}=P_{Wr}$.

$$\Pr(P_{W,t} = P_{wr}) = \Pr(v_{in} < v_t < v_f) = \exp\left[-\left(\frac{v_r}{c_t}\right)^{k_t}\right] - \exp\left[\left(-\left(\frac{v_f}{c_t}\right)^{k_t}\right)\right]$$
(3.8)

As the wind power function is continuous and linear in the interval $(v_{in} < v < v_f)$. Therefore the probability of wind power being in the interval $(0 < P_{W, t} < P_{Wr})$ is defined as:

$$f_{P_{W,t}}(P_{W,t}) = \left(\frac{k_t h v_{in}}{c_t P_{Wr}}\right) \left[\frac{(1 + h P_{W,t} / P_{Wr}) v_{in}}{c_t}\right]^{k_t - 1} \exp\left\{-\left[\frac{(1 + h P_{W,t} / P_{Wr}) v_{in}}{c_t}\right]^{k_t}\right\}$$
(3.9)

Where $h=(v_r-v_{in})/v_{in}$ and $f_{Pw,t}(P_{W,t})$ is continuous probability function of transform wind power variable. Since, the renewable sources have been investigated in DELD problem. Thus, it has the domain of time to represent time sequence *t*. This same approach is also employed for modeling of solar irradiance and solar power.

3.2Probabilistic Characterization of Solar Irradiance and PV Output Power

The solar irradiance to energy conversion is mostly dependent upon the solar insolation, temperature of solar cell and technical properties of different PV module. However, the geographical location (latitude and altitude) and climate condition (cloud cover) also greatly affect the solar insolation reaching on ground. However, a

simplest approach for PV module output power being dependent upon solar insolation and temperature is considered. It is calculated as follows [59]:

$$T_{c,t} = T_A + S_t \left(\frac{N_{OT} - 20}{0.8} \right)$$
(3.10)

$$I_t = S_t [I_{SC} + K_i (T_C - 25)]$$
(3.11)

$$V_t = V_{OC} - K_v \times T_{c,t} \tag{3.12}$$

$$P_{S,t}(S_t) = N \times FF \times V_t \times I_t \tag{3.13}$$

$$FF = \frac{V_{MPP} \times I_{MPP}}{V_{OC} \times I_{SC}}$$
(3.14)

Where $T_{c,t}$ and S_t are cell temperature °C and solar irradiance at time interval t respectively; T_A is ambient temperature; K_v is voltage temperature coefficient V/°C; K_i is current temperature coefficient A/°C; N_{OT} is nominal operating temperature of cell in °C. The other parameters FF, I_{SC} and V_{OC} are defined as fill factor, short circuit current (A) and open-circuit voltage (V) respectively. The I_{MPP} and V_{MPP} are the current and voltage at the maximum power point respectively. $P_{S,t}(S_t)$ is generated solar output power of the PV module during time interval t.

The solar irradiance exist the partial predictability and discontinuity as zero availability at night. Therefore, a Beta distribution function is utilized to model the solar radiation defined as [17].

$$f_{S,t}(S_t) = \begin{cases} \frac{\Gamma(a_t + b_t)}{\Gamma(a_t)\Gamma(b_t)} \left(\frac{S_t}{S_{\max,t}}\right)^{a_t - 1} \left(1 - \frac{S_t}{S_{\max,t}}\right)^{b_t - 1} & 0 \le \left(\frac{S_t}{S_{\max,t}}\right) \le 1, a_t > 0, b_t > 0 \\ 0 & else \end{cases}$$
(3.15)

Where $f_{S,t}(S_t)$ is Beta probability distribution function and S_t is solar irradiance in kW/m². a_t and b_t are two shape parameters of the Beta distribution function at time interval *t*. The parameters of Beta distribution can be evaluated by using the mean (μ_t) and standard deviation (σ_t) as follows:

$$a_t = \mu_t^2 \left(\frac{1 - \mu_t}{\sigma^2} - \frac{1}{\mu_t} \right)$$
(3.16)

$$b_t = a_t \left(\frac{1}{\mu_t} - 1\right) \tag{3.17}$$

It is noted that Beta distribution variable lie in the range of (0, 1). Therefore, a nominal value of solar irradiance is (S_t/S_{max}) considered. Similarly, solar power

variable is also normalized. The transforms solar power variable also follows Beta distribution. It is modeled into solar power variable as [18]:

$$f_{PV,t}(P_{PV,t}) = \begin{cases} \frac{1}{P_{PV}^{\max}} \frac{\Gamma(a_t + b_t)}{\Gamma(a_t)\Gamma(b_t)} \left(\frac{P_{PV,t}}{P_{PV,t}^{\max}}\right)^{a_t - 1} \left(1 - \frac{P_{PV,t}}{P_{PV,t}^{\max}}\right)^{b_t - 1} & 0 \le \left(\frac{P_{PV,t}}{P_{PV,t}^{\max}}\right) \le 1, a_t > 0, b_t > 0 \\ 0 & else \end{cases}$$
(3.18)

Where $P^{max}_{PV,t}$ is the maximum generated solar power at time interval *t*. $f_{PV,t}(P_{PV})$ is transform solar power probability distribution function. $\Gamma(\bullet)$ is gamma function.

3.3Probabilistic Cost Model for Wind and Solar Power

Wind and solar based power generation are stochastic in nature. Thus, the scheduling of these sources may differ from their actual available generation. This mismatch of power can result in overestimation or underestimation conditions. Generally, the overestimation and underestimation terms related to the condition of under generation and over generation of power generation. The overestimation condition occurs when available power generation is less than forecasted power. Therefore, the operator looks for alternative source to purchases the power from it. Similarly, the underestimation situation incur in the operation when available power is more than the forecasted power, thus surplus power get loss and paid in the form penalty cost to the farm operator. Therefore, to consider the cost model of renewable sources a probabilistic cost model is suggested as investigated in [22], which calculate the expected cost using probabilistic distribution function under the condition of over generation and under generation. The procedural approaches for both sources are equivalent as described below.

3.3.1 Wind Power Cost Model

The availability of wind power from wind turbine is un-deterministic and random in nature. As the wind speed predictability is uncertain at any instant of time. Therefore, the operator continuously faces the situation of underestimation and overestimation of wind power availability, which impacts the scheduling of wind power. Therefore, in wind power generation, the overestimation defined as when generated wind power is below than the scheduled power. On conversely, when available power is more than the scheduled power it is termed as underestimation. The overestimation and underestimation cost terms are expressed in similar manner as in [22]: $k_{r,Wj}(P_{Wj,t} - P_{Wj,av,t})$ and $k_{p,Wj}(P_{Wj,av,t} - P_{Wj,})$, where $k_{p,Wj}$ is penalty (underestimation) coefficient for additional wasted wind power from wind farm and $k_{r,Wj}$ is reserve (overestimation) coefficient cost for purchasing power from another source on unavailability of power from wind farm. In order to simplicity of integral presentation, the subscript "*j*, *t*" is dropped in $P_{Wj,t}$ and $P_{Wj,av,t}$. The overestimation analysis of wind power is as follows. Assuming forecasted and actual available wind power of wind farm are $P_{Wl}(0 \le P_{Wl} < P_{Wr})$ and $P_{W,av}$ respectively. The average value of integral can be expressed as in [60]:

$$E(P_{W,o}) = s_1 + s_2 \tag{3.19}$$

Where

$$s_1 = P_{W1} \Pr(P_{W,av} = 0)$$
 (3.20)

$$s_{1} = P_{W1} \left\{ 1 - \exp\left[-\left(\frac{v_{in}}{c}\right)^{k} \right] + \exp\left[-\left(\frac{v_{f}}{c}\right)^{k} \right] \right\}$$
(3.21)

$$s_{2} = \int_{0}^{P_{W1}} (P_{W1} - P_{W}) f_{P_{W}}(P_{W}) dP_{W}$$

= $\int_{0}^{P_{W1}} P_{W1} f_{P_{W}}(P_{W}) dP_{W} - \int_{0}^{P_{W1}} P_{W} f_{P_{W}}(P_{W}) dP_{W}$
= $H_{1} - H_{2}$ (3.22)

Where $f(P_W)$ is probability distribution function of wind power. The parameters *c* and *k* are scale and shape parameter of Weibull *pdf* and $h=(v_r/v_{in}-1)$.

$$f_{P_{w}}(P_{W}) = \left(\frac{khv_{in}}{P_{Wr}c}\right) \left[\frac{(1+hP_{W}/P_{Wr})v_{in}}{c}\right]^{k-1} \exp\left\{-\left[\frac{(1+hP_{W}/P_{Wr})v_{in}}{c}\right]^{k}\right\}$$
(3.23)

To evaluate the H_1 and H_2 , using variable substitution $z = [(1 + hP_W / P_{W_r})v_{in} / c]^k$.

$$H_{1} = \int_{0}^{P_{W1}} P_{W1} \left(\frac{khv_{in}}{P_{Wr}c} \right) \left[\frac{(1+hP_{W}/P_{Wr})v_{in}}{c} \right]^{k-1} \exp\left\{ -\left[\frac{(1+hP_{W}/P_{Wr})v_{in}}{c} \right]^{k} \right\} dP_{W}$$
(3.24)

The term H_1 converted into (3.25) with modified integration limit c_{ll} and c_{ul} . Here c_{ll} and c_{ul} are assigned as lower and upper integration limit for reader convenience. These are $c_{ll}=\exp\{-[v_{in}/c]^k\}$ and $c_{ul}=\exp\{-[(1+hP_{W1}/P_{Wr})v_{in}/c]^k\}$.

$$H_{1} = P_{W1} \int_{c_{u}}^{c_{u}} \exp(-z) dz$$
 (3.25)

Now, it can be easily integrated as:

$$H_{1} = P_{W1} \exp\left\{-\left[\frac{v_{in}}{c}\right]^{k}\right\} - P_{W1} \exp\left\{-\left[\frac{(1+hP_{W1}/P_{Wr})v_{in}}{c}\right]^{k}\right\}$$
(3.26)

The term H_2 is processed through integrating by parts

$$H_{2} = \int_{0}^{P_{W1}} P_{W} \left(\frac{khv_{in}}{P_{Wr}c} \right) \left[\frac{(1+hP_{W} / P_{Wr})v_{in}}{c} \right]^{k-1} \exp\left\{ - \left[\frac{(1+hP_{W} / P_{Wr})v_{in}}{c} \right]^{k} \right\} dP_{W}$$
(3.27)

$$H_{2} = P_{W} \int_{0}^{P_{W1}} f_{P_{W}}(P_{W}) dP_{W} - \int_{0}^{P_{W1}} 1 \int_{0}^{P_{W1}} f_{P_{W}}(P_{W}) dP_{W} dP_{W}$$
(3.28)

$$H_{2} = P_{W} \exp(-z) \Big|_{c_{ul}}^{c_{ul}} - \frac{P_{Wr}c}{khv_{in}} \int_{c_{ul}}^{c_{ul}} \exp(-z) z^{\binom{1-k}{k}} dz$$
(3.29)

Finally, we have

$$H_{2} = -P_{W1} \exp\left\{-\left[\frac{(1+hP_{W1}/P_{Wr})v_{in}}{c}\right]^{k}\right\} - \frac{P_{Wr}c}{khv_{in}}\Gamma\left(\frac{1}{k}\right)\left[\Gamma\left(c_{ul},\frac{1}{k}\right) - \Gamma\left(c_{ll},\frac{1}{k}\right)\right] \quad (3.30)$$

Here $\Gamma(y)$ and $\Gamma(\alpha, x)$ are gamma and incomplete gamma function. The definition of gamma and incomplete gamma function are stated as follows [61]:

$$\Gamma(\mathbf{y}) = \int_0^\infty t^{y-1} \exp(-t) dt \tag{3.31}$$

$$\Gamma(\alpha, x) = \int_{x}^{\infty} t^{\alpha - 1} \exp(-t) dt$$
(3.32)

It is noted that many computational software packages like Matlab supports the mathematical functions. Thus it can be easily evaluated. Putting H_1 and H_2 term together in (3.22) becomes (3.33).

$$s_{2} = P_{W1} \exp\left\{-\left[\frac{v_{in}}{c}\right]^{k}\right\} - \frac{P_{Wr}c}{khv_{in}}\Gamma\left(\frac{1}{k}\right)\left[\Gamma\left(\left[\frac{(1+hP_{W1}/P_{Wr})v_{in}}{c}\right]^{k}, \frac{1}{k}\right) - \Gamma\left(\left[\frac{v_{in}}{c}\right]^{k}, \frac{1}{k}\right)\right] (3.33)$$

Now, putting the s_1 and s_2 terms in (3.19) gives. It is average scale of overestimation term. Finally, the average cost of overestimation can be obtained by multiplying the reserves cost coefficient with expected value of overestimation. It is expressed as follows: $k_{r,W}E(P_{W,o})$.

$$E(P_{W,o}) = P_{W1}\left\{1 - \exp\left[-\left(\frac{v_{in}}{c}\right)^{k}\right] + \exp\left[-\left(\frac{v_{f}}{c}\right)^{k}\right]\right\} + P_{W1}\exp\left\{-\left[\frac{v_{in}}{c}\right]^{k}\right\} - \frac{P_{Wr}c}{khv_{in}}\Gamma\left(\frac{1}{k}\right)\left[\Gamma\left(\left[\frac{(1+hP_{W1}/P_{Wr})v_{in}}{c}\right]^{k},\frac{1}{k}\right) - \Gamma\left(\left[\frac{v_{in}}{c}\right]^{k},\frac{1}{k}\right)\right]$$
(3.34)

The analysis of underestimation term can be carried out in the similar manner as for overestimation term. The expected value of underestimation term is stated as follows:

$$E(P_{W,u}) = s_3 + s_4 \tag{3.35}$$

Where

$$s_3 = (P_{Wr} - P_{W1}) \Pr(P_{W,av} = P_{Wr})$$
(3.36)

$$s_{3} = P_{W1} \left\{ \exp\left[-\left(\frac{v_{r}}{c}\right)^{k} \right] - \exp\left[-\left(\frac{v_{f}}{c}\right)^{k} \right] \right\}$$
(3.37)

$$s_{4} = \int_{P_{W^{1}}}^{P_{W^{r}}} (P_{W} - P_{W^{1}}) f_{P_{w}}(P_{W}) dP_{W}$$

$$= \int_{P_{W^{1}}}^{P_{W^{r}}} P_{W} f_{P_{w}}(P_{W}) dP_{W} - \int_{P_{W^{1}}}^{P_{W^{r}}} P_{W^{1}} f_{P_{W}}(P_{W}) dP_{W}$$

$$= G_{1} - G_{2}$$

(3.38)

Solving the terms G_1 and G_2 , using variable substitution $y = [(1+hP_W/P_{Wr})v_{in}/c]^k$.

$$G_{1} = \int_{P_{W1}}^{P_{Wr}} P_{W}\left(\frac{khv_{in}}{P_{Wr}c}\right) \left[\frac{(1+hP_{W}/P_{Wr})v_{in}}{c}\right]^{k-1} \exp\left\{-\left[\frac{(1+hP_{W}/P_{Wr})v_{in}}{c}\right]^{k}\right\} dP_{W}$$
(3.39)

The term G_1 is assessed as similar to H_2 . The modified lower and upper integration limits are $d_{ll} = \exp\{-[(1+hP_{W1}/P_{Wr})v_{in}/c]^k\}$ and $d_{ul} = \exp\{-[(1+h)v_{in}/c]^k\}$ respectively.

$$G_{1} = P_{W} \int_{P_{W1}}^{P_{W1}} f_{P_{W}}(P_{W}) dP_{W} - \int_{P_{W1}}^{P_{W1}} 1 \int_{P_{W1}}^{P_{W1}} f_{P_{W}}(P_{W}) dP_{W} dP_{W}$$
(3.40)

$$G_{1} = -P_{W} \exp(-y) \Big|_{d_{II}}^{d_{uI}} - \frac{P_{Wr}c}{khv_{in}} \int_{d_{II}}^{d_{uI}} \exp(-y) y^{(1-k/k)} dy$$
(3.41)

$$G_{1} = -P_{Wr} \exp\left\{-\left[\frac{(1+h)v_{in}}{c}\right]^{k}\right\} + P_{W1} \exp\left\{-\left[\frac{(1+hP_{W1}/P_{Wr})v_{in}}{c}\right]^{k}\right\} - \frac{P_{Wr}c}{khv_{in}}\Gamma\left(\frac{1}{k}\right)\left[\Gamma\left(d_{ul},\frac{1}{k}\right) - \Gamma\left(d_{ll},\frac{1}{k}\right)\right]$$
(3.42)

The term G_2 is evaluated as similar to H_1 . Now putting the G_1 and G_2 together in (3.38) gives s_4 .

$$G_{2} = \int_{P_{W1}}^{P_{Wr}} P_{W1}\left(\frac{khv_{in}}{P_{Wr}c}\right) \left[\frac{(1+hP_{W}/P_{Wr})v_{in}}{c}\right]^{k-1} \exp\left\{-\left[\frac{(1+hP_{W}/P_{Wr})v_{in}}{c}\right]^{k}\right\} dP_{W}$$
(3.43)

$$G_2 = P_{W1} \int_{d_U}^{d_{wl}} \exp(-y) dy$$
 (3.44)

$$G_{2} = -P_{W1} \exp\left\{-\left[\frac{(1+h)v_{in}}{c}\right]^{k}\right\} + P_{W1} \exp\left\{-\left[\frac{(1+hP_{W1}/P_{Wr})v_{in}}{c}\right]^{k}\right\}$$
(3.45)

$$s_{4} = (P_{W1} - P_{Wr}) \exp\left\{-\left[\frac{(1+h)v_{in}}{c}\right]^{k}\right\} - \frac{P_{Wr}c}{khv_{in}}\Gamma\left(\frac{1}{k}\right) \begin{bmatrix}\Gamma\left(\left[\frac{(1+h)v_{in}}{c}\right]^{k}, \frac{1}{k}\right)\\-\Gamma\left(\left[\frac{(1+h)V_{in}}{c}\right]^{k}, \frac{1}{k}\right)\end{bmatrix} (3.46)$$

Finally, the average value of underestimation is stated in (3.47). The average cost of overestimation can be calculated by multiplying the penalty cost coefficient with average value of underestimation. It is defined as follows: kp, $_{WE}(P_{W,u})$.

$$E(P_{W,u}) = P_{W1}\left\{\exp\left[-\left(\frac{v_r}{c}\right)^k\right] - \exp\left[-\left(\frac{v_f}{c}\right)^k\right]\right\} + (P_{W1} - P_{Wr})\exp\left\{-\left[\frac{(1+h)v_{in}}{c}\right]^k\right\} + \frac{P_{Wr}c}{khv_{in}}\Gamma\left(\frac{1}{k}\right)\left[\Gamma\left(\left[\frac{(1+h)v_{in}}{c}\right]^k, \frac{1}{k}\right) - \Gamma\left(\left[\frac{(1+h)P_{W1}}{c}, \frac{P_{Wr}}{c}\right]^k, \frac{1}{k}\right)\right]$$
(3.47)

3.3.2 Solar Cost Model with Incomplete Beta function

The solar module output depends upon the solar irradiance intensity and temperature. Hence, the power output may be varying at any instant of time. Thus, the operator required to evaluate the underestimate and overestimate of solar power availability similar to wind power availability as in [22]. This access the impact of solar partial predictability on scheduling cost of solar power. The overestimation and underestimation cost terms for solar can be expressed as follows: $k_{r,PVk}(P_{PVk,t} - P_{PV,av,t})$ and $k_{p,PVk}(P_{PV,av,t} - P_{PVk})$, where $k_{p,PVk}$ is penalty (underestimation) for excess wasted solar power from solar farm and $k_{r,PVk}$ is reserve (overestimation) cost for purchasing power from another source on deficit power from solar farm.

The overestimation analysis of solar power as follows. The expected value $E(\bullet)$ of overestimation can be terms expressed as in (3.48).

$$E(P_{PV,0}) = \int_{0}^{P_{PVk,t}} (P_{PVk,t} - P_{PV}) f_{PV,t}(P_{PV}) dP_{PV}$$
(3.48)

$$E(P_{P_{V,0}}) = \int_{0}^{P_{P_{V,k}}} P_{P_{V,k}} f_{P_{V,t}}(P_{P_{V}}) dP_{P_{V}} - \int_{0}^{P_{P_{V,k}}} P_{P_{V}} f_{P_{V,t}}(P_{P_{V}}) dP_{P_{V}}$$
(3.49)

$$f_{PV,t}(P_{PV}) = \frac{1}{P_{PV}^{\max}} \frac{1}{B(a,b)} \left(\frac{P_{PV}}{P_{PV,t}^{\max}}\right)^{a_t - 1} \left(1 - \frac{P_{PV}}{P_{PV,t}^{\max}}\right)^{b_t - 1}$$
(3.50)

Where $f_{PV,t}(P_{PV})$ is two parametric a_t and b_t beta distribution function. B(a, b) is beta function. Substituting (3.50) in (3.49) this becomes.

$$E(P_{P_{V,0}}) = \int_{0}^{P_{P_{V,t}}} P_{P_{V,t}} \frac{1}{P_{P_{V}}^{\max}} \frac{1}{B(a,b)} \left(\frac{P_{P_{V}}}{P_{P_{V,t}}^{\max}}\right)^{a_{t}-1} \left(1 - \frac{P_{P_{V}}}{P_{P_{V,t}}^{\max}}\right)^{b_{t}-1} dP_{P_{V}} - \int_{0}^{P_{P_{V,t}}} P_{P_{V}} \frac{1}{P_{P_{V}}^{\max}} \frac{1}{B(a,b)} \left(\frac{P_{P_{V}}}{P_{P_{V,t}}^{\max}}\right)^{a_{t}-1} \left(1 - \frac{P_{P_{V}}}{P_{P_{V,t}}^{\max}}\right)^{b_{t}-1} dP_{P_{V}}$$
(3.51)

The above mentioned (3.51) is expanded expression of (3.49). In order to simplify the integral substituting $x=P_{PV}/P^{max}_{PV}$ and $pv_1=P_{PVk,t}/P^{max}_{PV}$. This converted into following form.

$$E(P_{PV,0}) = \int_{0}^{PV_{1}} P_{PVk,t} \frac{1}{B(a_{t},b_{t})} x^{a_{t}-1} (1-x)^{b_{t}-1} dx - P_{PV}^{\max} \int_{0}^{PV_{2}} \frac{1}{B(a_{t},b_{t})} x^{a_{t}} (1-x)^{b_{t}-1} dx \quad (3.52)$$

According to the definition of Regularized Beta Function (RBF) [62].

$$I_{y}(a,b) = \frac{B_{y}(a,b)}{B(a,b)} = \frac{1}{B(a,b)} \int_{0}^{y} y^{a-1} (1-y)^{b-1} dt \qquad 0 \le y \le 1, a > 0, b > 0 \qquad (3.53)$$

Where $I_y(a, b)$ is regularized beta function. $B_y(a, b)$ and B(a, b) are incomplete and complete Beta functions.

In order to convert (3.52) in standard regularized beta function we finally have:

$$E(P_{PV,0}) = \int_{0}^{PV_{1}} P_{PVk,t} \frac{1}{B(a_{t},b_{t})} x^{a_{t}-1} (1-x)^{b_{t}-1} dx - P_{PV}^{\max} \cdot \frac{a}{a+b} \int_{0}^{PV_{2}} \frac{1}{B(a_{t}+1,b_{t})} x^{a_{t}} (1-x)^{b_{t}-1} dx \quad (3.54)$$

As many computational platforms supports RBF like Matlab. It can be expressed as (3.54). Here "lower" signifies the integration of function 0 to pv_1 . It represents the average scale of overestimation term. Thus the average cost of overestimation can be obtained by multiplication of reserve cost coefficient $k_{r,PV}$ i.e., $k_{r,PV} E(P_{PV,o})$.

$$E(P_{PV,o}) = P_{PVk}I_{PV_1}(PV_1, a_t, b_t, 'lower') - P_{PV}^{\max} \frac{a}{a+b}I_{PV_1}(PV_1, a_t+1, b_t, 'lower')$$
(3.55)

In the similar way analysis of underestimation can be done. The average value of underestimation term is expressed as in (3.56).

$$E(P_{PV,u}) = \int_{P_{PVk,t}}^{P_{PVk}} (P_{PV} - P_{PVk}) f_{PV,t}(P_{PV}) dP_{PV}$$
(3.56)

$$E(P_{P_{V,u}}) = \int_{P_{PVk,t}}^{P_{PVk,t}} P_{PV} f_{PV,t}(P_{PV}) dP_{PV} - \int_{P_{PVk,t}}^{P_{PVk,t}} P_{PVk,t} f_{PV,t}(P_{PV}) dP_{PV}$$
(3.57)

$$E(P_{P_{V,u}}) = \int_{P_{PVk,t}}^{P_{PVk,t}} P_{PV} \frac{1}{P_{PV}^{\max}} \frac{1}{B(a,b)} \left(\frac{P_{PV}}{P_{PV,t}^{\max}}\right)^{a_{t}-1} \left(1 - \frac{P_{PV}}{P_{PV,t}^{\max}}\right)^{b_{t}-1} dP_{PV} - \int_{P_{PVk,t}}^{P_{PVk,t}} P_{PVk,t} \frac{1}{P_{PV}^{\max}} \frac{1}{B(a,b)} \left(\frac{P_{PV}}{P_{PV,t}^{\max}}\right)^{a_{t}-1} \left(1 - \frac{P_{PV}}{P_{PV,t}^{\max}}\right)^{b_{t}-1} dP_{PV}$$
(3.58)

The simpler expression of integral can be derived by substituting $x = P_{PV}/P_{PV}^{\text{max}}$ and $pv_2 = P_{PVI,t}/P_{PV}^{\text{max}}$ in (3.58). It becomes.

$$E(P_{PV,u}) = P_{PV}^{\max} \int_{PV_2}^{1} \frac{1}{B(a_t, b_t)} x^{a_t} (1-x)^{b_t - 1} dx - \int_{PV_2}^{1} \frac{1}{B(a_t, b_t)} x^{a_t - 1} (1-x)^{b_t - 1} dx \qquad (3.59)$$

This is modified to turn into the standard RBF. So we have

$$E(P_{PV,u}) = P_{PV}^{\max} \cdot \frac{a}{a+b} \int_{PV_2}^{1} \frac{1}{B(a_t+1,b_t)} x^{a_t} (1-x)^{b_t-1} dx - \int_{PV_2}^{1} \frac{1}{B(a_t,b_t)} x^{a_t-1} (1-x)^{b_t-1} dx \quad (3.60)$$

$$E(P_{PV,u}) = P_{PV}^{\max} \cdot \frac{a}{a+b} I_{PV_2}(pV_2, a_t+1, b_t, 'upper') - P_{PVk} I_{PV_2}(pV_1, a_t, b_t, 'upper')$$
(3.61)

Here "upper" represents the function integration from pv_2 to 1 and (3.61) is average value of underestimation term. So average cost of underestimation term is stated as $k_{p,PV} E(P_{PV,u})$.

These following cost models of wind and solar power are incorporated in the objective formulation of stochastic DELD system with renewable sources. The detail mathematical modeling is presented in the next chapter.

CHAPTER 4 PROPOSED PROBLEM FORMULATION

This chapter presents a mathematical modeling of stochastic DELD problem with renewable sources using the various practical constraints such as valve-point loading effect, ramp rate limits and POZs etc. The objective cost function is consists of thermal generators, wind and solar powered generator. The cost functions of wind and solar involves proportional cost, in addition underestimation and overestimation effects are considered by suggesting a probabilistic cost model.

In practical mathematical modeling of ED operation, the generator exhibit various constraints such as valve-point loading effect due to multi-fuel valves operation, ramp rate limits and POZs arises due to physical and stability limitations. A detail description about these constraints is discussed below.

In ELD operation, the fuel cost function of thermal generator is usually expressed in quadratic form. However, generator operating with the large steam turbine has had the multi-fuel opening valves. It is used to operate sequentially to match the increased generation. This opening of valves results in the sudden heat rate rise and causes throttling losses rapidly [38].



Figure 4.1 Fuel cost function with and without valve-point loading effect.

Thus, the valve point-loading effect introduces ripples that lead to non-convexity and dis-continuity. Thus, a sinusoidal function is used to model to exhibit of the steam injection to steam turbine through multi-valve of steam turbines. Therefore, a fuel cost function in quadratic form is modified with an additional term of sinusoidal function to account the valve-point loading effect as depicted in Fig. 4.1.



Figure 4.2 Different generation states due to up and down ramp rate limits

It is general assumption in static ELD that, the generators schedule is adjusted instantly. However, under real time operation, the generators' ramp rate limit contains the operating range of the generator between the two consecutive operating periods during the transitions states [63-64] shown in Fig. 4.2. It is due to generator physical and stability constraints thus, the generation limits of thermal generators keep on changing with respect to their ramp rate limits on successive schedule. This modifies the generation limits at each state operation. Though, in SELD, it is usually not taken into context, where generators are schedule for one operating period without knowledge of prior state. Therefore, ramp rate limits is preferably incorporated in DELD operation, where generators are schedule for various continuous periods.

The POZs are those spans of the entire operating range of generator where operation is obstructed on the circumstance of the physical limitation due to steam valve or vibration in shaft bearing of the generators. It may be due to faults in the machines or associated auxiliaries, which leads to instability region in the generator operating ranges [65]. Thus, overall this results in intermittent operation of generator over its entire operating range. Therefore, these zones must be avoided for economic production [66].


Figure 4.3 Fuel cost function with prohibited operating zones.

Thus, it can be seen from the Fig. 4.3, POZs lead to discontinuity in the fuel cost function of the generator. Hence, in order to account these all practical constraints, it is adequate to consider the ELD problem into DELD form instead of SELD. As mentioned in the previous chapter that hybrid generation taking active part into modern power system. Therefore, wind and solar powered source also are incorporated in DELD system. Further, the impacts of these sources are analyzed by considering their operating cost as well.

4.1 Mathematical Modeling of Stochastic DELD with Renewable Sources

DELD is major optimization operation in power system. It aims to obtain the optimal schedule of generators with minimal total operating cost for pre-scheduled load demand in account of satisfying the multi-constraints of the generator and network over the time horizon. Since, here renewable sources are also incorporated in the system. Thus, the total operating cost comprises of the operating fuel cost of thermal generator, the operating cost of wind generator and the operating cost of solar powered generating system. The formulated operating cost for the Stochastic DELD system is modeled as [24].

Minimize
$$F = \sum_{t=1}^{H} \left(\sum_{i=1}^{N_G} F(P_{Gi,t}) + \sum_{j=1}^{M_W} F(P_{Wj,t}) + \sum_{k=1}^{L_{PV}} F(P_{PVk,t}) \right)$$
 (4.1)

Where, $F(P_{Gi,t})$ is operating fuel cost of the thermal units, $F(P_{Wj,t})$ is operating cost of wind farm and $F(P_{PVk,t})$ is operating cost of the solar farm. N_G , M_W and L_{PV} stand for numbers of conventional thermal generator, wind powered generator and solar powered generators respectively. *H* denotes total number of time interval.

The first term of the objective function formulates thermal generator fuel cost charge. It is modeled by non-convex function consist of quadratic and sinusoidal function to consider the valve-point loading effect [38].

$$F(P_{G_{i,t}}) = \sum_{i=1}^{N_G} (a_i + b_i P_{G_{i,t}} + c_i P_{G_{i,t}}^2) + |e_i \sin(f_i (P_{G_i}^{\min} - P_{G_{i,t}}))|$$
(4.2)

Where a_i , b_i , c_i are the fuel cost coefficients of the *i*th generator, and e_i and f_i are of the valve-point loading coefficients, $P_{Gi,t}$ is the active power of the *i*th generator at time interval *t*.

The second term accounts the operating cost of wind powered generator. This term is consists of three parts [24].

$$F(P_{Wj,t}) = \sum_{j=1}^{M_{W}} F_{Wj}(P_{Wj,t}) + \sum_{j=1}^{M_{W}} F_{P,Wj,t}(P_{Wj,av,t} - P_{Wj,t}) + \sum_{j=1}^{M_{W}} F_{r,Wj,t}(P_{Wj,t} - P_{Wj,av,t})$$
(4.3)

In this, the first part is proportional term defined as direct cost. This cost derived from wind-powered generator. However, the existence of this part depends upon who possess the wind farm. If the system operator owns the wind farm, then this part will not exist because wind power does not require any fuel. However, if the system operator wants to revenue for initial layout of wind energy conversion system (WECS), then there will be cost involved. Furthermore, for non-utility WECS the cost will be based on the special contractual agreements. Therefore, wind generation participation for supplying the power requires to sets the upper and lower limits for the optimal operation [67]. As, in the recent years, the wind power generation has emerged as substantial energy source, therefore, in general sense it is factual to consider the wind cost proportional to scheduled wind power, regardless who owns the generation facilities. If the operator is paying to wind farm owner for power, then direct cost will be involved

$$F(P_{W_{j,t}}) = F_{W_{j,t}} P_{W_{j,t}}$$
(4.4)

Where $F(P_{Wj,t})$ is the direct cost function for j^{th} wind-powered generator at time interval *t*. F_{Wj} is direct cost coefficient for j^{th} wind-powered generator and $P_{Wj,t}$ is scheduled wind power variable. This is cost for purchasing the scheduled power from wind farm owner. Thorough research reveals that the wind generation operational cost is around 57% of the total thermal cost [68].

The second part is associated with the penalty cost for not utilizing all the available wind power. However, the existence of this term also depend upon the, who possess the generation facilities. If, the wind-powered generator owns by system operator, then the penalty cost function may not be exist. Otherwise, it will be related by a linear difference between available (actual) and scheduled wind power respectively. It is modeled as

$$F_{P,Wj,t}(P_{Wj,av,t} - P_{Wj,t}) = k_{P,Wj}(P_{Wj,av,t} - P_{Wj,t})$$

= $k_{P,Wj} \int_{W_{i,t}}^{P_{Wrj}} (P_W - P_{Wj,t}) f_{P_{w,t}}(P_W) dP_W$ (4.5)

Where $F_{P,Wj,t}$ is penalty cost function at time interval *t*. $k_{P,Wj}$ is the penalty cost coefficient of under estimation, $P_{Wj,av,t}$ is the available (actual) wind power from the *j*th wind generator and $f_{Pw,t}(P_W)$ is probability distribution function of wind generated power at time interval *t*. The third part includes reserve requirement cost. It models the overestimation term, when the available wind power is in deficient amount than the scheduled wind power.

$$F_{r,Wj,t}(P_{Wj,t} - P_{Wj,av,t}) = k_{r,Wj}(P_{Wj,t} - P_{Wj,av,t})$$

= $k_{r,Wj} \int_{0}^{P_{Wj,t}} (P_{Wj,t} - P_{W}) f_{P_{w,t}}(P_{W}) dP_{W}$ (4.6)

Where $F_{r,Wj,t}$ is the reserve cost function at time sequence *t*. $k_{r,Wj}$ is the reserve cost coefficient of overestimation and $P_{Wj,av,t}$ is the available wind power from the *j*th wind generator at time interval *t*. The second and third parts are employed to consider the changeability of wind power. It measures wind power variability in terms of underestimation and overestimation. The underestimation is defined as when available power is more than scheduled wind power. Similarly, the overestimation, when available wind power is less than scheduled wind power.

The third term of the objective function is operating cost of solar powered PV modules. This term includes three parts [24]. The solar cost function is modeled

similar to wind cost function. The total cost of solar power can be expressed as follows:

$$F(P_{Wj,t}) = \sum_{k=1}^{L_{PV}} F_{PVk}(P_{PVk,t}) + \sum_{k=1}^{L_{PV}} F_{P,PVk,t}(P_{PVk,av,t} - P_{PVk,t}) + \sum_{k=1}^{L_{PV}} F_{r,PVk,t}(P_{PVk,t} - P_{PVk,av,t})$$
(4.7)

The first part is direct cost from solar-generated power. It accounts the cost proportional to scheduled solar-generated power.

$$F(P_{PVk,t}) = F_{PVk} P_{PVk,t}$$

$$(4.8)$$

Where $F(P_{PVk,t})$ is direct cost function for k^{th} solar-generated power at time interval *t*. It is payment made for using the power from the PV system operator. F_{PVk} and $P_{PVk,t}$ are solar power cost coefficient and schedule solar power variable respectively. The analysis is carried out in same way as for wind cost function.

The second part is penalty cost function associated with for not using all the available power from PV systems. It is a linear difference between available solar power and scheduled solar power respectively. It is expressed as:

$$F_{P,PVk,t}(P_{PVk,av,t} - P_{PVk,t}) = k_{P,PVk}(P_{PVk,av,t} - P_{PVk,t})$$

= $k_{P,PVk} \int_{P_{PVk,t}}^{P_{PVk}} (P_{PV} - P_{PVk}) f_{PV,t}(P_{PV}) dP_{PV}$ (4.9)

Where $F_{P,PVk,t}$ is penalty cost function at time interval *t*. $k_{P,PVk}$ is the penalty cost coefficient of under estimation (over generation), $P_{PVk,av,t}$ is the available solar power from the *k*th PV-powered generator and $f_{PV,t}(P_{PV})$ is probability distribution function of solar power at time interval *t*. The third part stimulates the reserve cost when the available solar power is less than the scheduled solar power.

$$F_{r,PVk,t}(P_{PVk,t} - P_{PVk,av,t}) = k_{r,PVk}(P_{PVk,t} - P_{PVk,av,t})$$

= $k_{r,PVk} \int_{0}^{P_{PVk,t}} (P_{PVk,t} - P_{PV}) f_{PV,t}(P_{PV}) dP_{PV}$ (4.10)

Where $K_{r,PVk}$ is the reserve cost coefficient of overestimation. $F_{r,PVk,t}$ is reserve requirement cost function. It accounts the overestimation (under generation) factor that available power is less than scheduled solar power. Thus, the operator has to manage power from the other source.

The various constraints subjected to stochastic DELD problem are:

4.1 Power Balance Constraint

The total power generation from all the power sources must satisfy the total power demand and network power loss under balance condition. Further, the network power loss can be evaluated by employing the B-coefficient loss formula [69]. The power balance equation can be defined as

$$\sum_{i=1}^{N_G} P_{Gi,t} + \sum_{j=1}^{M_W} P_{Wj,t} + \sum_{k=1}^{L_{PV}} P_{PVk,t} = P_{load,t} + \sum_{i=1}^{N_G} \sum_{l=1}^{N_G} P_{Gi,t} B_{jl,t} P_{Gl,t} + \sum_{i=1}^{N_G} P_{i,t} B_{i0,t} + B_{00,t}$$
(4.11)

Where $P_{load,t}$ is power demand at time interval *t*. $B_{jl,t}$ is the transmission loss coefficient matrix $i=1, 2, ..., N_G$, $l=1, 2, ..., N_G$, and t=1,2,...,H. $B_{i0,t}$ is the *i*th element of the loss coefficient vector. $B_{00,t}$ is the constant coefficient. It is noted that the problem is formulated for DELD system. Thus, the power balance equation must be satisfied for each interval load deamnd.

4.2Ramp Rate Limits

In DELD problems, the generator output of thermal unit restricts its smooth and instantaneous operation due to its ramp rate limits [63]. This changes generation limits at each state operation. Hence, the output of the *i*th thermal unit at time *t* affects the next state t+1. Therefore, to account the ramp rate limits the generator value transition can be expressed as follows:

If generation increases,
$$P_{Gi,t} - P_{Gi,t-1} \le UR_i$$
 $i = 1, ..., N_G; t = 1, ..., H$ (4.12)

If generation decreases, $P_{G_{i,t-1}} - P_{G_{i,t}} \le DR_i \ i = 1, ..., N_G; t = 1, ..., H$ (4.13)

Where $P_{G_{i,t}}$ and $P_{G_{i,t-1}}$ are active output power at *t* and *t*-1 states. UR_i and DR_i stand for the up and down ramp limit of the *i*th generator (MW/time-period).

4.3Power Inequality Constraint

The power limits of each generator should be within its boundary limits for feasible and stable operation. Therefore, the thermal units limits between its upper and lower limits can be expressed as:

$$P_{Gi}^{\min} \le P_{Gi,t} \le P_{Gi}^{\max} \quad i = 1, \dots, N_G; t = 1, \dots, H.$$
(4.14)

However, the thermal generator subjected to ramp rate limits due to physical and stability concern. Thus, for each time state thermal generator minimum and maximum

power limits should be revised. Therefore adjusted inequality constraints for each thermal generator in each interval can be expressed as:

$$\max(P_{Gi}^{\min}, P_{Gi,t-1} - DR_i) \le P_{Gi,t} \le \min(P_{Gi}^{\max}, P_{Gi,t-1} + UR_i) \ i = 1, \dots, N_G; t = 1, \dots, H.$$
(4.15)

The generation limits of the renewable sources must be constrained to defined upper and lower limits. Since, it is based on the system operator agreement for optimal operation of power system [70]. The generation limits of renewable sources are kept independent of ramp rate limits. Therefore the generation limits of these sources are between the minimum and maximum limits.

$$P_{W_i}^{\min} \le P_{W_i} \le P_{W_i}^{\max} ; \ j = 1, \dots, N_W; t = 1, \dots, H$$
(4.16)

$$P_{PVk}^{\min} \le P_{PVk} \le P_{PVk}^{\max} ; \ k = 1, \dots, N_{PV}; t = 1, \dots, H$$
(4.17)

4.4Prohibited Operating Zones

In the thermal generator operation, the generator operates in the intermittent manner over its entire operating range due to POZs. This make the generator fuel cost function discontinous with the disjoint convex sub-regions [63-64] between the generator minimum and maximum limits. Thus, the generation limits for the *i*th generator with *j* number of POZs zones can be defined as:

$$P_{Gi}^{\min} \le P_{Gi,t} \le P_{Gi,t,1}^{L} \\ P_{Gi,m-1}^{U} \le P_{Gi,t} \le P_{Gi,t,m}^{L} \\ P_{Gi,t,N_{PZi}}^{U} \le P_{Gi,t} \le P_{Gi}^{\max} \\ \end{cases}; \ i \in \{1, \dots, N_{GPZ}\}, m \in \{2, 3, \dots, N_{PZj}\}$$
(4.18)

Where, the superscripts *L* and *U* define the minimum and maximum limit of POZs of *i*th unit. N_{GPZ} is total number of generators with POZs zones; N_{PZj} is the total number of POZs for the *i*th generator.

Since, DELD is highly non-linear, non-convex complex optimization problem with discontinuous decision variables. Therefore, to solve it an optimization technique is proposed in next chapter.

CHAPTER 5 PROPOSED OPTIMIZATION TECHNIQUE

This chapter presents a recently developed metaheuristic technique called Fireworks algorithm (FWA). It is swarm intelligent based optimization technique inspired by the splendid sparks of the firework. The comprehensive study on FWA reveals that it performs exceptionally well on the non-shifted functions having optima at the origin of the search space. However, its performance degrades heavily on shifted function owing to intrinsic limitations of FWA. Moreover, the several engineering optimization problems deals in shifted optima. Therefore, an improved version of FWA, Improved FWA with Chaotic Sequence Operator (IFWA-CSO) is proposed in this chapter.

5.1 Fireworks Algorithm

FWA is a non-biological swarm intelligent algorithm. It mimics the explosion process of the fireworks. In analogy with the real firework and lighten the night sky, when the firework (individual) is set off to the potential search space, the shower of the sparks fill the local space around it [47]. In FWA, the explosion process employs two type of mechanism, first, more sparks with minimum explosion amplitude and second is less sparks with the large explosion amplitude. These features are designed in such a way that it stimulates the simultaneous exploration and exploitation to perform the global and local search adaptively. However, to further enhance the diversity of tentative solutions another type of spark, Gaussian sparks is also created. It is applied on the selected sparks. Furthermore, the distance based selection operator also conserves the diversified individual to the next generation.

FWA initializes with a predefined number of randomly generated fireworks in the problem search space. The explosion amplitude and number of sparks for each firework is evaluated by their functional value [48]. Then, fireworks exploded and crafts different types of sparks within their potential space. Finally, the predefined number of fireworks selected among all original fireworks and their generated sparks keeping in the mind to survive the best firework individual to the next generation. The basic framework of FWA is depicted in Fig. 5.1.



Figure 5.1 The flowchart of Fireworks Algorithm

5.2 Fireworks Explosion

The fireworks explosion exhibit two specific behaviors through firework explosion display such as good and bad explosion. For the good explosion, firework generates numerous sparks in close proximity of the explosion center. This gives spectacular display of fireworks explosion. On the other hand the bad explosion generates few scattered sparks around the explosion center in large area. These two manners are depicted in Fig. 5.2.



Figure 5.2 Two types of firework explosion

Thus, these two ways can be utilize from algorithm point of view. A good firework symbolizes the fireworks in the promising region with concentrating sparks.

It also indicates that fireworks might be in the close vicinity of the optimal solution. Thus, it is employed to perform the local search of the firework. On the contrary, a bad firework creates few scattered sparks in the large radius of the exploded firework, suggesting that the firework is far away from the optimal location. Hence, this strategy can be opted for global search.

5.3Brief Overview about FWA

The FWA algorithm characterization involves many operators such as explosion operator, explosion sparks, mapping operator and selection. The brief explanation of each operator is described as follows:

5.3.1 Explosion Operator

The firework explosion is characterized by good and bad explosion. Good explosion (firework) generates more sparks in close vicinity of the firework. On the other hand, the bad explosion (firework) generates less sparks with larger search radius. In this way, FWA provides simultaneous exploitation and exploration of the search space by good and bad fireworks, respectively. Therefore, in order to mimics these features of firework, the number of sparks s_i and their explosion amplitudes A_i are governed by the following model:

$$s_{i} = m \cdot \frac{y_{\max} - f(x_{i}) + \xi}{\sum_{i=1}^{n} (y_{\max} - f(x_{i})) + \xi} ; \quad i \in \{1, 2, \dots, n\}$$
(5.1)

$$A_{i} = \hat{A} \cdot \frac{f(x_{i}) - y_{\min} + \xi}{\sum_{i=1}^{n} (f(x_{i}) - y_{\min}) + \xi} ; i \in \{1, 2, \dots, n\}$$
(5.2)

Where y_{max} and y_{min} are max $(f(x_i))$ and min $(f(x_i))$ values of the objective function respectively. *m* and \hat{A} are the limiting values of total sparks and the explosion amplitude, respectively. ξ is a very small real number to counter zero-division error. To avoid overwhelming effects of maximum sparks for good firework, the boundary conditions for s_i are defined as below:

$$s_{i} = \begin{cases} round(\alpha.m) & ;s_{i} < (\alpha.m) \\ round(\beta.m) & ;s_{i} > (\beta.m), \alpha < \beta < 1 \\ round(s_{i}) & ;else \end{cases}$$
(5.3)

Where α and β are algorithm specific values and *round* command is used to set the value to the nearest integer.

5.3.2 Generation of Explosion Sparks

When a firework exploded in space, the sparks so generated are with different amplitudes in all possible random directions. These sparks are then starts diverging from the location of the firework. This explosion process of the *i*th firework to generate sparks in its near vicinity by adding the offset displacement $\Delta x_{i,j}$ on randomly selected dimensions. The offset displacement is defined as given below.

$$x_{i,j} = x_{i,j} + \Delta x_{i,j}; \ \Delta x_{i,j} = A_i \times rand(-1,1)$$
(5.4)

In case, the sparks fall-out of the potential space, it is mapped to potential space using the mapping operator. For more details, the Algorithm.1 of [47] may be referred.

5.3.3 Generation of Specific (Gaussian) Sparks

In order to enhance the diversity in population, the specific sparks are generated using the Gaussian mutation operator, named as Gaussian sparks. The Gaussian sparks are generated for certain randomly selected fireworks. For this purpose, the Gaussian having mean and standard deviation each unity is selected while following the Gaussian (Normal) distribution. The Gaussian sparks generate tentative solutions in the surroundings of the selected fireworks. The expression for the Gaussian spark is defined as follows:

$$x_{i,j} = x_{i,j} \times \text{Gaussian}(1,1)$$
(5.5)

Where $x_{i,j}$ represents the *i*th individual at the *j*th dimension. Gaussian (1, 1) is a number drawn from normal distribution with mean value 1 and standard deviation 1. For further detail, the Algorithm.2 of [47] may be referred.

5.3.4 Mapping Operator

Whenever the individuals fall out of the potential space, they become infeasible. Such infeasible individuals are kept within the potential space using the mapping operator. The mapping operator is defines as follows:

$$x_{i,j} = x_j^{\min} + \left| x_{i,j} \right| \% (x_j^{\max} - x_j^{\min})$$
(5.6)

Where operator % stand for modulo operation (reminder of division) and x_j^{max} and x_j^{min} are upper and lower bounds of the problem. However, the mapping operator has

its own disadvantages as it can easily drag an infeasible spark location close to origin, benefiting the functions having optima near the origin of the search space [47].

5.3.5 Selection

At the end of each iteration *n* fireworks locations are selected to set off the fireworks in the next iteration. Thus, it required adequate diversity strategy operator for fireworks selection. For this the distance based selection operator [71] is considered. Thus, for the firework x_i , the selection probability is defined as:

$$P(x_i) = \frac{R(x_i)}{\sum_{k \in p} R(x_k)}$$
(5.7)

$$R(x_i) = \sum_{k \in p} d(x_i, x_k) = \sum_{k \in p} ||x_i - x_k||$$
(5.8)

Where, p is set of all current fireworks and both types of generated sparks excluding the best firework. The (5.7) indicate that the firework or sparks in low crowded regions will have a higher probability to be selected for the next iteration than fireworks or sparks in crowded regions [48]. Since the best firework must be passed to next iteration. Therefore the elitism is used to preserves the best firework.

FWA is most promising optimization for non-shifted function. However, its performance on the shifted functions suffers severely as many engineering problem deal with shifted optima. Therefore, to enables its applicability for all type of optimization problem, several modifications are suggested in the proposed method.

5.4Proposed Improved Fireworks Algorithm using Chaotic Sequence Operator (IFWA-CSO)

In FWA, the sparks of the given firework are generated by selecting number of dimensions randomly, quite irrespective of the fitness of fireworks. This may cause over diversity in population and thus results in slow convergence. Furthermore, its distance based selection operator increases CPU time on account of large number of distance calculation among the individuals [48]. Another limitation of FWA is that it causes insignificant explosion amplitude prevents the explosion for best firework. This deteriorates its local search potential especially at the anaphase of the algorithm. Finally, the mapping and Gaussian mutation operators of FWA have inherent tendency to map/create tentative solutions towards the origin of the search space [48].

The conventional FWA therefore suffers badly when dealing with problem where the global optima exist far away from the origin. Thus, in order to improve the inadequacy of conventional FWA following some features has been introduced in proposed IFWA method. IFWA is proposed by suggesting Limiting Mapping Operator (LMO), Adaptive Dimension Selection Operator (ADSO), Non-uniform Mutation Operator (NMO) and Chaotic Sequence Operator (CSO) as described below.

5.4.1 Limiting Mapping Operator

The function of the mapping operator is to keep the fireworks within the problem search space whenever they tend to fall out of it during the evolutionary process. This could be achieved in a random fashion as in EFWA of [48]. But, it hampers all previous efforts of the algorithm in selecting this value for the dimension. Therefore, LMO is proposed where the dimensions violating the boundary limits are intended to set at the boundary limits as defined below.

$$x_{i,j} = \begin{cases} x_j^{\min} & ; x_{i,j} < x_j^{\min} \\ x_j^{\max} & ; x_{i,j} > x_j^{\max} \end{cases}$$
(5.9)

Where $x_{i,j}$ is *i*th individual at *j*th dimension of problem. x_j^{max} and x_j^{min} are upper and lower limits of the problem. The proposed LMO not only utilizes all earlier processing effort of the algorithm but also causes fast convergence.

5.4.2 Adaptive Dimension Selection Operator

In FWA, both explosion and specific sparks are created through randomly selecting dimensions. This causes better fit fireworks may undergo wild variations whereas less fit fireworks may faces less variations among their dimensions. It may lead to over diversity in population or retards the pace of algorithm, whatsoever, the convergence of the algorithm suffers badly. Therefore, fitness based operator ADSO is proposed which select the number of dimensions of the given firework by its fitness value i.e., higher the fitness, more will be the selected dimensions and vice-versa. The mathematical modeling proposed for ADSO is derived from the amplitude explosion of FWA as given below.

$$D_{i} = 1 + \hat{D} \cdot \frac{f(x_{i}) - y_{\min} + \xi}{\sum_{i=1}^{n} (f(x_{i}) - y_{\min}) + \xi} ; i \in \{1, 2, \dots, n\}$$
(5.10)

Where y_{\min} is the min ($f(x_i)$) value of the objective function. \hat{D} is algorithm parameter to control the dimension number. Care has been taken while generating sparks of fireworks so it could generate sparks by selecting at least one dimension.

5.4.3 Non-uniform Mutation Operator

Ref. [48] has shown that the Gaussian spark operator of FWA creates many sparks near to the origin or out of the search space regardless of function optimality. Any spark found outside the search space is being mapped near to the origin by the mapping operator. Moreover, if firework is located near to the origin of the search space then it cannot get away from this location [48]. This adversely affects the performance of algorithm when applied to optimization problems having global optimum far away from the origin. Therefore, NMO is proposed in IFWA to replace Gaussian sparks which has been taken from [72]. NMO provides sparks having dynamically decreasing offset with iterations as given by (5.11). The equation reveals that NMO facilitates global searching during initial phase and then gradually shifts into the local searching at the anaphase of the algorithm.

$$x_{i,j} = \begin{cases} x_{i,j} + \Delta(iter, x_j^{\max} - x_{i,j}) & \text{; if } round(rand) = 0 \\ x_{i,j} - \Delta(iter, x_{i,j} - x_j^{\min}) & \text{; if } round(rand) = 1 \end{cases}$$
(5.11)

Where, $(x_j^{\text{max}}, x_j^{\text{min}})$ is upper and lower limits of the variable $x_{i,j}$ and $\Delta(iter, y)$ is non-uniform function which can be determined using (5.12). It gives the function value in the range [0, y] such that $\Delta(iter, y)$ approaches towards zero as iteration progresses.

$$\Delta(iter, y) = y \times (1 - r.\exp(1 - \frac{iter}{iter\max})^b)$$
(5.12)

Where, r is a uniform random number [0, 1], *iter*max is predefined maximum iteration count and b is a dependent parameter on the iteration number. The value of b is chosen as 5 experimentally [72].

5.4.4 Chaotic Sequence Operator

Chaos system has wide application in the area of science due to the disorder behaviour. It is non-linear dynamic system appeared to be the deterministic initially, but later seem to be random and sensitive to the initial condition. It has various applications in the engineering field [73-74]. In [73] author used chaotic sequence

with mutation factor in the differential evolution for ED problem. Caponetto *et al.* [74] employed the various chaotic sequences instead of the random numbers in the evolutionary algorithm. Shengsong *et al.* [75] presented a chaotic sequence application to solve the optimal power flow using a chaotic hybrid algorithm. For the iterator logistic map of degree 2, a non-linear polynomial is described as follows [74]:

$$\gamma_{l} = \mu . \gamma_{l-1} . (1 - \gamma_{l-1}) \tag{5.13}$$

The behaviour of the system is controlled by control parameter μ ranges between [0, 4] and γ_l is chaotic parameter at *l*-state used to determine the pattern of the logistic map, whether it follows a constant value, sustained oscillation between limited sequence of size or behave chaotically in unpredictable manner. The deterministic behaviour of (5.13) has been found with μ =4 and $\gamma_0 \notin$ [0, 0.25, 0.50, 0.75, 1] experimentally [74].

While generating sparks of fireworks in FWA, the amplitudes of sparks are randomized. However, the amplitude of the given firework depends upon its function value whereas the relative magnitudes of dimensions may vary through wide limits. It implies that the impact of the offset displacement does not remain same among all dimensions of the firework. Higher the magnitude of the dimension lesser will be the impact of the offset displacement and vice-versa. Therefore, the dimensions of the fireworks in IFWA are allowed to modulate by suggesting the following model.

$$x_{i,j} = \gamma_l (x_{i,j} + \Delta x_{i,j}); \ \Delta x_{i,j} = A_{i,j} \times rand(-1,1)$$
(5.14)

The model described by (5.14) is different than that of FWA in two senses. First, the offset displacement attribute to selected dimensions of the given firework are different. Second, the chaotic parameter governs whole dimension rather the offset displacement alone. This model therefore provides versatility to sparks of fireworks which enhances diversity in population. So both exploration and exploitation potential of the search space is improved.

In addition to aforementioned modifications suggested in IFWA, the explosion amplitude of the best firework is kept a minimum value as per following model [48]. This is essential to keep the best firework active especially during the anaphase of the algorithm.

$$A_{i,j} = \begin{cases} A_j^{\min} & \text{if } A_{i,j} < A_j^{\min} \\ A_{i,j} & else \end{cases}$$
(5.15)

$$A_{j}^{\min}(iter) = A_{j}^{init} - \frac{A_{j}^{init} - A_{j}^{final}}{iter\max} \sqrt{(2 \times iter\max - iter) \times iter}$$
(5.16)

Where A_j^{init} and A_j^{final} are the initial and final amplitude of the explosion which can be fixed as fraction of the search space. *iter* and *itermax* shows current iteration and maximum iteration count.

5.4.5 Selection Operator

The selection operator of conventional FWA is majorly responsible for most of the runtime on account of distance based selections [48]. It happens on account of large number of distance calculations which is huge figure for a large dimensional problem. This makes it computationally demanding. Therefore, probability based Roulette Wheel Selection is suggested in IFWA. In this selection procedure a probability distribution proportional to the fitness is created. The fireworks are then selected by sampling the distribution function given by [76].

$$\varphi(x_i) = \frac{y_{\max} - f(x_i)}{\sum_{i=1}^{p} (y_{\max} - f(x_i))}$$
(5.17)

Where *p* is the total number of individuals, including original fireworks, both explosion and specific sparks and $\varphi(x_i)$ is the probability that x_i will be selected.

In order to measures the effectiveness of each suggested modification, different variants have been formed and predecessor modifications are included in successive variants. Finally, all previous modifications are included in IFWA-CSO termed as proposed method. The following four variants are as follows:

- (a) IFWA-I: FWA with LMO
- (b) IFWA-II: IFWA-I with ADSO
- (c) IFWA-III: IFWA-II with NMO
- (d) IFWA-CSO: IFWA-III with CSO

5.5Constraint Handling Algorithm

In IFWA, during the initialization or sparks generation process, it is likely that the firework individual may not satisfy the various constraints in the stochastic DELD problem and require constraint handling strategy. In this context, the penalty function

method is most popular method. It transforms the constraint problem into unconstraint problem and conveniently used to handle individual during evolution process by punishing infeasible solution to ensure the feasible ones are favored. However, this method suffers from large computation demanding time due to multiple run for the fine tuning of the penalty factor, which degrades the efficiency of the algorithm [77]. Therefore, to overcome this drawback an efficient heuristic approach is proposed for solving the various constraints of the stochastic DLED problem. In the proposed method, the objective(s) are optimized by equally maintaining the violation limits. This makes the entire individual feasible and actively part into optimization. The procedure of proposed algorithm is as follows:

Step 1: Set the dispatch interval index t=1.

Step 2: Set the *count*=1

Step 3: Adjust the generation limits of each conventional generating units at each interval as per following (5.18) & (5.19)

$$P_{G_{i,t}}^{\min} = \begin{cases} P_{G_{i}}^{\min} & \text{if } t = 1\\ \max(P_{G_{i,t-1}} - DR_{i}, P_{G_{i}}^{\min}) & else \end{cases}$$
(5.18)

$$P_{G_{i,t}}^{\max} = \begin{cases} P_{G_i}^{\max} & \text{if } t = 1\\ \min(P_{G_{i,t-1}} + UR_i, P_{G_i}^{\max}) & else \end{cases}$$
(5.19)

Where $P_{Gi,t}^{max}$ and $P_{Gi,t}^{min}$ are the generation limits of the conventional generator at each interval *t*. Then, the infeasible solution of each thermal generator unit at each interval will be modified as below

$$P_{G_{i,t}} = \begin{cases} P_{G_{i}}^{\min} & \text{if } P_{G_{i,t}} < P_{G_{i,t}}^{\min} \\ P_{G_{i,t}} & \text{if } P_{G_{i,t}}^{\min} < P_{G_{i,t}} < P_{G_{i,t}}^{\max} \\ P_{G_{i}}^{\max} & \text{if } P_{G_{i,t}} > P_{G_{i,t}}^{\max} \end{cases}$$
(5.20)

The power limits of wind and solar powered presumed to be independent of ramp rate limit. Therefore, for renewable generator violating the boundary limits will be adjusted by (4.16) & (4.17).

Step 4: Calculate the residual (mismatch of the power) using the power balance equation.

$$D_{err,t} = \sum_{i=1}^{N_G} P_{Gi,t} + \sum_{j=1}^{M_W} P_{Wj,t} + \sum_{k=1}^{L_{PV}} P_{PVk,t} - P_{load,t} - P_{loss,t}$$
(5.21)

If $D_{err,t}=0$, then go to step 6, otherwise go to step 5.

Step 5: Select the generating unit (conventional unit or wind or solar power unit) randomly, and then the value of selected generator is modified as follows:

$$P_{i,t} = \begin{cases} P_{i,t} - \min\{D_{err,t}, (P_{i,t} - P_{i,t}^{\min}) \times rand\}, if \ D_{err,t} > 0 \\ P_{i,t} - \max\{D_{err,t}, (P_{i,t} - P_{i,t}^{\max}) \times rand\}, if \ D_{err,t} < 0 \end{cases}$$
(5.22)

If $P_{i,t}$ does not violate the boundary limits, go to Step 6. Otherwise modified it by (5.20) or (4.16) or (4.17), and then go to Step 6 Step6: *count=count*+1, if *count* > *count*max, return to previous hour and rearrange the

schedule. If residual D_{err} reach in predefined tolerance level, then go to Step 7 Step 7: Set t=t+1, modify the generation limits of conventional generator for the next dispatch interval according to (5.18) & (5.19).

5.6Firework Individuals Initialization

The fireworks are initialized by first randomly generating in the multi-dimensional search space. $X=[x_1, x_2, x_3, ..., x_n]^T$, where *n* is size of the population and $x_1, x_2, x_3, ..., x_n$ represents the firework (individual). Each firework individual is consists of decision vector. Since, the total number of generating units $(N_G+M_W+L_{PV})$ and number of dispatch interval are *T*. Therefore, the decision vector size will be $(N_G+M_W+L_{PV})^*T$. The appearance of decision vector will be expressed as $x_i=[U_{i,1}, U_{i,2}, U_{i,3}, ..., U_{i,T}]$. where *T* represents the time interval and $U=[P_{Gi,1}, P_{Gi,2,...,}, P_{Gi,NG}, P_{Wi,1,...,}, P_{Wi,MW}, P_{PV_{i,1,...,}}, P_{PV_{i,1,...,}}, P_{PV_{i,1,...,}}, P_{W_{i,1,WV}}$. Here, N_G, M_W and L_{PV} are the units of the thermal generator, wind powered generator and solar powered generator respectively. These all decision variables are uniformly generated between their upper and lower limits as expressed below.

$$P_{G_{i,j}} = P_{G_{i,j}}^{\min} + rand_{i,j} \times (P_{G_{i,j}}^{\max} - P_{G_{i,j}}^{\min})$$
(5.23)

$$P_{W_{i,j}} = P_{W_{i,j}}^{\min} + rand_{i,j} \times (P_{W_{i,j}}^{\max} - P_{W_{i,j}}^{\min})$$
(5.24)

$$P_{PV_{i,j}} = P_{PV_{i,j}}^{\min} + rand_{i,j} \times (P_{PV_{i,j}}^{\max} - P_{PV_{i,j}}^{\min})$$
(5.25)

Thus, initial population of generating schedule, over time horizon is expressed as in (5.26).

$$X = \begin{bmatrix} U_{1,1} & U_{1,2} & \cdots & U_{1,T} \\ U_{2,1} & U_{2,2} & \cdots & U_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ U_{n,1} & U_{n,2} & \cdots & U_{n,T} \end{bmatrix}$$
(5.26)

After initialization, the firework individuals satisfy problem constraints defined by equations and infeasible individuals are adjusted by using a constraint handling algorithm as aforementioned.

5.7Pseudo Code of Proposed IFWA-CSO

```
Randomly select n location for fireworks
while (iter<ietrmax)</pre>
  Set off fireworks at n locations;
  for each firework x<sub>i</sub>
     Calculate the explosion amplitude A_i, number of sparks \hat{s}_i and adaptive
     dimension D_i according to (5.1), (5.2) and (5.10);
     Obtain the location of sparks for each firework x_i;
     for each firework x<sub>i</sub>
        for i=1:round(rand × D_i)
           set z^{k}=round(rand), k=1,2,...,dimension;
           if (z^{k} = = 1)
              x_{i,j} = \gamma_l(x_{i,j} + A_{i,j} \times rand(-1,1));
             end if
        end for
     end for
  end for
  for k=1:m̂
     randomly select the firework x_i
    for selected firework x_i
        for j=1:round(rand \times D<sub>i</sub>)
           set z^{k}=round(rand), k=1,2,\ldots, dimension;
           if (z^{k}==1)
             x_{i}^{k} = x_{i}^{k} + \Delta(iter, y);
          end if
        end for
     end for
     Select the best location and keep it for next explosion generation;
     Select (n-1) locations from the two types of sparks and the current
     fireworks using selection operator;
```

end

The validation of proposed IFWA-CSO and its predecessor variants are conducted in the simulation results section in the next chapter. The detailed analysis of performance of each suggested modification is carried-out on the different standard systems in the ranges of small to large range dimensional problem.

CHAPTER 6 SIMULATION RESULTS

In this chapter the validation of the proposed IFWA-CSO is performed. For this first testing is done on some well-known benchmark function, and then applied on the power system optimization problem of economic operation in static and dynamic states with variety of constraints. The various different cases are implemented and presented with the numerical results and compared with the recently established algorithms reported in literature. For experimental MATLAB platform is used.

6.1 Selection of Parameters

FWA performs quite well with less number of fireworks [47]. Thus, in FWA number of fireworks remains intact between ranges 3 to 5. However, the number of the sparks *m* is subject of the dimensionality and complexity of the problem. The rest parameters include number of specific sparks \hat{m} , α , β , explosion amplitude \hat{A} , A_j^{init} , A_j^{final} . A new parameter, number of dimension \hat{D} is introduced in the proposed method. It depends upon the dimensionality of the firework individual. For the testing of the benchmark functions, the parameters chosen are n=5, m=50, $\hat{m}=5$, a=0.04, $\beta=0.8$, $\hat{A}=40$, $A_j^{init}=(x^{\max}_j - x^{\max}_j) \times 0.02$ and $A_j^{final}=(x^{\max}_j - x^{\max}_j) \times 0.001$ same as reported in [47]. It is to validate the IFWA-CSO with the standard FWA and EFWA method. The additional parameter, number of dimension \hat{D} is set to 10 in IFWA-CSO method experimentally. The maximum iteration *iter*max is set to 5000 for all. The various parameters used for the power systems of economic operation test systems are listed in Table 6.1.

	Parameter				,	Value	
N				5			
ŵ				5			
A				0.04			
В				0.8			
Â			40				
A_i^{init}			$(x^{max}_{j} x^{min}_{j}) \times 0.5$				
A_{jf}^{inal}				(x^{max})	$(x^{min}_{j}) \times 0.001$		
System	ELD s	system	Ι	DELD system RDELI		RDELD	system
Parameter	40-units	140-units	5-uni	its	15-units	7-units	17-units
<i>iter</i> max	2500	5000	500	0	2500	2500	1500
m	90	90	50		90	90	90
Ď	10	20	20		50	20	50

Table 6.1 Control parameters of FWA and proposed FWA variants

6.2 Validation of Proposed IFWA-CSO on Benchmark Functions

The benchmark functions are essential to validate and measure the performance of the algorithm. These functions are categorized into various classes on the basis of their shape such as: Schwefel 1.2 is uni-modal function with single minima; Generalized Rosenbrock is multi-modal; Generalized Schwefel 2.26, Generalized Rastrigin, Generalized Griewank and Penalized Function P16 are classes into a complex multi-modal high-dimensional problems with many local minimum optima and a single global optimum; Six-hump Camel-back is two dimensional multi-modal problem with fewer local minima and two global minima, symmetric about the origin. Similarly, Goldstein-Price is low-dimensional multi-modal problems with fewer local minimum optima and one global optimum [78]. Here, for the validation, eight benchmark functions have been considered for the study. The definition of these function are defined in Table 6.2 [47, 78] and their optimum and boundary limits are known to be a priori.

Function	Expression	D	Search
			space
Schwefel 1.2	$f_1 = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} x_j \right)^2$	30	[±100] ^D
Generalized Rosenbrock*	$f_2 = \sum_{i=1}^{D-1} \left\{ 100 \left(x_{i+1} - x_i^2 \right)^2 + \left(x_i - 1 \right)^2 \right\}$	30	[±30] ^D
Generalized Schwefel 2.26*	$f_3 = 418.9829 * D - \sum_{i=1}^{D} x_i \sin(\sqrt{x_i})$	30	[±500] ^D
Generalized Rastrigin	$f_4 = \sum_{i=1}^{D} \left\{ x_i^2 - 10\cos(2\pi x_i) + 10 \right\}$	30	[±5.12] ^D
Generalized Griewank	$f_5 = \frac{1}{4000} \sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	[±600] ^D
Penalized function P16*	$\mu(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a \le x_i \le a \\ k(-x_i - a)^m & x_i > a \end{cases}$	30	[±50] ^D
	$f_6 = 0.1 \left\{ \sin^2(3\pi x_i) + \sum_{i=1}^{D-1} (x_i - 1)^2 \left\{ 1 + \sin^2(3\pi x_{i+1}) \right\} + (x_D - 1)^2 \times \left\{ 1 + \sin^2(2\pi x_D) \right\} \right\}$		
	$+\sum_{i=1}^{D}\mu(x_i, 5, 100, 4)$		
Six-hump Camel-back*	$f_7 = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	[±5] ^D
Goldstein- Price*	$f_8 = \left\{ 1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2) \right\} \times$	2	$[\pm 2]^{D}$
	$\left\{30 + \left(2x_1 - 3x_2\right)^2 \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2\right)\right\}$		

Table 0.2 Benchmark function	Table 6.2	Benchmark	functions
------------------------------	-----------	-----------	-----------

* Shifted function

A set of 30 independent trails run is presented in Table 6.3. It presents the statistical results in terms of mean and standard variation (STD). It can be seen from the table that the performance of standard FWA on the non-shifted functions f_1 , f_4 and

 f_5 is superior to EFWA and proposed IFWA-CSO method is giving equivalent result to FWA. However, this exceptional performance of FWA is due to its operator rather than algorithm intelligence [48]. On the contrary, shifted function f_3 performance get worse on standard FWA, this slightly improved in EFWA, while IFWA-CSO getting significantly better value with negligible STD. It is noted that in low dimensional shifted functions f_7 and f_8 show equivalent result in all methods. On the contrary, for the higher dimensional problem FWA and EFWA performance on the shifted functions deteriorates severely. Although, for a shifted function f_2 , it is found that FWA perform better than EFWA and IFWA-CSO. It may be due to that the optimum point close or near to origin (1.0^D) [48]. Therefore, the proposed IFWA-CSO performed extremely well on the shifted function in addition, adequate performance on non-shifted function. This shows the robustness of the proposed method on all type of functions irrespective of their optima. Therefore, proposed IFWA-CSO method is suitable optimizer for the shifted function as power system optimization problem.

deviation for benchmark functions						
	FWA		EFWA		Proposed IFWA-CSO	
F	Mean	STD	Mean	STD	Mean	STD
f_1	0.000000	0.000000	0.235746	0.083984	0.000530	0.000436
f_2	22.081905	8.474207	101.005459	129.277549	30.197884	15.580127
f_3	-11844.093386	545.381796	-11198.171052	555.879034	-12569.484439	0.000707
f_4	0.000000	0.000000	0.401931	0.765849	0.000000	0.000000
f_5	0.000000	0.000000	0.096446	0.049704	0.000000	0.000000

0.000037

0.000000

0.000000

0.000002

-1.031628

3.000000

0.00000

0.000000

0.000000

0.000066

-1.031628

3.000000

Table 6.3 Comparison of the results of different algorithm mean and standard deviation for benchmark functions

6.3 Application of Proposed IFWA-CSO Method

0.010997

0.000000

0.000008

0.008011

-1.031628

3.000003

In this section the application of the developed variants of FWA and proposed IFWA-CSO method is applied for economic operation in the power system. For this, three different types of test systems are considered. The first system is static ELD (SELD), generally called as ELD system to optimize the fuel cost of thermal generator while satisfying the various system and generator constraints. The next is DELD system to obtain the optimal scheduling of the thermal generator over the time horizon with optimized fuel cost of thermal generator and the third system is proposed stochastic DELD system. It involves a scenario based one unit of wind and solar powered generation in dynamic states. The wind and solar unit are modeled by suitable probability distribution function and their operating charge is modeled by

considering a direct cost function. Further, their variability cost is modeled by probabilistic cost function. It optimizes the total cost of operation; consist of the fuel cost of thermal generator as well as renewable sources operating cost. For the convenience, these systems are abbreviated as static ELD (S), dynamic ELD (D) and renewable dynamic ELD (R) respectively. Each system is further bi-furcated. The details of each case study of each system are defined below:

Case S₁: 40-unit generating system with valve-point loading effect in ELD problem.

Case S_2 : 140-unit generating system with valve-point loading effect, ramp rate limits and prohibited operating zones in ELD problem.

Case D₁: 5-unit generating system with valve-point loading effect, ramp rate limits and transmission loss in DELD problem.

Case D₂: 15-unit generating system with ramp rate limits, prohibited operating zones and transmission loss in DELD problem.

Case R_1 : 5-unit generating system with valve-point loading effect, ramp rate limits, prohibited operating zones and transmission loss and one unit of each wind and solar powered generator in RDELD problem.

Case R_2 : 15-unit generating system with valve-point loading effect, ramp rate limits and transmission loss and one unit of each wind and solar powered generator in RDELD problem.

Case study S_1 : This case study consists of 40-units of thermal generators with nonconvex fuel cost function to account the valve-point loading effect [38, 80]. The detail data is taken from [80]. The total power demand is set to be 10500. The application results of the FWA [47], EFWA [48] and proposed FWA variants are presented and compared with other established meta-heuristic techniques in Table 6.4. The results are presented in terms of best, average and worst. Furthermore, a statistical error analysis is performed to establish the proposed method and the results are presented after the sample of 100 trails in Table 6.4. The standard deviation (STD) measures the diversity of the sample from the average value. Similarly, coefficient of variation (COV) measures the dispersion of the samples. It expresses the accuracy of the method of producing the repetitive value. It represents central tendency of samples. It can be observed from the table that COV of each proposed variants is successively reduced in order of the suggested modification. However, in the proposed method the COV deviate a bit. It is observed that the performance of proposed method is not up to that level. The reason for this could be over diversity due to CSO operator, which dis-allowing the individuals to converge. Thus, it indicates that the proposed method lag in the small range dimensional problem.

Method	Best fuel cost	Average fuel	Worst fuel Cost	STD	COV
	(\$/hr)	cost (\$/hr)	(\$/hr)		
PSO-LRS [81]	122035.795	123382	125740.63	-	-
NPSO [81]	121704.739	122221.37	122995.098	-	-
NPSO-LRS [81]	121664.431	122981.591	122209.319	-	-
APSO [82]	121663.522	122153.673	122912.396	-	-
SOH-PSO [64]	121501.14	121853.57	122446.3	-	-
QPSO [83]	121448.21	124793.4	-	-	-
ABC [84]	121441.03	121995.82	-	-	-
ICA-PSO [85]	121422.1	-	-	-	-
θ-PSO [46]	121420.903	121509.842	121852.425	-	-
DE/BBO [86]	121420.895	-	-	-	-
HHS [87]	121415.592	121615.854	-	-	-
CCPSO [80]	121412.536	121445.327	121525.493	-	-
IPSO [85]	121412.866	121509.522	121546.842	-	-
IABC-LS [86]	121412.73	-	121471.61	-	-
NAPSO [87]	121412.57	-	-	-	-
CSA [41]	121412.536	121520.411	121810.254	-	-
aBBOmDE[66]	121414.8734	121487.8532	121568.3254	-	-
DCPSO [38]	121412.535	121423.131	121516.886	-	-
ORCSA [92]	121412.5355	121472.45	121596.18	-	-
<i>θ</i> -MBA [91]	121491.0662	121491.066	121491.066	-	-
FWA	127077.2331	129081.5599	130633.4888	701.6016	0.5435
EFWA	121500.2888	121750.3721	122150.5890	158.1281	0.1299
IFWA-I	121486.1761	121780.6196	122434.5042	179.0485	0.1470
IFWA-II	121414.7643	121568.1210	121969.9267	114.1673	0.0939
IFWA-III	121414.7887	121563.4020	121969.8765	92.5862	0.0762
IFWA-CSO	121424.8314	121572.1485	121927.5762	113.9440	0.0937

Table 6.4 Comparisons results for case study S_1

A qualitative analysis is carried-out by drawing a set of sample convergence characteristics for the case study S_1 . It can be seen from the Fig.6.1 that the FWA is very slow in convergence and become constant after few iteration. Thus, it shows inability of the FWA on the shifted function. This improves slightly in EFWA but, not much sufficient. However with the proposed modifications, it can be observed that all proposed variants follow nearly same path with narrow width, which indicate that all variants flight particle with good speed. For, a large resolution of FWA variants and proposed IFWA-CSO method, the enlarge view of Fig. 6.1, is presented in Fig. 6.2.

However, in later stage it continuously avoids the local trapping that allows the particle to reach in the optimal region in less iteration.



Figure 6.1 Convergence characteristics for case study S₁



Figure 6.2 Enlarged view of Figure 6.1

Case study S_2 : In this case study, the effectiveness of the proposed algorithm is investigated on the large-scale generating system consists of 140 thermal units with ramp rate limits, valve-point loading effect and POZs. The detailed data for this system is taken from [80]. The expected power demand is set to 49342 MW. The results of standard FWA [47], EFWA [48] and its proposed modifications are presented with other existing population based optimizations in Table 6.5 and the efficacy of proposed method is presented after 100 trails. The statistical results of FWA and EFWA and proposed variants are presented for the analysis purpose. It can be observed from the table that the FWA performance is severely affected. This indicates non-suitability of FWA on the large dimensional shifted functions. Further, this marginally improved by EFWA, which overcome the paradigm of the FWA on the shifted function. But, the result of EFWA shows the insufficiency on the largescale problem. However, with the proposed variants the results subsequently upgraded. In FWA-I, FWA-II and IFWA-III best optimal solution is on reach, but the average fuel cost differs. However, with all conjunction in the proposed technique, the results obtained are similar to recently publish technique with negligible COV. It shows the potential of proposed method getting better quality solution. Thus, each subsequent modification made over FWA play important role in elevating the proposed technique. This proves the efficacy of the proposed technique to generate a solution in a narrow range for a large-scale complex combinatorial problem.

Methods	Best fuel cost	Average fuel cost	Worst fuel cost	STD	COV
	(\$/hr)	(\$/hr)	(\$/hr)		
GSO [42]	1728151.17	1745515.00	1753229.56	-	-
CCPSO [80]	1657962.73	1657962.73	1657962.73	-	-
CTPSO [80]	1657962.73	1657964.06	1658002.79	-	-
CQGSO [42]	1657962.73	1657962.74	1657962.78	-	-
DCPSO [38]	1657962.720	1657962.720	1657962.720	-	-
FWA	1672303.503	1677475.819	1682637.393	2100.215	0.125
EFWA	1661464.532	1663398.530	1666528.633	1024.622	0.062
IFWA-I	1658512.996	1659269.966	1660227.466	456.084	0.027
IFWA-II	1657962.734	1658210.812	1659027.823	279.000	0.017
IFWA-III	1657962.746	1658103.449	1658342.686	100.696	0.006
IFWA-CSO	1657962.79	1657984.60	1658094.73	34.742	0.002

Table 6.5 Comparisons results for case study S₂



Figure 6.3 Convergence characteristics for case study S₂

A set of convergence characteristics for the best fuel cost is obtained during a sample trial for the case study S_2 is shown in Fig. 6.3. It can be observed from the figure that FWA is slow on convergence owing to intrinsic limitation of FWA. Though, it marginally better in EFWA. However, with the proposed modifications, it is shown that the convergence characteristics become more rapid than conventional FWA and EFWA. Further, to visualize the convergence of FWA's variants and proposed IFWA-CSO method in large resolution an enlarge view of Fig. 6.3, is presented in Fig. 6.4 and 6.5. It can be shown from enlarge view that the proposed

variants take less iteration to reach the optimal region. A detailed analysis of each suggested modifications over FWA are discussed in technique aspects in the later section.



Figure 6.4 Enlarged view of Figure 6.3



Figure 6.5 Enlarged view of Figure 6.3

Case study D₁: In this case study, 5-units of thermal generators with valve point loading, ramp rate limits and B-coefficients for DELD system are considered. The detail data is taken from [93]. The load demand is pre-scheduled for 24-hours. The results of conventional FWA [47], EFWA [48] and its proposed variants with other metaheuristics are listed in Table 6.6. The optimal total fuel cost obtained by the proposed IFWA-CSO for 24 hours' time duration is \$43078.322. The total network power loss is 194.313MW and to measure the competence of algorithms, the solution quality is presented after 50 sample trial. The result shows that FWA is performing very poorly on the low-scale problem. It may be due that the DELD problem is heavily constrained and of huge search space problem. However, the EFWA is unexpectedly performing better than IFWA-I, IFWA-II, IFWA-III in term of the average and worst fuel cost, but deviate in terms of the best obtained optimal fuel cost. Although, IFWA-I, IFWA-II, IFWA-III performance subsequently improved,

but, not at par level than EFWA. Similarly, it is found that the proposed IFWA-CSO performance is not satisfactory, though it obtained the best optimal cost.

Method	Best fuel cost	Average fuel	Worst fuel Cost	STD	COV
	(\$)	cost (\$)	(\$)		
TVAC-IPSO [94]	43136.561	43185.664	43302.233	-	-
EAPSO [11]	43820	44082	44982	-	-
DE [95]	43213	43813	44247	-	-
ABC [14]	44046.83	44064.73	44218.64	-	-
AIS [44]	44385.4 44	44758.8	45553.8	-	-
ICA [96]	43117.055	43144.472	43209.533	-	-
CMAES [97]	43526	43915	44191	-	-
FAIPSO [98]	43048	43091	43149	-	-
MTLA [12]	43048.4	43077.9	43128.5	-	-
FWA	44312.205	45092.397	46565.37	415.4978	0.9214
EFWA	43250.398	43813.424	44304.273	245.4416	0.5602
IFWA-I	43471.436	44105.076	45172.179	433.3374	0.9825
IFWA-II	43203.663	43843.698	44481.706	353.762	0.8069
IFWA-III	43210.834	43715.768	44419.377	305.9599	0.6999
IFWA-CSO	43078.324	43799.587	44644.796	377.3985	0.8616

Table 6.6 Comparisons results for case study D₁



Figure 6.6 Convergence characteristics for case study D₁

A set of sample convergence characteristics for case study D_1 is shown in Fig. 6.6. It can be observed from the figure that FWA and EFWA suffer from slow convergence. However, with the proposed modification convergence characteristics become more rapid than conventional FWA and EFWA. To visualize the early projectile of the best particle is shown in Fig. 6.7. It can be shown from the enlarge view that the proposed variants take less iteration to reach in the optimal region.



Figure 6.7 Enlarged view of Figure 6.6

The total power loss obtained for optimum generating schedule during 24-hours is shown in Table 6.7. It can be seen that EFWA is giving minimum loss. In the proposed variants the proposed IFWA-CSO is giving optimum loss.

Table 6.7 Total power loss using proposed IFWA-CSO and its variants for case study D_1 during 24 hours

Method	Power Loss (MW)	Method	Power Loss (MW)
FWA	194.7314	IFWA-II	195.6062
EFWA	193.4160	IFWA-III	195.3342
IFWA-I	196.2064	IFWA-CSO	194.3131

Case study D₂: In this case, 15-units of thermal generators with ramp limits, B-coefficients and POZs are considered. The relevant data is taken from [99] and load demand is predefined for 24 hours. The obtained results of FWA [47], EFWA [48] and proposed variants with other reported metaheuristics are tabulated in Table 6.8 and the quality solutions are presented after 50 trials. The total optimum fuel cost obtained for 24 hours' time duration is \$759132.266 by employing the proposed IFWA-CSO algorithm. The total power loss obtained for best fuel cost is 640.731 MW. It can be observed from the table that the IFWA-II, IFWA-III and proposed IFWA-CSO are giving comparable and much better results than other published methods i.e. PSO [100], SFEP [99], CASDHS [13]. The existing FWA algorithms may also be qualitatively compared with proposed variants of FWA on the basis of standard deviation (SD) and coefficient of variation (COV) of their respective sampled solutions.

Method	Best fuel cost	Average fuel cost	Worst fuel Cost	STD	COV
	(\$)	(\$)	(\$)		
PSO [100]	774131.000	-	-	-	-
SFEP [99]	783411.000	-	-	-	-
CASDHS [13]	759689.220	759766.233	759845.736	-	-
FWA	764537.966	765162.012	765771.271	277.550	0.036
EFWA	761281.540	761722.613	762050.141	189.093	0.025
IFWA-I	759753.985	759925.218	760181.521	85.155	0.011
IFWA-II	759132.816	759135.767	759155.185	3.915	0.000
IFWA-III	759132.266	759134.708	759154.331	3.480	0.000
IFWA-CSO	759132.263	759134.315	759154.179	3.163	0.000

Table 6.8 Comparisons results for case study D₂



Figure 6.8 Convergence characteristics for case study D₂



Figure 6.9 Enlarge view of Figure 6.8

To analyze the qualitative behavior, a set of convergence characteristics is shown in Fig. 6.8. It can be seen from the figure FWA and EFWA continuously avoiding the local trapping, but slow in convergence. It is marginally better in the IFWA-I. However, IFWA-II, IFWA-III and proposed IFWA-CSO are giving fast convergence. An enlarge view of Fig. 6.8 is shown in Fig. 6.9. It shows that the IFWA-CSO flight the particle in global region with better exploration capability in early state and improved the exploitation competence in later state.

It has been observed that the proposed variants of FWA significantly reduce the fuel cost of thermal generator. Thus, the optimal schedule is also improves. This also reflects note-worthy decrease in the power loss of the generator. The total power loss for 24- hours for best schedule in each method is tabulated in Table 6.9. Further, the power loss schedule of best operating schedule of FWA, EFWA and its proposed variants is shown in Fig. 6.10 for each time state.

Table 6.9 Total power loss using proposed IFWA-CSO and its variants for case study D_2 during 24 hours

Method	Power Loss (MW)	Method	Power Loss (MW)
FWA	731.769	IFWA-II	641.383
EFWA	676.505	IFWA-III	640.884
IFWA-I	652.902	IFWA-CSO	640.731



Figure 6.10 Power loss schedule for best generating schedule for 24-hours in case study D2

6.4 Stochastic DELD System with Renewable Sources

In this system, the renewable sources (one wind and one solar powered source) are incorporated with the conventional power system. The wind and solar are modeled by considering the hourly wind speed and solar irradiance data, then fit into their respective *pdf* and related generated parameters is tabulated in Table 6.10. It measures the potential of the wind and solar potential that show the variability over period of time. The relevant data of hourly wind speed and solar irradiance is taken from NREL [101]. The associated parameters of wind energy conversion device, cut-in v_{in} , rated v_r and cut out v_f wind speed are 5 m/s, 25 m/s and 45 m/s respectively. Similarly, solar energy conversion device, PV module specifications are listed in Table 6.11. The

analysis is carried out on two conventional dynamic systems with the incorporation of wind and solar unit in the case study R_1 and R_2 .

	Weibull parameter		Beta parameter	
Time interval	C_t	k_t	a_t	b_t
1	9.78	4.60	0	0
2	9.64	4.48	0	0
3	9.74	4.35	0	0
4	9.71	4.11	0	0
5	9.41	4.15	0	0
6	9.34	4.23	0	0
7	9.20	4.11	0	0
8	8.78	4.06	0	0
9	8.65	3.87	1.04	4.29
10	8.94	3.74	2.16	4.81
11	10.23	3.71	2.98	5.11
12	11.28	4.00	3.40	4.55
13	12.00	4.21	3.24	4.06
14	12.37	4.73	3.22	3.64
15	12.59	4.49	2.75	3.81
16	12.40	4.74	2.61	4.01
17	12.03	4.59	2.05	5.23
18	11.35	4.31	1.03	6.40
19	10.77	4.28	0	0
20	10.09	4.35	0	0
21	9.67	4.99	0	0
22	9.53	5.36	0	0
23	9.71	5.41	0	0
24	9.78	5.13	0	0

Table 6.10 Hourly Weibull and Beta distribution parameter

Table 6.11 Specifications for PV module

Module characteristics	Unit
Watt peak (<i>PV^{max}</i>)	340 W
Open circuit voltage (V_{oc})	46 V
Short circuit current (I _{sc})	9.78 A
Voltage at maximum power (V_{MPP})	37.8 V
Current at maximum power (I_{MPP})	8.99 A
Fill factor (FF)	0.755
Nominal cell operating temperature (N_{OT})	46℃
Current temperature coefficient (K_i)	0.047 mA/℃
Voltage temperature coefficient (K_v)	0.335 mV/°C

Case study R₁: This case study consists of a scenario based one wind power and one solar power source of each capacity of 80 MW and 30 MW with dynamic conventional power system of 5-unit thermal systems. The total penetration of renewable sources is kept to 18% of average load demand. The load demand is assumed to be same as conventional system forecasted for 24-hours. The relevant data of conventional 5-units thermal system is taken from [93]. The related cost coefficient of wind power, direct cost coefficient (F_{W_i}), penalty cost coefficient (k_{P,W_i}) and reserve cost coefficient $(k_{r,Wi})$ are assumed to be 2(\$/MWhr), 1(\$/MWhr) and 4(\$/MWhr) respectively. Similarly, the following cost coefficient of the solar power, direct cost coefficient (F_{PVk}), penalty cost coefficient ($k_{P,PVK}$) and reserve cost coefficient ($k_{r,PVK}$) are 2.1(\$/MWhr), 1(\$/MWhr) and 4(\$/MWhr) respectively. It is noted that the cost coefficients of wind and solar power are based on the system average thermal fuel cost that to make comparable cost of wind power as well as solar power. The optimal fuel cost is found to be \$42687.07387 for 24 hours using the proposed IFWA-CSO algorithm. The result using proposed IFWA-CSO is tabulated in Table 6.12. The total cost of considered system is break-up into three parts, thermal fuel cost, wind cost and solar cost, tabulated in Table 6.13. The contribution of thermal, wind and solar power generator are found to be 92.81%, 6.25% and 0.09% respectively. It observed that the thermal fuel cost reduced by 8.03% with integration of wind and solar unit and 0.8% reduction in the total operating cost of the system. This results in the saving of \$361.2499. However, the reduction in the total cost of operation is negligible. But, on the contrary thermal fuel cost reduction is significant, which show promising growth of non-conventional resources. It is noted that the relative analysis is carried out with the conventional dynamic system.

Case study R₂: This case study involves the 15-units thermal dynamic system with the incorporation of wind and solar unit capacity of 400 MW and 100 MW each. The relevant data of dynamic conventional 15-unit thermal system is taken from [99]. The total penetration of renewable sources is assumed to be around 20% of the average load demand. The total demand is predicted for 24-hours. The cost coefficients of wind and solar unit are defined in similar way as in case R₁. The cost coefficients of wind power, direct cost coefficient (F_{Wj}), penalty cost coefficient ($k_{P,Wj}$) and reserve cost coefficient ($k_{r,Wj}$) are 8(\$/MWhr), 1.5(\$/MWhr) and 10(\$/MWhr) respectively. The solar cost coefficients, direct cost coefficient (F_{PVk}), penalty cost coefficient

 $(k_{P,PVK})$ and reserve cost coefficient $(k_{r,PVK})$ are as follows 9(\$/MWhr), 1.5(\$/MWhr) and 11(\$/MWhr) respectively. The optimal operating cost by employing proposed IFWA-CSO for 24 h time duration is \$758723.68840. The total power loss is 587.7652MW. The result of the proposed method is tabulated in Table 6.12 and the break-up cost listed in Table 6.13. The break-up of each type of resources (thermal, wind and solar) are 95.06%, 4.61% and 0.32% respectively. It observed that thermal fuel cost reduced by 5.22% with integration of wind and solar unit and 0.29% reduction in the total operating cost of the system. This results in the saving of \$2213.8184.

Table 6.12 Quality solutions using proposed IFWA-CSO method for case study $R_1 \& R_2$

Case	Best fuel cost (\$)	Average fuel cost (\$)	Worst fuel Cost (\$)	STD	COV
R ₁	42687.07387	43649.08423	44995.79792	669.35548	1.53349
R ₂	756918.44460	756971.10834	757018.91608	30.73427	0.00406

Table 6.13 The break-up of total cost in term of fuel cost, wind cost and solar cost for case study $R_1 \& R_2$

Case	Fuel cost (\$)	Wind cost (\$)	Solar cost (\$)	Total cost (\$)
R ₁	39619.03160	2666.14151	401.90044	42687.07355
R ₂	719521.46450	34921.78398	2475.19855	756918.44703

6.5 Analysis

In this section, the impact on the scheduling of renewable sources output power due to associated parameter with the wind and solar power is analyzed. As, the wind and solar power cost coefficients are based on assumption, thus selecting their suitable values are a cumbersome task. Therefore, an analysis has been carried-out on the various parameters such as Weibull c parameter, reserve cost coefficients and penalty cost coefficient.

A. Optimal wind power output as function of the Weibull c parameter

The wind power direct cost function is derived from proportional term. It is based on the scheduling amount of power. So, as direct cost coefficient increases, wind schedule start decreases. Therefore, a suitable value should be chosen for optimal operation. Similarly, the reserve and penalty cost coefficient will also affect the schedule. Here, for the analysis the optimal value of the parameters for case study R₁ are defined as, $F_{Wj}=2$, $k_{P,Wj}=1$, $k_{r,Wj}=4$. As, wind power is function of wind speed, therefore, it is sensible to measure the effect of wind speed profile on the wind power schedule. Therefore, the generated Weibull parameter c for each time interval is employed. Since, Weibull parameter c is unit of wind speed. It indicates that the higher the values of c higher proportion of wind speed means high wind power generation. A wind power optimum schedule for scenario based stochastic DELD consist of 5-unit thermal generators and each unit of wind and solar is plotted in Fig. 6.11. It can be seen from figure the as parameter c value is increased, the amount of wind schedule is also increased. However, in some portion the power gets reduced, it may be due to the cost parameter, by drawing economic benefits from the other sources.



Figure 6.11 Optimal wind power output as function of the Weibull c parameter for the case study R1

Similarly, same generated c parameter values are employed for stochastic DELD system of 15-units and one unit of wind and solar with the wind cost parameter $F_{Wj}=8$, $k_{P,Wj}=1.5$, $k_{r,Wj}=10$.



Figure 6.12 Optimal wind power output as function of the Weibull c parameter for case study R2

It can be seen from the Fig. 6.12 optimum wind schedule power is closely follows the wind speed profile at each interval. If parameter c is decreases, the schedule amount of wind power is also decreases or vice-versa. However, if wind direct cost increases, the amount of wind power will be decrease even if power is available in greater proportion. Thus, for more economic operation there should be proper correlation between direct cost and wind speed profile.

B. Optimal wind and solar power output as function of the reserve cost coefficient

The reserve cost is required for purchasing the power from the conventional sources, to bear the mismatch power, when the available power is less than scheduled power. It is related by the constant called reserve cost coefficient. To analyze the reserve cost coefficient effect independently the penalty cost coefficient is assumed to be zero. The analysis is carried out on the stochastic DELD system of 15-units and one unit of wind and solar with the wind cost parameter $F_{Wi}=8$ and reserve cost coefficient is varied from 2 to 14. It is noted that the optimum schedule is obtained for whole system. However, for the convenience conventional units' schedules are omitted from the figure. The optimal schedule of wind power obtained for the different values of $k_{r,W}$ are plotted in the Fig. 6.13. It can be seen from the figure as the value of reserve cost coefficient is increased the schedule wind power is also get reduced. Therefore, to reduce the mismatch of power between available and forecasted power, operator will approach conservatively in the scheduling of wind power, as high amount of cost will be paid for overestimating the wind power. Furthermore, this analysis also indicate that in the high value of speed region the wind output power is scheduled in greater amount than other lower speed region.





Similarly, solar optimal schedule is plotted in the Fig. 6.14 with the direct cost coefficient (F_{PVk})=9 and the penalty cost coefficient is assumed to be zero. Since, the solar availability is limited. Thus, the scheduling output power from it causes

discontinuity. Thus, the solar capacity is assumed to be small to avoid large disturbance in the conventional thermal generator during their ramp up and down operation. It can be observed from the figure solar power schedule decreased with increase in the reserve cost coefficient. Thus, it concludes that for the higher value of the reserve cost coefficient the scheduled power should be low, because it deviate the economic operation at high cost value.





After analyzing the effect on the scheduling of the wind and solar power due to change in the Weibull parameter *c* and reserves cost coefficient, now the effect of penalty cost coefficient on the scheduling of the output power is investigated. The employed system is same as in previous section. For the independent analysis the reserve cost coefficient is assumed to be zero. The direct cost of wind and solar power is assumed to F_{Wj} =8 and F_{PVk} =9 respectively. The obtained optimum schedule for wind power is shown in the Fig. 6.15. It can be observed that the scheduled power is nearly operating at its maximum limits. Further, increasing the value of penalty cost coefficient, the wind power tends to approach high value to diminish the effect of underestimation. Similarly, the solar optimum schedule shown in Fig. 6.16 is also showing the same characteristics. It indicates that for the large value of the penalty cost coefficient, the scheduled power should be high to avoid the penalty. It is also noted that the obtained schedule of wind and solar power will also be affected by their respective direct cost coefficient. Therefore, for the optimal operation the value
of penalty cost coefficients should be kept low because, it is not sensible to schedule the output power at their maximum limits due to their low capacity factor.



Figure 6.15 Optimal wind power output as function of the penalty cost coefficient for case study R_2



Figure 6.16 Optimal solar power output as function of the penalty cost coefficient for case study R2

6.6 Discussion

The proposed method is validated on the benchmark functions and found to be adequate for all type of optimization problem. Further, it is applied on the three different power system optimization problems. The significance of the each suggested modifications made over FWA are subsequently investigated in the solution technique analysis. Similarly, the renewable sources impact is analyzed with the conventional power system.

6.1.1 Methodology Aspects

In this a Stochastic DELD problem is formulated with the incorporation of wind and solar based power source. The uncertainty related to the renewable sources is considered by using the suitable probability distributions. It is modeled by generating the hourly distribution to consider the time domain as in DELD problem. For the modeling of the wind speed and solar insolation Weibull and Beta probability distribution function are employed respectively and then these transformed into power variable using transformation variable theory. This measures the potential of renewable sources hourly in DELD problem. It is noted that the cost coefficient plays a major role in the scheduling of the wind and solar power. For this an analysis is carried out on the various parameters independently, it observed that the Weibull parameter c scalability reflects the amount of the scheduled power. Further, the scheduling of wind and solar power is strongly dependent upon the reserve and penalty cost coefficient. If reserve cost coefficient is increased the schedule power is decreased, while on the other hand schedule power is decreased with the increase in the penalty cost coefficient.

6.1.2 Solution Technique Aspects

In this section, the suggested modifications made in FWA are analyzed in largescale optimization problem. For this case study S_2 is considered for the study. A set of convergence characteristics for the best total cost obtained during a sample trial for S2 is shown in Fig. 6.3. It can be observed from the figure that FWA is slow on convergence due to its severe limitations on the shifted function [48]. This limitation overcome in EFWA, but its convergence is found to be improved marginally for this large-scale optimization problem, as it can be seen from figure. IFWA-I converges better with incorporation of LMO to restrict over diversity associated with EFWA. This further improved using fitness-based ADSO in IFWA-II. ADSO provides directed search by controlled dimension selection as it creates fitness governed sparks. Although IFWA-II enables better flight of individuals in global region, but the exploitation potential weakens during anaphase of the algorithm. The NMO incorporated in IFWA-III maintains better balance between exploration and exploitation of the search space so enhances convergence especially during anaphase of the algorithm. The chaotic sequence operator employed in IFWA-CSO enables the algorithm to search new solution points. So it does not trapped in the clusters of suboptimal solutions present in the problem search space of large-scale optimization problems.

The enlarge view of Fig. 6.3, is presented in Fig. 6.4 and 6.5. It can be observed from the figures that the FWA and its variants are continuously avoiding the local trapping, but far away from global region. The proposed IFWA-CSO method better

flights the individual into the global region and then continuously searching the optimal solution till maximum iteration exhausted. This is on account of enhanced exploration and exploitation potentials developed in are enhanced in IFWA-CSO and thus, it finds better quality solutions.

The obtained best operating generating schedules correspond to the best operating cost using the proposed IFWA-CSO method for all case studies are included in the Appendix-A.

The conclusions and future scope are discussed in the next chapter.

CHAPTER 7 CONCLUSIONS AND FUTURE SCOPE

The trending development of the renewable technologies and their break-even point occurrence in power generation has emerged the hybrid generation and getting much importance in the modern power system. Therefore, a stochastic analysis is carried out with the incorporation of wind and solar power. For this, a probabilistic model is presented to account the uncertainty related to the wind speed and solar irradiation and transform them into power variable using proper probability distribution functions. In addition, the renewables sources operating cost is also incorporated to analyze the economic factor. The renewable sources generation is related by direct cost function and their variability and unpredictability is measured by considering the underestimation and overestimation terms. A newly developed swarm intelligence based algorithm FWA is introduced inspired by the explosion of fireworks. However, FWA performance on the shifted functions severely affected. Therefore, to improve its feasibility in all type of optimization problem a new algorithm, IFWA-CSO is proposed. The major conclusions drawn from the dissertation are:

- A stochastic DELD problem is formulated with the incorporation of wind and solar based powered generation. The cost function of wind and solar based power generation are also included in the objective(s) to consider the economic factor.
- The potential of the wind and solar power generator is modeled by Weibull and Beta *pdf*. The generated *pdf* parameters measure expected power generation and a probabilistic cost model is formulated using these distribution functions.
- An IFWA-CSO algorithm is proposed based on explosion of fireworks. It is improved version of standard FWA, with its efficacy in all type of optimization problems. IFWA-CSO is proposed by suggesting Limiting Mapping Operator (LMO), Adaptive Dimension Selection Operator (ADSO), Non-uniform Mutation Operator (NMO) and Chaotic Sequence Operator (CSO).
- The proposed IFWA-CSO is verified on the well-known benchmark functions. The result shows that proposed method performs remarkably well on the shifted functions as well as adequate performance on the non-shifted function. This proving it a suitable optimizer for the power system optimization problem.

- The effectiveness of proposed variants of FWA and proposed IFWA-CSO are measured on various ED and DELD problem with the variety of generator and network constraints. Moreover, the scalability of dimension of the problem is also considered. The result shows the efficacy of the proposed method handling the ED and heavily constraint DELD problem. The comprehensive study reveals that the proposed IFWA-CSO perform remarkably better on the large-scale optimization problem, which shows its superiority over FWA, EFWA and other metaheuristics.
- Two scenarios based stochastic DELD system with the integration of renewable sources (one wind and one solar) are optimized using proposed IFWA-CSO. Further, their analyses are carried out based on the different wind and solar parameters. It reveals that the variations in the various wind and solar parameter have greater impact on the scheduling of output power. It is observed that the higher value of Weibull parameter *c* have greater amount of the scheduled power. Similarly, the reserve cost coefficient increment reduced the scheduled power on the contrary scheduled power increase with the increase in the penalty cost coefficient.

Future Scope

This work can be further extended by applying the proposed algorithm on different optimization problems. The proposed problem can be extended into the dynamic pricing system in the competitive real energy market. The stochastic approach can be used for real time economic dispatch by applying it into smaller time frame (minute). Furthermore, this problem can be incorporated with the stochastic load model, environmental issues and spinning reserve etc.

PUBLICATIONS

Following papers have been communicated out of this thesis work.

- Vipin C. Pandey, Vinay K. Jadoun, Nikhil Gupta, Khaleequr. R. Niazi and Anil Swarnkar, "Improved Fireworks Algorithm with Chaotic Sequence Operator for Non-convex Economic Load Dispatch", Engineering Applications of Artificial Intelligence, Elsevier.
- Vipin C. Pandey, Vinay K. Jadoun, Nikhil Gupta, Khaleequr. R. Niazi and Anil Swarnkar, "Economic Load Dispatch with Renewable Energy Sources using Improved Fireworks Algorithm", IET Renewable Power Generation.

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APPENDIX-A

Table A.1 Optimal Generating Schedule for Case Study S₁ using proposed IFWA-CSO.

Unit	Power (MW)						
1	110.905	11	168.798	21	523.313	31	190.000
2	110.831	12	94.000	22	523.328	32	190.000
3	97.393	13	214.758	23	523.309	33	190.000
4	179.765	14	394.276	24	523.315	34	164.926
5	87.903	15	394.284	25	523.360	35	200.000
6	140.000	16	304.525	26	523.312	36	200.000
7	259.606	17	489.361	27	10.000	37	110.000
8	284.600	18	489.309	28	10.000	38	110.000
9	284.599	19	511.265	29	10.000	39	110.000
10	130.000	20	511.319	30	96.327	40	511.315

Table A.2 Optimal Generating Schedule for Case Study S₂ using proposed IFWA-CSO.

Unit	Power (MW)						
1	119.000	41	3.0000	81	542.000	121	175.000
2	164.000	42	3.0000	82	56.0000	122	2.000
3	190.000	43	250.000	83	115.000	123	4.000
4	190.000	44	250.000	84	115.000	124	15.00
5	190.000	45	250.000	85	115.000	125	9.000
6	190.000	46	250.000	86	207.000	126	12.000
7	490.000	47	250.000	87	207.000	127	10.000
8	490.000	48	250.000	88	175.000	128	112.000
9	496.000	49	250.000	89	175.000	129	4.000
10	496.000	50	250.000	90	180.424	130	5.000
11	496.000	51	165.000	91	175.000	131	5.000
12	496.000	52	165.000	92	575.400	132	50.000
13	506.000	53	165.000	93	547.500	133	5.000
14	509.000	54	165.000	94	836.800	134	42.000
15	506.000	55	180.000	95	837.500	135	42.000
16	505.000	56	180.000	96	682.000	136	41.000
17	506.000	57	103.000	97	720.000	137	17.000
18	506.000	58	198.000	98	718.000	138	7.0000
19	505.000	59	312.000	99	720.000	139	7.0000
20	505.000	60	308.589	100	964.000	140	26.000
21	505.000	61	163.000	101	958.000	-	-
22	505.000	62	95.0000	102	947.900	-	-
23	505.000	63	511.000	103	934.000	-	-
24	505.000	64	511.000	104	935.000	-	-
25	537.000	65	490.000	105	876.500	-	-
26	537.000	66	256.826	106	880.900	-	-
27	549.000	67	490.000	107	873.700	-	-
28	549.000	68	490.000	108	877.400	-	-
29	501.000	69	130.000	109	871.700	-	-
30	499.000	70	294.562	110	864.800	-	-
31	506.000	71	141.585	111	882.000		
32	506.000	72	365.908	112	94.000		

33	506.000	73	195.000	113	94.000	
34	506.000	74	217.549	114	94.000	
35	500.000	75	217.549	115	244.000	
36	500.000	76	258.663	116	244.000	
37	241.000	77	403.245	117	244.000	
38	241.000	78	330.000	118	95.000	
39	774.000	79	531.000	119	95.000	
40	769.000	80	531.000	120	116.000	

Table A.3 Optimal generating scheduling for case study D_1 15- for 24 hour using

Proposed IFWA-CSO.

	P1	P2	P3	P4	P5	PL	
Time (hr)	(MW)	(MW)	(MW)	(MW)	(MW)	(MW)	Fuel cost (\$/hr)
1	20.486	98.615	30.006	124.912	139.798	3.816	1249.858
2	10.000	96.636	67.931	124.847	139.703	4.117	1432.093
3	10.000	97.310	107.873	124.895	139.698	4.776	1392.410
4	10.095	98.558	112.675	174.889	139.797	6.014	1659.547
5	10.000	92.574	112.655	209.793	139.734	6.756	1587.896
6	40.000	105.455	112.686	209.819	147.952	7.912	1865.774
7	15.342	98.633	112.703	209.818	197.952	8.447	1916.596
8	12.395	98.548	112.955	209.838	229.522	9.258	1797.888
9	42.395	105.538	112.893	209.854	229.520	10.200	2013.897
10	64.089	98.489	112.650	209.813	229.520	10.559	1996.886
11	75.000	103.868	112.819	209.832	229.525	11.044	2038.030
12	75.000	124.669	112.707	209.810	229.535	11.720	2180.246
13	64.024	98.500	112.745	209.772	229.519	10.559	1997.518
14	49.622	98.537	112.671	209.818	229.520	10.168	1977.704
15	35.876	98.538	112.678	186.518	229.514	9.125	2010.612
16	10.000	98.536	112.656	136.518	229.523	7.233	1683.061
17	10.000	87.619	112.645	124.900	229.518	6.683	1615.341
18	10.000	98.530	112.671	165.231	229.519	7.951	1853.552
19	11.852	98.639	113.407	209.829	229.531	9.258	1798.977
20	23.192	98.542	153.407	209.816	229.520	10.478	2120.992
21	38.608	98.548	113.407	209.816	229.521	9.900	1947.838
22	10.000	98.542	112.667	209.806	181.899	7.914	1864.022
23	10.000	98.539	112.672	171.969	139.760	5.940	1655.926
24	10.000	80.249	112.635	124.846	139.758	4.488	1421.659

Table A.4 Optimal generating scheduling for case study D_2 for 24 hour using

proposed IFWA-CSO.

Time	P1	P2	P3	P4	P5	P6	P7	P8
(Hr)	(MW)							
1	304.740	335.080	130.000	130.000	335.004	287.999	366.000	162.000
2	384.740	295.625	130.000	130.000	215.004	364.856	445.999	62.000
3	380.794	296.116	130.000	130.000	150.000	429.596	465.000	60.000

4	376.008	285.078	130.000	130.000	150.005	455.473	465.000	60.000
5	380.318	335.144	130.000	130.000	150.000	459.984	465.000	60.000
6	394.452	337.327	130.000	130.000	150.000	460.000	465.000	60.000
7	407.695	338.273	130.000	130.000	150.000	460.000	465.000	60.000
8	448.668	395.775	130.000	130.000	157.297	459.967	465.000	60.000
9	455.000	455.000	130.000	130.000	237.285	460.000	465.000	60.000
10	455.000	455.000	130.000	130.000	304.372	460.000	465.000	60.000
11	455.000	455.000	130.000	130.000	345.483	460.000	465.000	60.000
12	455.000	455.000	130.000	130.000	347.996	460.000	465.000	60.000
13	455.000	455.000	130.000	130.000	344.040	460.000	465.000	60.000
14	454.993	454.995	130.000	130.000	381.960	460.000	465.000	60.000
15	455.000	455.000	130.000	130.000	461.953	460.000	465.000	60.000
16	455.000	455.000	130.000	130.000	469.997	459.999	465.000	60.000
17	455.000	455.000	130.000	130.000	438.378	460.000	465.000	60.000
18	455.000	454.998	130.000	130.000	361.484	460.000	465.000	60.000
19	455.000	455.000	130.000	130.000	246.969	460.000	465.000	60.000
20	455.000	455.000	130.000	130.000	200.020	460.000	465.000	60.000
21	449.443	392.274	130.000	130.000	150.000	460.000	465.000	60.000
22	393.236	335.339	130.000	130.000	150.000	460.000	465.000	60.000
23	385.120	299.456	130.000	130.000	150.000	455.172	465.000	60.000
24	381.441	296.673	130.000	130.000	150.000	455.532	465.000	60.000

Time	P9	P10	P11	P12	P13	P14	P15	PL
(hr)	(MW)	(MW)	(MW)	(MW)	(MW)	(MW)	(MW)	(MW)
1	25.000	34.000	46.275	52.283	25.000	15.000	15.000	27.382
2	25.000	25.000	49.745	51.303	25.000	15.000	15.000	19.274
3	25.000	25.000	48.004	49.908	25.000	15.000	15.000	18.418
4	25.000	25.000	47.398	50.503	25.000	15.000	15.000	18.465
5	25.000	25.000	50.970	51.077	25.000	15.000	15.000	19.494
6	25.000	25.000	50.924	53.084	25.000	15.000	15.000	19.788
7	25.000	25.000	52.121	52.946	25.000	15.000	15.000	20.034
8	25.000	25.001	58.601	54.873	25.000	15.000	15.000	22.184
9	25.000	45.288	79.972	80.000	25.000	15.000	15.000	26.545
10	25.000	58.306	80.000	80.000	25.000	15.000	15.000	29.679
11	25.000	74.743	80.000	79.993	25.000	15.000	15.000	32.221
12	25.000	74.365	80.000	79.990	25.000	15.000	15.000	32.351
13	25.000	73.055	80.000	80.000	25.000	15.000	15.000	32.096
14	25.000	87.795	80.000	80.000	25.000	15.000	15.000	34.743
15	25.010	138.283	80.000	79.995	25.000	15.000	15.000	42.242
16	25.000	127.346	80.000	80.000	25.000	15.000	15.000	42.343
17	25.000	107.820	79.999	80.000	25.000	15.000	15.000	39.197
18	25.000	79.791	80.000	80.000	25.000	15.000	15.000	33.273

19	25.000	35.797	80.000	80.000	25.000	15.000	15.000	26.767
20	25.000	25.001	76.035	72.880	25.000	15.000	15.000	24.937
21	25.000	25.000	57.324	54.917	25.000	15.000	15.000	21.958
22	25.000	25.000	51.216	51.930	25.000	15.000	15.000	19.722
23	25.000	25.000	48.670	51.443	25.000	15.000	15.000	18.861
24	25.000	25.000	48.119	50.991	25.000	15.000	15.000	18.756

Table A.5 Optimal generating scheduling for case study R_1 for 24 hour using

proposed IFWA-CSO. P4 P1 W P2 P3 P5

Time	P1	P2	P3	P4	P5	W	PV	PL
(Hr)	(MW)	(MW)	(MW)	(MW)	(MW)	(MW)	(MW)	(MW)
1	10.000	98.367	112.696	40.000	139.760	12.621	0.000	3.444
2	10.000	98.643	112.697	40.000	139.761	37.349	0.000	3.450
3	10.000	98.540	112.664	90.000	139.758	28.265	0.000	4.227
4	10.000	98.521	112.709	125.179	139.765	48.723	0.000	4.897
5	10.000	98.555	112.646	175.179	139.772	27.866	0.000	6.018
6	10.000	98.421	112.697	209.769	139.766	44.255	0.000	6.908
7	10.000	98.530	112.658	209.804	179.495	23.366	0.000	7.852
8	10.000	98.537	112.688	209.826	229.495	2.650	0.000	9.196
9	10.000	98.544	112.703	209.842	229.519	32.134	6.455	9.198
10	10.000	98.669	112.626	209.835	229.515	40.904	11.650	9.199
11	10.000	98.540	112.719	209.819	229.495	53.821	14.803	9.197
12	10.000	98.543	112.659	209.821	229.532	70.122	18.518	9.196
13	10.000	98.543	112.683	209.809	229.598	41.166	11.398	9.199
14	10.000	98.587	112.676	209.817	229.613	29.895	8.612	9.200
15	10.000	98.552	80.404	209.831	229.506	26.827	7.409	8.529
16	10.000	98.190	40.404	209.689	229.518	0.000	0.000	7.801
17	10.000	86.108	30.000	209.802	229.410	0.000	0.000	7.319
18	10.000	98.549	30.000	209.832	229.500	35.618	2.150	7.648
19	11.585	98.542	32.248	209.823	229.517	80.000	0.000	7.716
20	22.431	98.613	72.248	209.816	229.541	80.000	0.000	8.649
21	10.000	98.529	112.248	209.724	229.481	29.200	0.000	9.183
22	10.000	98.524	112.566	209.810	179.481	2.469	0.000	7.850
23	10.000	98.519	72.566	209.769	139.759	2.565	0.000	6.179
24	10.000	75.954	32.566	209.796	139.749	0.000	0.000	5.065

Table A.6 Optimal generating scheduling for case study R_2 for 24 hour using proposed IFWA-CSO.

Time (IIa)	P1	P2	P3	P4	P5	P6	P7	P8
Time (Hr)	(MW)	(MW)						
1	341.700	239.446	129.721	129.992	150.021	410.771	462.870	60.000
2	302.613	271.392	129.999	129.993	150.000	408.444	460.758	60.000
3	341.192	276.937	129.962	129.988	150.002	362.408	464.503	60.000
4	362.303	227.202	129.996	130.000	150.000	429.521	458.742	60.000

5	36	5.590	2	77.997	130.000	129.99	99	150.0	000	42	7.817	46	52.695	60.002
6	37	0.993	3	01.818	130.000	130.00	00	150.0	000	42	2.634	46	53.505	60.000
7	36	7.906	3	50.264	129.988	130.00	00	150.0	000	39	6.604	40	50.799	60.000
8	43	0.023	3	73.198	130.000	129.87	2	150.0	000	42	6.183	46	53.755	60.000
9	43	1.268	4	19.981	130.000	130.00	00	226.4	452	45	9.956	40	64.475	60.000
10	45	4.964	4	54.999	130.000	130.00	00	228.3	347	45	7.894	40	55.000	60.000
11	45	3.388	4	54.853	130.000	129.99	92	230.0	010	45	9.696	40	55.000	60.000
12	45	3.831	4	19.216	130.000	130.00	00	228.4	464	45	9.803	40	64.963	60.000
13	45	5.000	4	18.381	129.999	130.00	00	223.7	752	45	8.839	40	54.986	60.000
14	44	8.990	4	54.999	129.979	130.00	00	203.7	761	46	0.000	40	55.000	60.000
15	45	5.000	4	55.000	129.947	130.00	00	275.8	804	45	9.351	40	53.740	60.000
16	45	4.978	4	55.000	129.802	129.94	4	286.3	390	45	9.644	40	54.974	60.000
17	45	5.000	4	54.689	130.000	129.99	93	292.	130	45	9.982	40	55.000	60.000
18	45	5.000	4	54.987	129.946	129.93	34	256.8	833	42	8.436	40	54.817	60.000
19	45	1.604	4	05.936	130.000	130.00	00	172.4	411	45	9.997	40	54.992	60.000
20	44	3.026	4	19.207	129.947	130.00	00	150.0	000	42	6.751	40	65.000	60.000
21	43	1.494	2	99.207	129.970	129.87	7	150.0	000	42	6.678	40	52.498	60.000
22	39	5.518	2	66.274	129.999	129.84	2	150.0	002	41	3.537	40	53.873	60.000
23	30	7.836	3	38.105	129.961	129.70	00	150.0	000	40	3.370	43	59.541	60.000
24	38	0.754	2	29.165	129.990	129.96	52	150.0	000	40	1.098	46	55.000	60.003
Time	P9	P10		P11	P12	P13	Р	14	P15	;	W		PV	PL
(Hr)	(MW)	(MW)	(MW)	(MW)	(MW)	(]	MW)	(M	W)	(MW)	(MW)	(MW)
1	25.006	25.00	00	45.979	51.201	25.000	1:	5.000	15.0	000	125.9	935	0.000	16.640
2	25.000	25.00	00	45.183	45.736	25.000	1:	5.000	15.0	001	122.4	410	0.000	16.529
3	25.000	25.00	00	50.020	53.299	25.000	1:	5.000	15.0	000	119.4	137	0.000	16.747
4	25.000	25.00	00	40.508	45.537	25.000	1:	5.000	15.0	000	114.0)87	0.000	16.896
5	25.000	25.00	00	49.325	49.576	25.000	1:	5.050	15.0	000	107.7	740	0.000	17.791
6	25.000	25.00	04	46.301	52.197	25.000	1:	5.000	15.0	000	101.8	324	0.000	18.275
7	25.000	25.00	00	45.677	49.204	25.000	1:	5.000	15.0	000	104.3	352	0.000	18.793
8	25.046	25.00	00	56.347	50.589	25.000	1:	5.000	15.0	000	88.7	05	0.000	20.717
9	25.000	25.0	18	79.550	74.668	25.000	1:	5.000	15.0	000	86.6	68	7.510	24.547
10	25.000	25.52	27	80.000	76.958	25.010	1:	5.000	15.0	000	93.1	16	17.081	25.896
11	25.000	31.22	21	79.965	78.487	25.000	1:	5.000	15.0	000	131.0)93	25.313	26.018
12	25.000	25.00	00	73.028	74.898	25.000	1:	5.000	15.0	000	179.7	749	31.095	25.046
13	25.000	25.00	00	56.845	77.758	25.000	1:	5.000	15.0	000	192.1	196	32.138	24.893
14	25.000	25.00	00	80.000	68.869	25.000	1:	5.000	15.0	000	213.6	520	34.694	24.912
15	25.000	66.9′	74	79.904	78.785	25.000	1:	5.000	15.0	000	217.6	511	29.408	28.523
16	25.000	50.63	33	80.000	79.505	25.000	1:	5.000	15.0	000	218.0)98	29.697	28.664
17	25.000	25.00	00	75.261	77.468	25.000	1:	5.000	15.0	000	208.5	517	17.523	28.564
18	25.015	25.00	00	79.055	78.298	25.000	1	5.002	15.0	000	183.3	339	3.886	26.549
19	25.000	25.00	00	69.212	67.169	25.000	1	5.000	15.0	000	157.5	576	0.000	22.898
20	25.000	25.00	00	70.066	66.974	25.000	1:	5.000	15.0	000	140.1	09	0.000	22.079
21	25.000	25.00	00	77.351	54.357	25.000	1	5.000	15.0	000	124.7	789	0.000	19.221
22	25.041	25.00	00	46 649	44 865	25,000	1	5 000	15 (000	124 3	307	0.000	17 906

23	25.097	25.000	39.335	29.692	25.002	15.000	15.000	125.997	0.000	17.634
24	25.000	25.000	42.764	46.063	25.000	15.000	15.000	131.166	0.000	16.966

APPENDIX-B

a_i (\$/hr)	b_i (\$/MWhr)	$\frac{c_i}{(\$/(MW)^2hr)}$	e_i (\$/hr)	f_i (1/MW)	P_{min} (MW)	P_{max} (MW)
94.705	6.73	0.0069	100	0.084	36	114
94.705	6.73	0.0069	100	0.084	36	114
309.54	7.07	0.02028	100	0.084	60	120
369.03	8.18	0.00942	150	0.063	80	190
148.89	5.35	0.0114	120	0.077	47	97
222.33	8.05	0.01142	100	0.084	68	140
287.71	8.03	0.00357	200	0.042	110	300
391.98	6.99	0.00492	200	0.042	135	300
455.76	6.6	0.00573	200	0.042	135	300
722.82	12.9	0.00605	200	0.042	130	300
635.2	12.9	0.00515	200	0.042	94	375
654.69	12.8	0.00569	200	0.042	94	375
913.4	12.5	0.00421	300	0.035	125	500
1760.4	8.84	0.00752	300	0.035	125	500
1728.3	9.15	0.00708	300	0.035	125	500
1728.3	9.15	0.00708	300	0.035	125	500
647.85	7.97	0.00313	300	0.035	220	500
649.69	7.95	0.00313	300	0.035	220	500
647.83	7.97	0.00313	300	0.035	242	550
647.81	7.97	0.00313	300	0.035	242	550
785.96	6.63	0.00298	300	0.035	254	550
785.96	6.63	0.00298	300	0.035	254	550
794.53	6.66	0.00284	300	0.035	254	550
794.53	6.66	0.00284	300	0.035	254	550
801.32	7.1	0.00277	300	0.035	254	550
801.32	7.1	0.00277	300	0.035	254	550
1055.1	3.33	0.52124	120	0.077	10	150
1055.1	3.33	0.52124	120	0.077	10	150
1055.1	3.33	0.52124	120	0.077	10	150
148.89	5.35	0.0114	120	0.077	47	97
222.92	6.43	0.0016	150	0.063	60	190
222.92	6.43	0.0016	150	0.063	60	190
222.92	6.43	0.0016	150	0.063	60	190
107.87	8.95	0.0001	200	0.042	90	200
116.58	8.62	0.0001	200	0.042	90	200

Table B.1 Input data of 40-Units thermal generators.

116.58	8.62	0.0001	200	0.042	90	200
307.45	5.88	0.0161	80	0.098	25	110
307.45	5.88	0.0161	80	0.098	25	110
307.45	5.88	0.0161	80	0.098	25	110
647.83	7.97	0.00313	300	0.035	242	550

Table B.2 Input data of 140-Units thermal generators.

<i>a_i</i> (\$/hr)	<i>bi</i> (\$/MWhr)	$\frac{c_i}{(\$/(MW)^2hr)}$	<i>e</i> _{<i>i</i>} (\$/hr)	f_i (1/MW)	P_{min} (MW)	P_{max} (MW)	UR _i (MW)	DR_i (MW)
1220.65	61.242	0.032888	0	0	71	119	30	120
1315.12	41.095	0.008280	0	0	120	189	30	120
874.288	46.31	0.003849	0	0	125	190	60	60
874.288	46.31	0.003849	0	0	125	190	60	60
1976.47	54.242	0.042468	700	0.08	90	190	150	150
1338.09	61.215	0.014992	0	0	90	190	150	150
1818.3	11.791	0.007039	0	0	280	490	180	300
1133.98	15.055	0.003079	0	0	280	490	180	300
1320.64	13.226	0.005063	0	0	260	496	300	510
1320.64	13.226	0.005063	600	0.055	260	496	300	510
1320.64	13.226	0.005063	0	0	260	496	300	510
1106.54	14.498	0.003552	0	0	260	496	300	510
1176.5	14.651	0.003901	0	0	260	506	600	600
1176.5	14.651	0.003901	0	0	260	509	600	600
1176.5	14.651	0.003901	800	0.06	260	506	600	600
1176.5	14.651	0.003901	0	0	260	505	600	600
1017.41	15.669	0.002393	0	0	260	506	600	600
1017.41	15.669	0.002393	0	0	260	506	600	600
1229.13	14.656	0.003684	0	0	260	505	600	600
1229.13	14.656	0.003684	0	0	260	505	600	600
1229.13	14.656	0.003684	0	0	260	505	600	600
1229.13	14.656	0.003684	600	0.05	260	505	600	600
1267.89	14.378	0.004004	0	0	260	505	600	600
1229.13	14.656	0.003684	0	0	260	505	600	600
975.926	16.261	0.001619	0	0	280	537	300	300
1532.09	13.362	0.005093	0	0	280	537	300	300
641.989	17.203	0.000993	0	0	280	549	360	360
641.989	17.203	0.000993	0	0	280	549	360	360
911.533	15.274	0.002473	0	0	260	501	180	180
910.533	15.212	0.002547	0	0	260	501	180	180
1074.81	15.033	0.003542	0	0	260	506	600	600
1074.81	15.033	0.003542	0	0	260	506	600	600
1074.81	15.033	0.003542	600	0.043	260	506	600	600
1074.81	15.033	0.003542	0	0	260	506	600	600

1278.46	13.992	0.003132	0	0	260	500	660	660
861.742	15.679	0.001323	0	0	260	500	900	900
408.834	16.542	0.002950	0	0	120	241	180	180
408.834	16.542	0.002950	0	0	120	241	180	180
1288.82	16.518	0.000991	0	0	423	774	600	600
1436.25	15.815	0.001581	600	0.043	423	769	600	600
669.988	75.464	0.902360	0	0	3	19	210	210
134.544	129.544	0.110295	0	0	3	28	366	366
3427.91	56.613	0.024493	0	0	160	250	702	702
3751.77	54.451	0.029156	0	0	160	250	702	702
3918.78	54.736	0.024667	0	0	160	250	702	702
3379.58	58.034	0.016517	0	0	160	250	702	702
3345.3	55.981	0.026584	0	0	160	250	702	702
3138.75	61.52	0.007540	0	0	160	250	702	702
3453.05	58.635	0.016430	0	0	160	250	702	702
5119.3	44.647	0.045934	0	0	160	250	702	702
1898.42	71.584	0.000044	0	0	165	504	1350	1350
1898.42	71.584	0.000044	1100	0.043	165	504	1350	1350
1898.42	71.584	0.000044	0	0	165	504	1350	1350
1898.42	71.584	0.000044	0	0	165	504	1350	1350
2473.39	85.12	0.002528	0	0	180	471	1350	1350
2781.71	87.682	0.000131	0	0	180	561	720	720
5515.51	69.532	0.010372	0	0	103	341	720	720
3478.3	78.339	0.007627	0	0	198	617	2700	2700
6240.91	58.172	0.012464	0	0	100	312	1500	1500
9960.11	46.636	0.039441	0	0	153	471	1656	1656
3672	76.947	0.007278	0	0	163	500	2160	2160
1837.38	80.761	0.000044	0	0	95	302	900	900
3108.4	70.136	0.000044	0	0	160	511	1200	1200
3108.4	70.136	0.000044	0	0	160	511	1200	1200
7095.48	49.84	0.018827	0	0	196	490	1014	1014
3392.73	65.404	0.010852	0	0	196	490	1014	1014
7095.48	49.84	0.018827	0	0	196	490	1014	1014
7095.48	49.84	0.018827	0	0	196	490	1014	1014
4288.32	66.465	0.034560	0	0	130	432	1350	1350
13813	22.941	0.081540	1200	0.03	130	432	1350	1350
4435.49	64.314	0.023534	0	0	137	455	1350	1350
9750.75	45.017	0.035475	1000	0.05	137	455	1350	1350
1042.37	70.644	0.000915	0	0	195	541	780	780
1159.9	70.959	0.000044	0	0	175	536	1650	1650
1159.9	70.959	0.000044	0	0	175	540	1650	1650
1303.99	70.302	0.001307	0	0	175	538	1650	1650

1156.19	70.662	0.000392	0	0	175	540	1650	1650
2118.97	71.101	0.000087	0	0	330	574	1620	1620
779.519	37.854	0.000521	0	0	160	531	1482	1482
829.888	37.768	0.000498	0	0	160	531	1482	1482
2333.69	67.983	0.001046	0	0	200	542	1668	1668
2028.95	77.838	0.132050	0	0	56	132	120	120
4412.02	63.671	0.096968	0	0	115	245	180	180
2982.22	79.458	0.054868	1000	0.05	115	245	120	180
2982.22	79.458	0.054868	0	0	115	245	120	180
3174.94	93.966	0.014382	0	0	207	307	120	180
3218.36	94.723	0.013161	0	0	207	307	120	180
3723.82	66.919	0.016033	0	0	175	345	318	318
3551.41	68.185	0.013653	0	0	175	345	318	318
4322.62	60.821	0.028148	0	0	175	345	318	318
3493.74	68.551	0.013470	0	0	175	345	318	318
226.799	2.842	0.000064	0	0	360	580	18	18
382.932	2.946	0.000252	0	0	415	645	18	18
156.987	3.096	0.000022	0	0	795	984	36	36
154.484	3.04	0.000022	0	0	795	978	36	36
332.834	1.709	0.000203	0	0	578	682	138	204
326.599	1.668	0.000198	0	0	615	720	144	216
345.306	1.789	0.000215	0	0	612	718	144	216
350.372	1.815	0.000218	0	0	612	720	144	216
370.377	2.726	0.000193	0	0	758	964	48	48
367.067	2.732	0.000197	0	0	755	958	48	48
124.875	2.651	0.000324	0	0	750	1007	36	54
130.785	2.798	0.000344	0	0	750	1006	36	54
878.746	1.595	0.000690	0	0	713	1013	30	30
827.959	1.503	0.000650	0	0	718	1020	30	30
432.007	2.425	0.000233	0	0	791	954	30	30
445.606	2.499	0.000239	0	0	786	952	30	30
467.223	2.674	0.000261	0	0	795	1006	36	36
475.94	2.692	0.000259	0	0	795	1013	36	36
899.462	1.633	0.000707	0	0	795	1021	36	36
1000.37	1.816	0.000786	0	0	795	1015	36	36
1269.13	89.83	0.014355	0	0	94	203	120	120
1269.13	89.83	0.014355	0	0	94	203	120	120
1269.13	89.83	0.014355	0	0	94	203	120	120
4965.12	64.125	0.030266	0	0	244	379	480	480
4965.12	64.125	0.030266	0	0	244	379	480	480
4965.12	64.125	0.030266	0	0	244	379	480	480
2243.19	76.129	0.024027	0	0	95	190	240	240

2290.38	81.805	0.001580	600	0.07	95	189	240	240
1681.53	81.14	0.022095	0	0	116	194	120	120
6743.3	46.665	0.076810	1200	0.043	175	321	180	180
394.398	78.412	0.953443	0	0	2	19	90	90
1243.17	112.088	0.000044	0	0	4	59	90	90
1454.74	90.871	0.072468	0	0	15	83	300	300
1011.05	97.116	0.000448	0	0	9	53	162	162
909.269	83.244	0.599112	0	0	12	37	114	114
689.378	95.665	0.244706	0	0	10	34	120	120
1443.79	91.202	0.000042	0	0	112	373	1080	1080
535.553	104.501	0.085145	0	0	4	20	60	60
617.734	83.015	0.524718	0	0	5	38	66	66
90.966	127.795	0.176515	0	0	5	19	12	6
974.447	77.929	0.063414	0	0	50	98	300	300
263.81	92.779	2.740485	0	0	5	10	6	6
1335.59	80.95	0.112438	0	0	42	74	60	60
1033.87	89.073	0.041529	0	0	42	74	60	60
1391.33	161.288	0.000911	0	0	41	105	528	528
4477.11	161.829	0.005245	0	0	17	51	300	300
57.794	84.972	0.234787	0	0	7	19	18	30
57.794	84.972	0.234787	0	0	7	19	18	30
1258.44	16.087	1.111878	0	0	26	40	72	120

Table B.3 Prohibited operating zones of 140-units thermal generators

	Prohibite	Prohibited operating zones (MW)								
Unit	Zone 1	Zone 2	Zone 3							
8	[250, 280]	[305, 335]	[420, 450]							
32	[220, 250]	[320, 350]	[390, 420]							
74	[230, 255]	[365, 395]	[430, 455]							
136	[50,75]	[80, 95]								

Table B.4 Input data of 5-Units of thermal generators

a_i	b_i	C_i	e_i	f_i	P_{min}	P_{max}	UR_i	DR_i
(\$/hr)	(\$/MWhr)	$((MW)^{2}hr)$	(\$/hr)	(1/MW)	(MW)	(MW)	(MW)	(MW)
25	2	0.008	100	0.042	10	75	30	30
60	1.8	0.003	140	0.04	20	125	30	30
100	2.1	0.0012	160	0.038	30	175	40	40
120	2	0.001	180	0.037	40	250	50	50
40	1.8	0.0015	200	0.035	50	300	50	50

Table B.5 Load demand of 5-unit thermal generators

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Load (MW)	410	435	475	530	558	608	626	654	690	704	720	740
Hour	13	14	15	16	17	18	19	20	21	22	23	24

Load (MW)	704	690	654	580	558	608	654	704	680	605	527	463
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Table B.6 B-Coefficients matrix of 5-units thermal generators

	0.000 049	0.000 014	0.000 015	0.000 015	0.000 020
	0.000 014	0.000 045	0.000 016	0.000 020	0.000 018
B =	0.000 015	0.000 016	0.000 039	0.000 010	0.000 012
	0.000 015	0.000 020	0.000 010	0.000 040	0.000 014
	0.000 020	0.000 018	0.000 012	0.000 014	0.000 035

Table B.7 Input data of 15-Units thermal generators

a_i (\$/hr)	b_i (\$/MWhr)	$\frac{c_i}{(\$/(MW)^2hr)}$	P_{min} (MW)	P _{max} (MW)	UR _i (MW)	DR_i (MW)	P_0 (MW)
671	10.1	0.000299	150	455	80	120	394.44
574	10.2	0.000183	150	455	80	120	450.27
374	8.8	0.001126	20	130	130	130	50.111
374	8.8	0.001126	20	130	130	130	113.36
461	10.4	0.000205	150	470	80	120	426.35
630	10.1	0.000301	135	460	80	120	207.1
548	9.8	0.000364	135	465	80	120	286.51
227	11.2	0.000338	60	300	65	100	262.88
173	11.2	0.000807	25	162	60	100	94.579
175	10.7	0.001203	25	160	60	100	133.78
186	10.2	0.003586	20	80	80	80	66.78
230	9.9	0.005513	20	80	80	80	29.9
225	13.1	0.000371	25	85	80	80	46.25
309	12.1	0.001929	15	55	55	55	15.01
323	12.4	0.004447	15	55	55	55	51.49

Table B.8 Load demand of 15-units thermal generators

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Load												
(MW)	2236	2215	2226	2236	2298	2316	2331	2443	2651	2728	2783	2785
Hour	13	14	15	16	17	18	19	20	21	22	23	24
Load												
(MW)	2780	2830	2953	2950	2902	2803	2651	2584	2432	2312	2261	2254

Table B.9 Prohibited operating zones of 15-units thermal generators

Unit	Prohibited operating zones (MW)					
2	[185 225]	[305 335]	[420 450]			
5	[180 200]	[305 335]	[390 420]			
6	[230 255]	[365 395]	[430 455]			
12	[30 40]	[55 65]				

0.00140	0.00120	0.00070	0.00070 -0.		-0.00		-0.00010		-0.00010	-0.00010	
0.00120	0.00150	0.00130	0.	.00000	-0.00050		-0.00020		0.00000	0.00010	
0.00070	0.00130	0.00760	-0.	.00010	-0.00130		-0.00090		-0.00010	0.00000	
-0.00010	0.00000	-0.00010	0.	.00340	-0.00070		-0.00040		0.00110	0.00500	
-0.00030	-0.00050	-0.00130	-0.	.00070	0.00900		0.00140		-0.00030	-0.00120	
-0.00010	-0.00020	-0.00090	-0.	.00040	0.0	0140	0.00160		0.00000	-0.00060	
-0.00010	0.00000	-0.00010	0.	.00110	-0.0	0030	0.00000		0.00150	0.00170	
-0.00010	0.00010	0.00000	0.	.00500	-0.00120		-0.00060		0.00170	0.01680	
-0.00030	-0.00020	-0.00080	0.	.00290	-0.00100		-0.00050		0.00150	0.00820	
-0.00050	-0.00040	-0.00120	0.00320		-0.00130		-0.00080		0.00090	0.00790	
-0.00030	-0.00040	-0.00170	-0.00110		0.00070		0.00110		-0.00050	-0.00230	
-0.00020	0.00000	0.00000	0.00000		-0.00020		-0.00010		0.00070	-0.00360	
0.00040	0.00040	-0.00260	0.00010		-0.00020		-0.00020		0.00000	0.00010	
0.00030	0.00100	0.01110	0.	0.00010		-0.00240		70	-0.00020	0.00050	
-0.00010	-0.00020	-0.00280	-0.	-0.00260		-0.00030		30	-0.00080	-0.00780	
-0.00030	0.000	-0.0	00030	-0.	.00020		0.00040		0.00030	-0.00010	
-0.00020	-0.000	40 -0.0	00040	0.	.00000		0.00040		0.00100	-0.00020	
-0.00080	-0.001	20 -0.0	-0.00170		0.00000		-0.00260		0.01110	-0.00280	
0.00290	0.003	20 -0.0	-0.00110		0.00000		0.00010		0.00010	-0.00260	
-0.00100	-0.001	30 0.0	0.00070		-0.00020		-0.00020		-0.00240	-0.00030	
-0.00050	-0.000	80 0.0	0.00110		-0.00010		-0.00020		-0.00170	0.00030	
0.00150	0.000	90 -0.0	-0.00050		0.00070		0.00000		-0.00020	-0.00080	
0.00820	0.007	-0.0	-0.00230		-0.00360		0.00010		0.00050	-0.00780	
0.01290	0.011	60 -0.0	-0.00210		-0.00250		0.00070		-0.00120	-0.00720	
0.01160	0.020	-0.0	-0.00270		-0.00340		0.00090		-0.00110	-0.00880	
-0.00210	-0.002	70 0.0	0.01400		0.00010		0.00040		-0.00380	0.01680	
-0.00250	-0.003	40 0.0	0.00010		0.00540		-0.00010		-0.00040	0.00280	
0.00070	0.000	90 0.0	0.00040		-0.00010		0.01030		-0.01010	0.00280	
-0.00120	-0.001	10 -0.0	-0.00380		-0.00040		-0.01010		0.05780	-0.00940	
-0.00720	-0.008	80 0.0	0.01680		0.00280		0.00280		-0.00940	0.12830	

Table B.10 B-Coefficients matrix of 15-units thermal generators

 $B_{ij} {=} [\ {-0.0001} \ {-0.0002} \ {0.0028} \ {-0.0001} \ {0.0001} \ {-0.0003} \ {-0.0002} \ {-0.0002} \ {0.0006} \ {0.0039} \ {-0.0017} \ {-0.0000} \ {-0.0006} \ {-0.0064} \];$

B₀₀=[0.0055];